Economic Commission for Europe
Inland Transport Committee
World Forum for Harmonization of Vehicle Regulations
Working Party on Noise
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Item 7 of the provisional agenda
Regulation No. 117 (Tyre rolling noise and wet grip adhesion)

Proposal for Supplement 6 to the 02 series of amendments to Regulation No. 117

Submitted by the expert from the Russian Federation

The text reproduced below was prepared by the expert from the Russian Federation to elaborate on the concept of tyre deceleration (dω/dt) in the test technology. It is submitted to GRB for final consideration (ECE/TRANS/WP.29/GRB/57, para. 19) on the basis of an informal document (GRB-59-02) distributed at the fifty-ninth session of the Working Party on Noise (GRB). The modifications to the existing text of the UN Regulation are marked in bold for new or strikethrough for deleted characters.

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1 In accordance with the programme of work of the Inland Transport Committee for 2012–2016 (ECE/TRANS/224, para. 94 and ECE/TRANS/2012/12, programme activity 02.4), the World Forum will develop, harmonize and update Regulations in order to enhance the performance of vehicles. The present document is submitted in conformity with that mandate.
I. Proposal

Annex 6, paragraph 3.5., amend to read:

"3.5. Duration and speed.

When the deceleration method is selected, the following requirements apply:

(a) The deceleration $j$ shall be determined in **differential exact** $\frac{d\omega}{dt}$ or **discrete approximate** $\frac{\Delta\omega}{\Delta t}$ form, where $\omega$ is angular velocity, $t$ - time;

   If the differential form $\frac{d\omega}{dt}$ is used, then the recommendations of Appendix 5 to this Annex are to be applied.

(b) …"

Annex 6, insert a new Appendix 5, to read:

"Annex 6 – Appendix 5

Deceleration method: Measurements and data processing for deceleration value obtaining in differential form $\frac{d\omega}{dt}$.

1. Record dependency "distance-time" of rotating body decelerated from peripheral with a speed range such as 82 to 78 km/h or 62 to 58 km/h dependent on a PC or CV tyre in a discrete form (figure 1) for a rotating body:

\[ z = f(t_z) \]

where:

$z$ is a number of body revolutions during deceleration;
$t_z$ is end time of revolution number $z$ in seconds recorded with 6 digits after zero.

Figure 1
Note 1: The lower speed of the recording range may be reduced down to 60 (40) km/h.

2. Approximate recorded dependency by continuous, monotonic, differentiable function:

2.1. Choose the value nearest to the maximum of z dividable by 4 and divide it into 4 equal parts with bounds: 0, z₁(t₁), z₂(t₂), z₃(t₃), z₄(t₄).

2.2. Work out the system for 4 equations each of the form:

\[ z_n = A \ln \frac{\cos B (T_\Sigma - t_n)}{\cos B T_\Sigma} \]

where unknowns:

A is a dimensionless constant,
B is a constant in revolutions per second,
T_\Sigma is a constant in seconds,
m is the number of bounds shown in figure 1.

Insert in these 4 equations the coordinates of 4-th bound above.

2.3. Take constants A, B and T_\Sigma as the solution of the equation system of paragraph 2.2 above using iteration process and approximate measured data by formulae:

\[ z(t) = A \ln \frac{\cos B (T_\Sigma - t)}{\cos B T_\Sigma} \]

where:

z(t) is the current continuous angular distance in number of revolutions (not only integer values);
t is time in seconds.

Note 2: Other approximating functions z=f(t) may be used if their adequacy is proven.

3. Calculate the deceleration j in revolutions per second squared (s⁻²) by the formula:

\[ j = AB^2 \frac{\omega^2}{A} \]

where:

\( \omega \) is the angular speed in revolutions per second (s⁻¹).

For the case \( U_n = 80 \text{ km/h} \); \( \omega = 22.222/R \) (or \( R \)).
For the case \( U_n = 60 \text{ km/h} \); \( \omega = 16.666/R \) (or \( R \)).

4. Estimate the quality of approximation of measured data and its accuracy by parameters:

4.1. Standard deviation in percentages:

\[ \sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} \left( 1 - \frac{z_i(t)}{z} \right)^2} \times 100\% \]

4.2. Coefficient of determination
\[
R^2 = 1 - \frac{\sum_{i=1}^{n} [z - z(t_i)]^2}{\sum_{i=1}^{n} [z - \bar{z}]^2}
\]

where:

\[
z = \frac{1}{n} \sum_{i=1}^{n} z = \frac{1}{n} (1 + 2 + ... + n) = \frac{1 + n}{2}
\]

**Note 3:** The above calculations for this variant of the deceleration method for tyre rolling resistance measurement can be executed by the computer program “Deceleration Calculator” downloadable from the WP.29 website\(^2\) as well as any software which allows the calculation of nonlinear regression.

