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Measuring substitution bias and the CPI – theory and practice.

Abstract: Consumers react to changes in relative prices by reducing the quantities purchased of those goods and services with relatively higher price changes, in order to buy more of those with lower relative price changes over time. A true measure of inflation would capture these changes in purchasing patterns. However, due to data limitations and other practical considerations, it is possible for the Consumer Price Index (CPI) to incorporate these changes only with some delay.

Most CPIs, including the Australian CPI, use a fixed-base Laspeyres-type formula (or Lowe index) to measure the change in the price of purchasing the same quantity of particular goods and services in the current period as was purchased in a specified base period. This tends to overstate the price change of those goods and services with relatively higher price changes, resulting in substitution bias National Statistical Offices (NSOs) apply different methods to estimate substitution bias. This paper focuses on the estimation of upper level substitution bias, some considerations in the estimation and why other approaches could be sub-optimal.

Lower level[1] (elementary) substitution bias can be mitigated through more frequent lower level reviews or the use of multilateral[2] index methods (in the case of scanner[3] or transaction data), and is not discussed in this paper.

The paper examines the theory and methods applied by NSOs to estimate upper level substitution bias, and presents empirical results based on the Australian Bureau of Statistics' (ABS) methods of measuring upper level substitution bias in the Australian context. It highlights the use of annual expenditure weights and annual indexes as currently applied by the ABS, identifies some limitations in measuring upper level substitution bias using the monthly or quarterly official CPI, and concludes with a set of broad recommendations to facilitate comparison of upper level substitution bias across countries.

Key words: Upper level substitution bias, superlative index, Laspeyres-type, Paasche, Fisher

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[1]Also known as within - stratum bias, refers to bias due to substitution within an expenditure class (EC).
[2] Multilateral index numbers are often used for spatial price and output comparisons across economic entities (e.g. countries). In a temporal context, multilateral index numbers make price comparisons across more than two time periods.
[3] Detailed data on sales of consumer goods obtained by scanning the bar codes for individual products at electronic points of sale in retail outlets. The data can provide detailed information about quantities, characteristics and values of goods sold, as well as their prices. Scanner data constitute a rapidly expanding source of data with considerable potential for CPI purposes. They are increasingly used for purposes of hedonic analysis.

1. Introduction

Prior to December quarter 2017, the Australian Bureau of Statistics (ABS) had reviewed and updated the CPI weights every five or six years, once Household Expenditure Survey (HES) data were available. As a result, these weights were fixed for the period until the next HES. In order to provide an estimate of the upper level substitution bias in the fixed–weight Australian CPI, the ABS constructed a retrospective superlative index following each five or six yearly weight updates. From December quarter 2017, the CPI is re-weighted annually using both the HES and Household Final Consumption Expenditure (HFCE) data from the National Accounts (ABS 2017b).

Although there are other sources of bias in CPI, the analysis focuses on one type only: upper level item substitution bias. Therefore, the results of the analysis should not be taken to equate to the total bias in the CPI, which will be the cumulative impact of all sources of bias. This analysis can be conducted only retrospectively, when new expenditure data are available from the HES.

Upper level substitution bias is considered the most widely accepted source of CPI bias, and the kind with which economists are most familiar from textbook expositions of price index theory and practice (ILO 2004, paragraph 11.38).

This paper looks at the theory and practice of deriving estimates of upper level substitution bias, using an unchained and chained Fisher index as a target index spanning eighteen years, with four sets of annual expenditure weights from the HES (1998-99[4], 2003-04, 2009-10 and 2015-16). It also examines considerations or constraints in deriving the components required for the calculation of the benchmark Fisher index in practice.

The paper presents results in the context of the two approaches that have been used by the ABS. It summarises international practice from selected NSOs (see Appendix 1), and draws attention to the desirable target index recommended by the International Labour Organisation (ILO, 2004).

The results, based on the current ABS approach, present a more accurate estimate of true inflation. This is due to the original annual expenditure weights being used, and the period of the indexes and weights now aligning (from the same period). They compare closely with results from previous analysis, and are consistent with analysis and studies by other NSOs.

[4]These periods refer to the Australian financial year which starts from 1 July and ends on 30 June of the next calendar year.

2. Methodology

2.1 Deriving components of the superlative index

To estimate the amount of potential upper level substitution bias in the CPI, a benchmark index is often calculated and compared to an index comparable to the CPI or to the actual CPI. The benchmark index, usually a superlative index, takes into account any changes in consumer spending patterns.

