Balanced imputation for swiss cheese nonresponse

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Item nonresponse

- **Only one variable** is subject to nonresponse.

Sex	Height	Weight
M	175	68
F	160	55
M	180	?
F	165	?

Item nonresponse

- **Only one variable** is subject to nonresponse.

Sex	Height	Weight
M	175	68
F	160	55
M	180	?
F	165	?

- **All variables** of the survey are subject to nonresponse.

Sex	Height	$P_{t=1}$	$P_{t=2}$	$P_{t=3}$
H	175	68	67	68
F	160	55	58	?
H	180	70	?	?
F	165	?	?	?

Monotone

Sex	Height	Weight
H	175	68
F	160	?
H	?	70
?	165	?

Non Monotone

Swiss cheese nonresponse

Swiss cheese nonresponse (non monotone)

All variables of the survey contain missing values, without a particular pattern.

Treatments

- Donor imputation methods (Andridge & Little, 2010; Judkins, 1997).
- Iterative imputation methods: a sequence of regression models between the variables (Raghunathan *et al.* , 2001).

Swiss cheese nonresponse

Properties of interest for an imputation method

- Preserve the distributions of the variables;
- Preserve the relationships between variables;
- Impute by realistic values.

Swiss cheese nonresponse

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Balanced K-nearest neighbor imputation (Hasler & Tillé, 2016)

- Imputation for one variable;
- Donor imputation method (random);
  - Continuous and discrete variables;
  - Only one donor per nonrespondent;
- Imputation by near donors (neighbors);
- Balanced sampling;
- If the observed values were imputed, the estimators with imputed values and the estimators with the observed values would be the same.

Swiss cheese nonresponse

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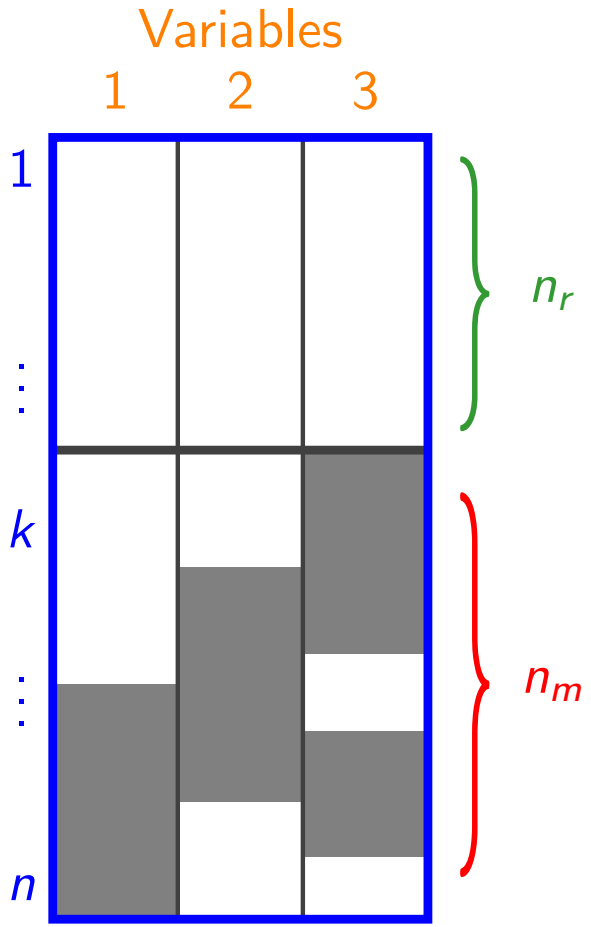
→ Extend this method for swiss cheese nonresponse !

Swiss cheese nonresponse

- Population U of size N.
- J variables of interest, $\mathbf{x}_k = (x_{k1}, \ldots, x_{kj}, \ldots, x_{kJ})^\top$.
- Sample s of size n.
- π_k, inclusion probabilities of the unit k.
- $s_r \subset s$, n_r completely observed units.
- $s_m = s - s_r$, $n_m = n - n_r$ units with missing values.
- Non monotone nonresponse.



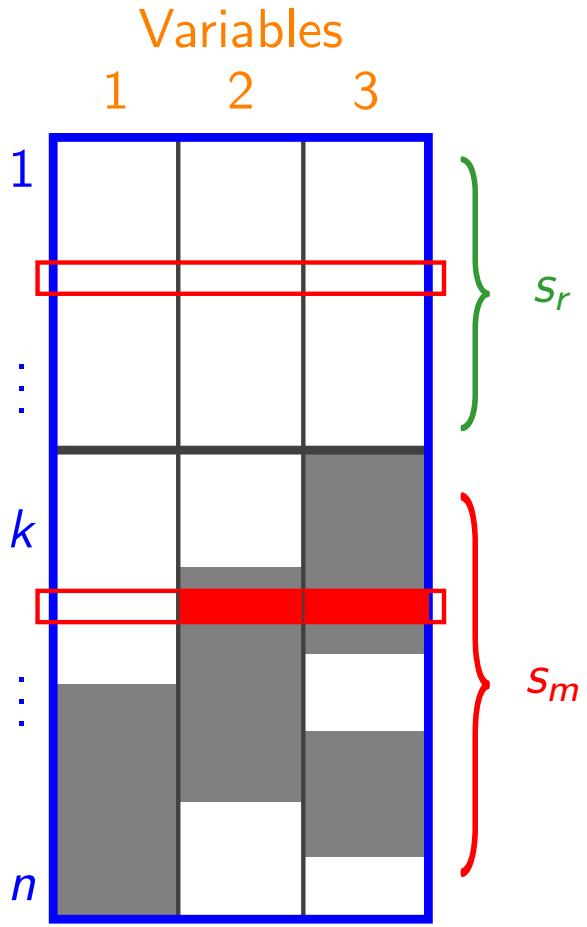
Requirements for the imputation methods

(i) Donor imputation method: select the donors in s_r.