## II. Justification

1. The proposed principle and its application in the computer program “Deceleration Calculator” is based on the exact relationship:

\[
j = \frac{d\omega}{dt} = \frac{d^2 z}{dt^2}
\]

2. Numerous experiments show that the formula in paragraph 2.3 of proposed Annex 6, Appendix 5 is very effective for experimental data approximation. This formula of constraint between current time \(t\) and current angular distance \(z\) results from the transformation of the dependence between retroreflective distance \(S\) and time \(T\) \([1], [2]\): (in common case \(T = T_z - t\)):

\[
S = A_m \ln \frac{1}{\cos B T_T}
\]

Retroreflective time \(T\) relates to the total deceleration time \(T_z\) by formula \(T = T_z - t\) (in local case \(T = T_z, |S = S_z|\)).

The second derivative of the function described by the formula in paragraph 2.3 of Annex 6, Appendix 5 is deceleration \(j\) in revolutions per second squared or \(s^{-2}\):

\[
j = \frac{d^2 z}{dt^2} = \frac{AB^2}{\cos^2 B (T_z - t)}
\]

3. There is no simplification or assumption between this formula and the formula of \(z\) in paragraph 2.3 of Annex 6, Appendix 5 because correspondent transformation is performed according to the rules of differential calculus of higher mathematics. Thus the need to measure and calculate speed is excluded.

4. The algorithm for determining parameters \(A, B\) and \(T_z\) includes the following steps:

4.1. The measuring time of each revolution of the rotating body which gives the experimental dependency as shown in figure 1:

\(^2\) To be indicated at a later stage.
4.2. Finding the value nearest to the maximum $z$ equals $n$, dividable by 4, dividing it into 4 equal parts and recording the coordinates of 4 points on the experimental curve (see figure 1).

4.3. Working out the equation system on the basis of the formula in paragraph 2.3 of Annex 6, Appendix 5 with a substitution of 4 point coordinates as shown in figure 1:

$$z_1 = A \ln \frac{\cos B(T_x - t_1)}{\cos BT_x}$$
$$...$$
$$z_4 = A \ln \frac{\cos B(T_x - t_4)}{\cos BT_x}$$

4.4. The pairwise transformations of the set of equations from paragraph 4.3 above gives a set of two equations:

$$\cos^2 B(T_x - t_1) = \cos BT_x \cos B(T_x - t_2)$$
$$\cos^2 B(T_x - t_3) = \cos B(T_x - t_4) \cos B(T_x - t_4)$$

Parameters $B$ and $T_x$ are determined in this set by an iteration process. Then parameter $A$ may be obtained from the fourth equation from the set of four equations above, multiplied by $2\pi$:

$$A = \frac{2\pi z_4}{\ln \frac{\cos B(T_x - t_4)}{\cos BT_x}}$$

Thus formulae $z = f(t)$ and $j = \frac{dz}{dt}^2$ become those with determined parameters and, thus, enabling subsequent applications. The first derivative of function $z = f(t)$ from paragraph 2.3 of Annex 6, Appendix 5 above is the angular speed $\omega$ in revolutions per second ($s^{-1}$):

$$\omega = \frac{dz(t)}{dt} = AB \frac{tgB(T_x - t)}{AB}$$

One can see from this that:

$$tgB(T_x - t) = \frac{\omega}{AB}$$

The next formula follows from geometry and the previous relation:

$$\cos^2 B(T_x - t) = \frac{1}{1 + tg^2 B(T_x - t)} = \frac{1}{1 + (\omega/AB)^2}$$

Substitution of this equality into formula $j$ in paragraph 2 of this justification yields:

$$j = AB^2 + \frac{\omega^2}{A}$$

This relationship is the main formula for the “Deceleration Calculator”.

5. The proposed derivative mathematical method approach associated to the used deceleration calculator provides for the approximation of an evaluated estimate close to 1.
III. Considerations by the Working Party on Braking and Running Gear (GRRF)³

During the sixty-seventh session of GRRF, the expert from the Russian Federation presented GRB-59-02 proposing an evaluation of the tyre rolling resistance test results using a mathematical algorithm — to be added as an alternative to the existing one in the text of the Regulation. He also presented the results of a validation study of the proposed algorithm for tyres of the C1 category performed in France, in cooperation with the Russian Federation (GRB-59-07 and GRB-59-08). GRRF noted that this proposal may be submitted by GRB to WP.29 as a supplement to UN Regulation No. 117. GRRF recommended further steps for a better understanding of the proposed method’s sensitivity, suggesting: (i) extending the validation study for tyres of categories C2 and C3 and (ii) verifying the application of the proposed alternative in the conditions of the conformity of production. The GRRF Chair invited the experts to provide their comments on the proposal to the GRB secretariat with a copy to the expert from the Russian Federation.

³ This section has been added by the secretariat as requested by GRB (ECE/TRANS/WP.29/GRB/57, para. 19).