The construction of a superlative benchmark index requires past and current information (expenditure weights or quantities). This is usually not available in the current period, and therefore the superlative index can be derived only retrospectively. This benchmark index can be a Fisher or Törnqvist[5] price index; in the case of this analysis, the Fisher is used[6].

This analysis, consistent with previous ABS studies, derives the Fisher index as a benchmark or target index. The two components of the Fisher index, the Laspeyres and Paasche indexes, are calculated between series reviews using expenditure weights (quantities) from the HES. This gives the unchained and chained[7] indexes over the analysis period.

2.1.1 True Laspeyres and Laspeyres-type (Lowe) index

The Laspeyres index uses expenditure weights with fixed underlying quantities from the price-reference period to derive a price index. These quantities are fixed until the next expenditure data, with different quantities, are available. In a true Laspeyres index, the period that provides the expenditure weights must coincide with the reference period for the prices.

The application of expenditure data from the period to which they pertain results in a true Laspeyres index. This is not practical and generally is not applied by NSOs in calculating the CPI, due to the time required to collect and implement the most recent expenditure data.

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[5] A symmetric index defined as the weighted geometric average of the price relatives in which the weights are simple arithmetic averages of the expenditure shares in the two periods. It is a superlative index. Also known as the Törnqvist–Theil price index.

[6] The Fisher index is used for consistency with previous ABS analysis and international practice.

[7] The chained index covers the entire eighteen-year analysis period and unchained indexes are between the series reviews or weight updates.

The Laspeyres-type index, also known as a Lowe index, uses a fixed basket approach but with price and quantities from different periods (unlike the Laspeyres index). As a result, the Lowe index is calculated using expenditure weights with underlying quantities from an earlier period (period b) than the price reference period (period 0). This is a result of price-updating[8] of the original expenditures due to the time required to collect and process expenditure data, such as HES. Consequently, the weights used in the Lowe formula are price-updated (ILO 2004). This is commonly applied by NSOs, including the ABS.

2.1.2 Paasche and Paasche-type index

The Paasche index uses current expenditure weights with current underlying quantity information. These are then applied to price changes to derive an index between the current and the price reference period. Due to the absence of the most recent expenditure weights or quantity information in every period, the Paasche index is not practical and would be difficult to produce in a timely manner.

The Paasche or Paasche-type index can be derived only retrospectively, when the most current expenditure data is available: a true Paasche index only in the period with actual expenditure weights pertaining to that period, and a Paasche-type index using modelled or proxy expenditure weights or quantities, as opposed to real expenditure or quantities from the latest HES or other data source.

2.1.3 Monthly, Quarterly and Annual Retrospective Indexes

The underlying quantities used in CPI expenditure weights are generally based on annual expenditure data from an annual HES or other household survey and therefore represent annual quantities. These are applied to price movements to derive the monthly or quarterly indexes as required by NSOs.

Diewert, Huwiler and Kohli (2008) support the construction of retrospective superlative indexes to assess the importance of substitution bias, whenever new data is available. Therefore, it is feasible to apply current expenditure weights to compile a retrospective index on a monthly, quarterly or annual frequency. This is occasionally undertaken by some NSOs for analysis or to derive estimates of upper level substitution bias.

[8] Price-updating revalue expenditure shares in the weight reference period at prices of the price reference period, which keeps the basket up to date and in line with the consumer expenditure trends.

Acquiring current expenditure weights or quantities for the Paasche index poses a challenge and requires a practical approach when HES data is unavailable. In the case of the ABS, the current period expenditure weights (or quantities) required for the Paasche component are derived linearly using expenditure weights between the two HES periods. This results in quarterly expenditures or quantities derived from annual expenditures or quantities, and similarly for monthly expenditure weights or quantities for a monthly index.

The use of monthly or quarterly modelled expenditure weights in upper level indexes may not be conceptually sound. This is because they are derived from annual expenditure weights with annual quantities, and may not accurately account for some seasonal patterns when estimating upper level substitution bias. This would be of particular interest if estimates are derived on a year-over-year monthly or year-over-year quarterly[9] indexes. For example, estimating bias using the same quarter each year, say June 2000 and June 2001, June 2002, June 2003, June 2004, June 2005 etc.

The ILO 2004 manual discusses and questions the use of annual expenditure weights and its impacts on month-to-month indexes, and provides recommendations on when to apply annual expenditure-weights for monthly or quarterly indexes (ILO 2004, paragraphs 4.66 and 4.84).