(ii) Only one donor per unit.

(iii) Donor selected in the set of K nearest neighbors of the unit with missing values.

(iv) Described in the following slides.



Matrix of imputation probabilities

(i) Donor imputation: select the donors in s_r:
Matrix of imputation probabilities $\boldsymbol{\psi} = (\psi_{ik})$, where $(i, k) \in s_r \times s_m$.

- ψ_{ik}: Probability that respondent i gives its values to nonrespondent k;
- $\psi_{ik} \geq 0$.

$$\boldsymbol{\psi} = \begin{pmatrix} \psi_{11} & \psi_{12} & \psi_{13} \\ \psi_{21} & \psi_{22} & \psi_{23} \\ \psi_{31} & \psi_{32} & \psi_{33} \end{pmatrix} = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \end{pmatrix}$$

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(iii) Donor selected in the set of K nearest neighbors of the unit with missing values:

$$\psi_{ik} = 0 \text{ if } i \notin \text{knn}(k)$$

where $\text{knn}(\ell) = \{j \in s_r | \, \text{rang}(d(j, \ell)) \leq K\}$ and $d(.,.)$ is a distance function.

Matrix of imputation probabilities

(iv) If the observed values of the nonrespondents were imputed, the total estimator of each variable should remain unchanged:

$$\sum_{k \in s_m} d_k r_{kj} \underbrace{\sum_{i \in s_r} \psi_{ik} x_{ij}}_{x^*_{kj}} = \sum_{k \in s_m} d_k r_{kj} x_{kj},$$

where $d_\ell = \pi_\ell^{-1}$ and $r_{\ell j}$ is 1 if unit ℓ responded to variable j, 0 otherwise, for $j = 1, \ldots, J$.

Matrix of imputation probabilities

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$$\sum_{k \in s_m} d_k r_{kj} \underbrace{\sum_{i \in s_r} \psi_{ik} x_{ij}}_{x^*_{kj}} = \sum_{k \in s_m} d_k r_{kj} x_{kj},$$

	Incomplete		Imputed	
	x_1	x_2	x_1	x_2
s_r	8	4	8	4
	6	2	6	2
s_m	?	3	*8*	*4*
	?	3	*6*	*2*
Total in s_m		6		6

Matrix of imputation probabilities

(iv) For $j = 1, \ldots, J$,

$$\sum_{k \in s_m} d_k r_{kj} \sum_{i \in s_r} \psi_{ik} x_{ij} = \sum_{k \in s_m} d_k r_{kj} x_{kj}$$

$$\sum_{i \in s_r} \left(\sum_{k \in s_m} d_k r_{kj} \psi_{ik} \right) r_{ij} x_{ij} = \sum_{k \in s_m} d_k r_{kj} x_{kj}.$$

Algorithm: ψ_{ik} are calculated by calibration:

$$\text{Initial weights} \quad \psi_{ik}^0 = \begin{cases} \frac{1}{K} & \text{if } i \in \text{knn}(k), \\ 0 & \text{otherwise.} \end{cases}$$

Iterations : calibrate, normalize.

Imputation matrix

Matrix of imputation probabilities

$$\psi = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \end{pmatrix}$$

Imputation matrix

$$\phi = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- ϕ_{ik}: 1 if unit i is the selected donor for unit k, 0 otherwise.
- Only one donor per nonrespondent, $\sum_{i \in s_r} \phi_{ik} = 1$.
- Requirement (iv): donors are selected in order to respect

$$\sum_{k \in s_m} \sum_{i \in s_r} \phi_{ik}\, d_k r_{kj} x_{ij} = \sum_{k \in s_m} \sum_{i \in s_r} \psi_{ik}\, d_k r_{kj} x_{ij} \left(= \sum_{k \in s_m} d_k r_{kj} x_{kj} \right).$$

Imputation matrix

Requirement (iv): donors are selected in order to respect

$$\sum_{k \in s_m} \sum_{i \in s_r} \phi_{ik}\, d_k r_{kj} x_{ij} = \sum_{k \in s_m} \sum_{i \in s_r} \psi_{ik}\, d_k r_{kj} x_{ij}.$$

- Stratified balanced sampling (Chauvet, 2009; Hasler & Tillé, 2014);
- n_m strata (nonrespondent) are created;
- One donor is selected per stratum.
- Inclusion probability used in the stratified balanced sampling is ψ_{ik};
- Balancing variable is $\psi_{ik}\, d_k r_{kj} x_{ij}$.

Imputation

Imputed value: $x^*_{kj} = \sum_{i \in s_r} \phi_{ik} x_{ij}$

Imputed total: $\widehat{X}_j = \sum_{k \in s_r} d_k x_{kj} + \sum_{k \in s_m} r_{kj} d_k x_{kj} + \sum_{k \in s_m} (1 - r_{kj}) d_k x^*_{kj}$

Discussion

- Determine K (not to large).
- Method for qualitative/quantitative variables.
- Possibility to force $\psi_{ik} = 0$ for any reason.
- Models and principles.
- R program.
- Variance estimation.

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Chauvet, Guillaume. 2009. Stratified Balanced Sampling. *Survey Methodology*, **35**, 115–119.

Hasler, Caren, & Tillé, Yves. 2014. Fast Balanced Sampling for Highly Stratified Population. *Computational Statistics and Data Analysis*, **74**, 81–94.

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Raghunathan, Trivellore E., Lepkowski, James M., van Hoewyk, John, & Solenberger, Peter W. 2001. A Multivariate Technique for Multiply Imputing Missing Values Using a Sequence of Regression Models. *Survey Methodology*, **27**(1), 85–95.