According to the ILO manual, "the annual chained Fisher indices should, however, normally be regarded as the more desirable target index to approximate, since this index will normally give better results if prices and expenditure shares are changing substantially over time" (ILO 2004, paragraph 22.41). It also supports the use of annual averages of indexes as a theoretically satisfactory method in some contexts (ILO 2004, paragraph 22.9).

Discussions towards the use of a common approach by most NSOs to calculate retrospective indexes or potential upper level substitution bias will provide the basis for further investigations, and an opportunity to compare these estimates across countries. A move to estimate annual chained retrospective superlative indexes using annual weights and annual indexes eliminates any inherent difficulties in comparison due to differences in the frequency of weight updates, and in the frequency of CPI release by NSOs.

[9] This derives CAGR between June quarters, relative to the price reference period for that set of weights.

2.2 The Australian CPI

The Australian CPI is an important economic indicator that measures price changes experienced by Australian households. It is compiled according to international standards, and is based on robust data collection and compilation methodologies. It is compiled and published quarterly.
From December quarter 2017, the CPI is re-weighted annually using both the HES and HFCE data from the National Accounts (ABS 2017b). Prior to this, the CPI was reviewed and re-weighted every five or six years using HES data only.

2.3 ABS approach to estimate the potential upper level substitution bias in the CPI.

The ABS' analysis of the potential upper level substitution bias in the fixed-weighted Australian CPI provides an estimate of the upward bias that can be attributed to consumer substitution between expenditure classes. This is the lowest level for which weighting information (from the HES) remains fixed between CPI reviews. Therefore, the analysis captures substitution bias from one expenditure class to another, but not within a given expenditure class.

The ABS continues to investigate and enhance its method to estimate the potential upper level substitution bias in the CPI, with the most recent analysis carried out in 2017. This has resulted in some enhancements to the previous method.

The previous method uses a quarterly Laspeyres-type, Paasche-type and Fisher-type indexes to estimate the potential upper level substitution bias between June 2000 and June 2011 (van Kints, M & Bishop, G, 2013). Section 3.1 discusses empirical results based on the most recent expenditure weights using this approach.

The current ABS methodology calculates annual true Laspeyres and Paasche–type indexes, instead of quarterly, using the original HES weights for each series of the CPI and annual estimates of price change. Under this approach, the Laspeyres index is a true Laspeyres and not a Laspeyres-type (Lowe) index as was previously the case.

For the Paasche index, the weights for each year ~~in~~ between HES periods are interpolated using a linear model, similar to the previous approach. The only difference is that the interpolated expenditure weights are derived from original HES, and represent annual quantities and prices. Analysis carried by de Haan, Balk and Hansen (2009) recommends this approach, to derive expenditure weights as a linear combination of benchmark years' expenditure weights for those years where expenditure data is unavailable, to approximate a retrospective superlative index. The geometric mean of the Laspeyres and Paasche produces the Fisher.

To estimate the overall average annual upper level substitution bias over the eighteen years, the indexes can be expressed as Compound Annual Growth Rates (CAGR).

$Laspeyres_{CAGR}$

$$= ((I_{L,2015\text{-}16} / I_{L,1998\text{-}99})^{(1/17)} - 1) * 100$$

$$= ((156.5/100.0)^{(1/17)} - 1) * 100$$

$$= 2.67\%$$

$Fisher_{CAGR}$

$$= ((I_{F,2015\text{-}16} / I_{F,1998\text{-}99})^{(1/17)} - 1) * 100$$

$$= ((150.9/100.0)^{(1/17)} - 1) * 100$$

$$= 2.45\%$$

The average annual upper level substitution bias is calculated as $Laspeyres_{CAGR}$ - $Fisher_{CAGR}$ = 2.67% - 2.45% = 0.22%. The CPI for the 1998-99 to 2015-16 period was potentially upwardly biased by 0.22 of a percentage point per year on average due to the upper level item substitution effect. These results are consistent with previous analysis and studies by other national statistical agencies (e.g. Shoemaker, 2013). See Appendix 1 for a detailed description of the current ABS method.

This analysis derives and compares potential estimates of upper level substitution bias in the Australian CPI by using the previous and current ABS methods. The estimates of bias from the time of re-weight are produced from the unchained indexes during the various expenditure weights updates. Refer to tables and graphs presented in section 3.

The ABS will update the CPI weights annually from December 2018 using HFCE data. The move to an annual re-weight, instead of a six-yearly weights update, reduces the amount of upper level bias in the Australian CPI. Results from ABS analysis estimate that the amount of upper level bias in the Australian CPI will be reduced by 0.15 of a percentage point on average annually (ABS 2016).

3. Empirical Results

3.1 ABS previous practice

The potential upper level substitution bias based on ABS' previous methodology, using updated expenditure weights (HES 2015-16) from time since re-weights are similar to those in the previous analysis (van Kints & Bishop, 2013). However, the estimates are slightly lower in most instances due to a low inflationary environment in the period since that last review.

The CPI (the actual published Australian CPI) tracks very well with the re-calculated[10] Laspeyres-type index over this period. They both show the impacts of very high prices in 2006 and 2011 for fruits and vegetables due to adverse conditions in these periods.

In these periods, some volatility was introduced into the CPI due to spikes in banana prices in 2006 and 2011 following tropical cyclones, where the price of bananas increased by over 300% temporarily. The arithmetic aggregation approach at the time assumed consumers still purchased the same quantity of bananas that they purchased when prices were at their pre-cyclone levels. In reality, consumers reduced the quantity of bananas they purchased and substituted with other fruits. The ABS now mitigates this through the use of multilateral methods and transaction data in the Australian CPI (ABS 2017a).

Table 1: Upper level substitution bias estimates using previous method from time since re-weight[11]

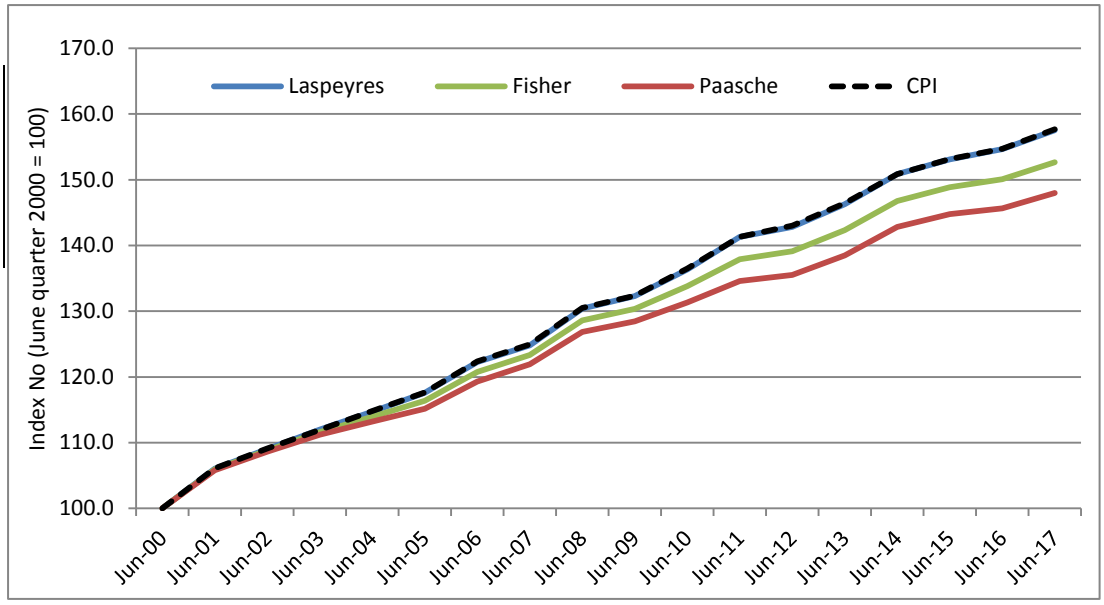
Time since re-weight	**Previous analysis (June 2000-June 2011)** (a)	**Laspeyres-type (June 2000-June 2017)** (b)	**CPI (June 2000-June 2017)**
1 Year (c)	0.16	0.17	0.24
2 Years	0.08	0.10	0.14
3 Years	0.12	0.11	0.11
4 Years	0.22	0.13	0.13
5 Years	0.25	0.17	0.17
6 Years (d)	0.24	0.18	0.18

a. Bias estimates from previous analysis (between June 2000 and June 2011)
b. This takes the average of the average annual item substitution bias for the period June quarter 2000 - June quarter 2005, June quarter 2005 - June quarter 2011 and the period June quarter 2011 – June quarter 2017
c. This figure includes the banana price increase in March 2006 and June 2011, due to cyclone Larry and cyclone Yasi respectively.
d. The six-year average annual item substitution bias is only based on the index numbers for June quarter 2005 to June quarter 2011, and June quarter 2011 to June quarter 2017.

[10]The re-calculated Laspeyres-type index applies the same methodology as in the published CPI using rounded index numbers. Differences are due to rounding and one-off impact of very high prices in the CPI during cyclones.

[11]The potential estimates of substitutions bias are derived as the difference in CAGR between the unchained Fisher-type index, the CPI and the Laspeyres-type.

Figure 1: Quarterly chained index



3.2 ABS current practice

The results based on the current methodology in table 1 show similar estimates of upper level substitution bias from the time since re-weight and overall, over the eighteen-year period. However, bias estimates for the first year are relatively small when compared to those from the CPI and Laspeyres-type index with price-updated weights.

Apart from the first year, the potential bias estimates using the current ABS method are slightly higher, ranging from 0.17 to 0.21 of a percentage point on average compared to those using the previous method, with Laspeyres-type ranging from 0.11 to 0.18 of a percentage point on average similar to the CPI (0.11 to 0.18). This indicates that the previous potential bias estimates in the CPI could have been slightly understated if a true Laspeyres index were used.

Of particular interest are the higher bias estimates in the first and second years for both the CPI and the re-calculated Laspeyres-type index, in which the weights are price-updated prior to implementation and other impacts as mentioned above. The bias estimates are quite similar in the later years but slightly higher with the current approach. A further analysis to isolate impacts of cyclones in 2006 and 2011 can provide an indication of whether price-updating could play a role in the bias estimates of the first and second years.

Table 2: Upper level substitution bias estimates (current method compared to CPI and previous method)

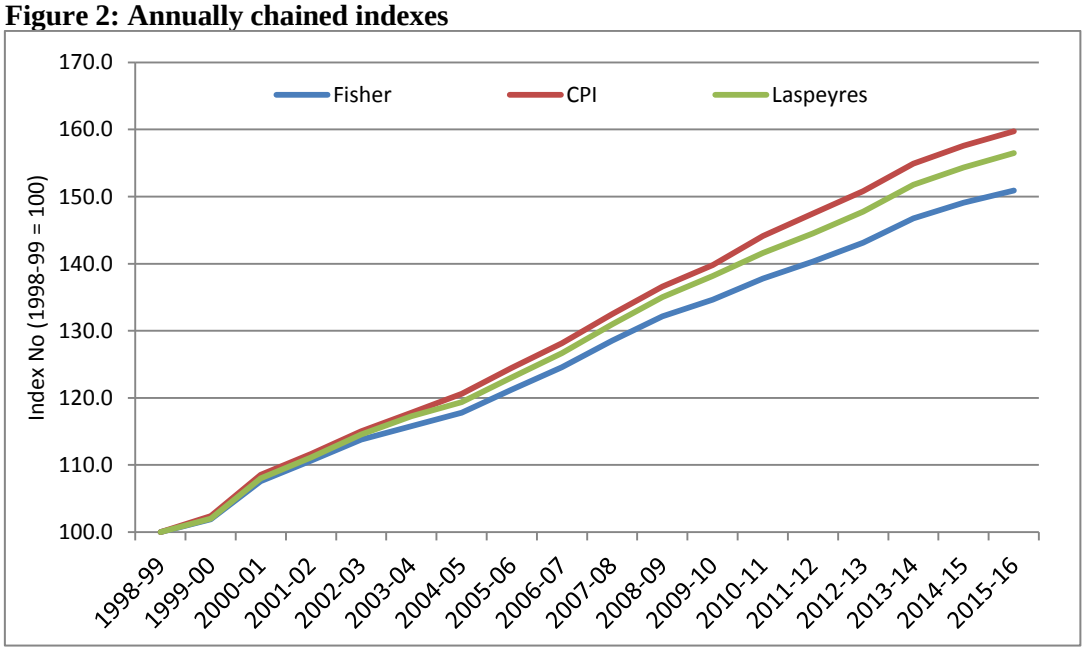
Time since re-weight	Laspeyres - type(June 2000-June 2017) (a)	CPI (June 2000-June 2017)	Laspeyres (ABS method)
1 Year (b)	0.17	0.24	0.11
2 Years	0.10	0.14	0.17
3 Years	0.11	0.11	0.17
4 Years	0.13	0.13	0.18
5 Years	0.17	0.17	0.21
6 Years(c)	0.18	0.18	0.20

a. This takes the average of the average annual item substitution bias for the period June quarter 2000 - June quarter 2005, June quarter 2005 - June quarter 2011 and the period June 2011 – June 2017.

b. This figure includes the banana price increase in March 2006 and June 2011 for the CPI and the Laspeyres-type, which was a result of cyclone Larry and cyclone Yasi respectively.

c. The six-year average annual item substitution bias is only based on the index numbers for June quarter 2005 to June quarter 2011, and June quarter 2011 to June quarter 2017.

Figure 2: Annually chained indexes



Ideally, the potential upper level substitution bias in the CPI should be derived by comparing the Fisher and the published All groups CPI. However, this poses a problem due to the delay and subsequent price-updating by NSOs when introducing new expenditure weights in the CPI. This results in differences in the price reference period in the first two years, as the published All groups CPI is still based on old expenditure weights.

Hence, this direct comparison of bias estimates from the CPI or the re-calculated Laspeyres-type index with estimates from the current ABS approach is misleading. This is shown in table 3, with high potential upper level substitution bias estimates in the first two years when the target Fisher index is compared to the CPI. Additionally, the high rates of bias seem to compound to higher rates in the later years because of the CAGR formula.

Table 3: Potential upper level substitution bias in the actual CPI.

Time since re-weight	CPI (a)	Laspeyres (ABS method)
1 Year	0.65	0.11
2 Years	0.53	0.17
3 Years	0.41	0.17
4 Years	0.35	0.18
5 Years	0.35	0.21
6 Years	0.34	0.20

(a) Old expenditure weights still in use for the first and second years

To isolate the impacts of the old weights in the first year (which are due to the delay in implementation of weights in the CPI), the first and second years after re-weight are excluded, by re-rebasing[12] the indexes to the second year after re-weight. This reduces the impact of the first year and second year, and the compounding effect due to the use of CAGR formula. Table 4 below shows very similar estimates between the CPI and Laspeyres, with slightly higher estimates for the CPI.

Excluding the bias estimates for the first and second years, the overall potential upper level substitution bias in the actual CPI and the true Laspeyres are quite similar: both around 0.20 of a percentage point on average annually over this period and close to the published estimate of 0.22 of a percentage point on average per year between 1998-99 and 2015-16. This shows that the bias estimates derived using the current ABS method better reflect the amount of upper level substitution bias in the actual CPI compared to the previous method.

Table 4: Potential upper level substitution bias in the actual CPI (Excl. year 1 and year 2)

Time since re-weight	CPI	Laspeyres (ABS method)
3 Years	0.17	0.15
4 Years	0.17	0.18
5 Years	0.23	0.23
6 Years	0.24	0.23

[12] Refers to changing the index reference period for a series of indices.

4. Summary and conclusions

The ABS has estimated the potential upper level substitution bias in the CPI using two methods. In the past, the ABS constructed a Laspeyres-type, Paasche-type and Fisher-type index at the All groups CPI using price-updated published CPI weights. As part of the most recent review of the Australian CPI, the ABS made some enhancements to the previous method.

The current ABS approach to estimating potential upper level substitution bias derives true Laspeyres and Paasche-type indexes at All groups from the EC level using annual expenditure weights and annual indexes. The difference between the Fisher and the true Laspeyres is the potential upper level substitution bias.

This approach is conceptually sound given the use of original annual expenditure weights (that represent annual quantities) and annual indexes that account for seasonal patterns, rather than price-updated expenditure weights and monthly or quarterly indexes. This approach aligns closely with recommendations on the construction of a target superlative index for analysis or estimation of upper level bias.

In the context of the ABS, the results present a more accurate estimate of true inflation, due to the original financial year weights being used and the period of the indexes and weights now aligning. The results are consistent with those from previous analysis and studies by other NSOs.

Currently, different NSOs apply different methods to estimate potential upper level substitution bias. This paper recommends NSOs consider a true Laspeyres and a Paasche-type, using annual expenditure weights (quantities) and annual indexes for a retrospective superlative index. A standard method by NSOs will enable comparisons across countries and further analysis of these estimates.

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6. Appendix

6.1 Appendix 1: Selected international country practice

Some NSOs, including the ABS, derive estimates of the potential upper level substitution bias in a CPI in a retrospective manner whenever new expenditure data is available. This is particularly the case with NSOs whose CPI expenditure weights are not updated annually. However, a variety of methods has been applied by different NSOs.

The Office for National Statistics (ONS) compiles the CPI on a monthly basis and has carried out analysis to calculate a monthly retrospective superlative CPI for the UK. This analysis calculated index numbers using annual weights as a proxy for monthly weights. The approach utilises original class level expenditure data, without price-updating, meaning that they correspond to the period they were collected (Clews, Sanderson, & Ralph, 2014). This is similar to the current ABS method that uses original weights and annual indexes (see Appendix 1).

Statistics New Zealand (SNZ) derives quarterly analytical retrospective indexes to estimate upper level substitution bias on an ongoing basis after the release of a new set of CPI expenditure weights. The current approach makes use of the latest price-updated annual weights between series reviews (SNZ 2014). This approach aligns closely with previous ABS methodology (van Kints & Bishop, 2013).

Quantitative measures of bias can be generated by comparing Laspeyres price indices to Fisher ideal, Törnqvist or other superlative indices (ILO 2004, paragraph 11.38). The studies examined in this paper derive these in different ways.

The results from Clews, Sanderson, & Ralph (2014), and SNZ (2014), derive estimates by comparing the superlative Fisher index and a Laspeyres-type (Lowe) index which replicates the CPI. However, other studies have derived upper level bias estimates using the actual CPI index series (Aizcorbe and Jackman, 1993). This shows there is currently no consensus, across NSOs, on the approach to calculating a retrospective index and quantifying potential upper level substitution bias in the CPI.

6.2 Appendix 2: Extract from the ABS Concepts Sources and Methods (ABS cat. no. 6461.0)

ESTIMATION OF UPPER LEVEL SUBSTITUTION BIAS

11.13 The ABS has constructed a retrospective superlative-type index to provide an estimation of potential item (upper level) substitution bias in the fixed-weight Australian CPI. While there are five main sources of bias in CPIs (described further in **Price index theory**), this analysis focuses on one type only - upper level item substitution bias - and therefore the results in the analysis should not be taken to equate to the total bias in the CPI, which will be the cumulative impact of all sources of bias. This analysis can only be conducted retrospectively, when new expenditure data are available.

11.14 Superlative indexes allow for substitution as they make use of weights for both the earlier and later periods under consideration (basically averaging across historical and current expenditures to derive a 'representative' set of weights for the period) whereas the Laspeyres index uses only base period weights.

11.15 The estimate of upper level substitution bias has been made at relatively high levels of aggregation. The analysis is calculated based on the amount of consumer substitution between expenditure classes as this is the lowest level for which reliable weighting information (from the HES and other alternative data sources) is available and the level at which the underlying quantity weights remain fixed between CPI reviews. Thus, the analysis captures substitution from one expenditure class to another, e.g. from beef and veal to poultry, but not within a given expenditure class, e.g. from beef to veal. The substitution within an expenditure class is called lower level substitution bias which is minimised through regular sample maintenance, sample reviews and choice of index formulas. In the December quarter 2017, the ABS implemented new methods, known as multilateral methods, to compile 28 ECs in the CPI. These methods utilise a census of products available in big datasets, and use expenditure data to weight products, mitigating the risk of lower level substitution bias.

11.16 The ABS enhanced the method to estimate upper level substitution bias as part of the 17th series review. This approach calculates financial year Laspeyres and Paasche-type indexes, using the HES weights for each series of the CPI and financial year estimates of price change.

11.17 Three superlative indexes have been constructed and linked together to form one continuous series. The first index was constructed on the 14th series CPI basis between 1998-99 and 2003-04, the second index was constructed on the 15th series CPI basis between 2003-04 and 2009-10, and the third constructed on the 16th series basis between 2009-10 and 2015-16.

11.18 Using the expenditure class weights at the weighted average of eight capital cities level, i) Laspeyres, ii) Paasche-type, and iii) superlative Fisher-type indexes have been calculated at the All groups CPI level[13]. The indexes have all been calculated with the base period 1998-99 = 100.0. The Fisher index is regarded as the best practical approximation of a 'true' (or 'ideal') price index, being the geometric average of the Laspeyres and Paasche indexes.

11.19 Under this approach, the Laspeyres index is a true Laspeyres, rather than a Lowe index as in the case of the published All groups CPI.

11.20 The Paasche-type index is retrospectively modelled using the HES weights, and a linear model to derive weights for financial years in between the re-weighting periods. The geometric mean of the Laspeyres and Paasche-type index approximates the Fisher.

11.21 The Laspeyres, Paasche-type and superlative Fisher-type indexes were constructed using the same structure as the All groups CPI published at the time to allow for direct comparison. The indexes from 1998-1999 to 2003-04 were derived using the 14th series classification consisting of 88 expenditure classes. The index numbers from 2003-04 to 2009-10 were derived using the 15th series classification consisting of 90 expenditure classes, and the index numbers from 2009-10 to 2015-16 were derived using the 16th series classification consisting of 87 expenditure classes.

11.22 Using these indexes, an estimate of upper level substitution bias in the CPI was obtained by subtracting the superlative (Fisher-type) index from the Laspeyres.

ANALYSIS OF THE UPPER LEVEL SUBSTITUTION BIAS

11.23 The analysis found the total upper level substitution bias of the All groups CPI (as measured by the difference between the Laspeyres index and the Fisher-type index) was 5.6 percentage points after 17 years due to the inability of the fixed-base index to take account of the item substitution effect. The Laspeyres index increased by a total of 56.5% from 1998-99 to 2015-16. The retrospective superlative index, calculated using the Fisher-type index, increased by 50.9% over the same period.

11.24 To estimate the average annual upper level substitution bias, the indexes can be expressed as Compound Annual Growth Rates (CAGR).

[13] For a description of the indexes, refer to **Price index theory** of the ABS CPI manual.

$Laspeyres_{CAGR}$

$$= ((I_{L,2015\text{-}16} / I_{L,1998\text{-}99})^{(1/17)} - 1) * 100$$
$$= ((156.5/100.0)^{(1/17)} - 1) * 100$$
$$= 2.67\%$$

$Fisher_{CAGR}$

$$= ((I_{F,2015\text{-}16} / I_{F,1998\text{-}99})^{(1/17)} - 1) * 100$$
$$= ((150.9/100.0)^{(1/17)} - 1) * 100$$
$$= 2.45\%$$

11.25 The average annual upper level substitution bias is calculated as $Laspeyres_{CAGR}$ - $Fisher_{CAGR}$ = 2.67% - 2.45% = 0.22%. The CPI for the 1998-99 to 2015-16 period was potentially upwardly biased by 0.22 of a percentage point per year on average due to the inability to take account of the upper level item substitution effect. These results are consistent with previous analysis and studies by other national statistical agencies.

11.26 The results show that the longer the period between re-weights, the larger the potential upper level item substitution bias effect on the index. Table 11.1 illustrates that the average annual substitution bias increases at a faster rate the longer the period between re-weights. The re-weighting periods in this analysis were 1998-99, 2003-04 and 2009-10.

11.1 AVERAGE ANNUAL ITEM SUBSTITUTION BIAS(a)

Time since re-weight	**$Laspeyres_{CAGR}$ - $Fisher_{CAGR}$**
1 year	0.11
2 years	0.17
3 years	0.17
4 years	0.18
5 years	0.21
(b)6 years	0.20

(a) This takes the average of the annual substitution bias for the 1998-99 to 2003-04, 2003-04 to 2009-10 and 2009-10 to 2015-16 periods.

(b) The six year average annual substitution bias is only based on the 2003-04 to 2009-10 and 2009-10 to 2015-16 periods.

11.27 This finding is consistent with the Statistics New Zealand (SNZ) analysis which showed that item substitution bias is considerably greater when NZ CPI weights are updated at six-yearly rather than three-yearly intervals[14].

[14] Analytical retrospective superlative index based on New Zealand's CPI: 2014, (Statistics New Zealand, 2014), available at http://archive.stats.govt.nz/browse_for_stats/economic_indicators/CPI_inflation/cpi-retrospective-superlative-index-2014.aspx

6.3 Appendix 3: Price index formulae

Laspeyres price index, $P_L(p^0, p^t, q^0) = \frac{\sum_{i=1}^{n} p_i^t q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0} = \sum_{i=1}^{n} (\frac{p_0^t}{p_i^0}) s_i^o$

Paasche price index, $P_P(p^0, p^t, q^t) = \frac{\sum_{i=1}^{n} p_i^t q_i^t}{\sum_{i=1}^{n} p_i^0 q_i^t} = \left\{ \sum_{i=1}^{n} \left(\frac{p_i^t}{p_i^0}\right)^{-1} s_i^t \right\}^{-1}$

Paasche − type price index, $P_P(p^0, p^t, q^t) = \frac{\sum_{i=1}^{n} p_i^t q_i^t}{\sum_{i=1}^{n} p_i^0 q_i^t} = \left\{ \sum_{i=1}^{n} \left(\frac{p_i^t}{p_i^0}\right)^{-1} \widehat{s_i^t} \right\}^{-1}$

Lowe price index, $P_{Lo}(p^0, p^t, q^b) = \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = \sum_{i=1}^{n} (\frac{p_0^t}{p_i^0}) s_i^{ob}$

Fisher price index, $P_F = \sqrt{P_L \times P_P}$

$$where\ s_i^{ob} = \frac{p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = \frac{p_i^b q_i^b \left(p_i^0 / p_i^b\right)}{\sum_{j=1}^{n} \left\lfloor p_j^b q_j^b \left(p_j^0 / p_j^b\right) \right\rfloor}, \qquad b < 0 < t$$

$$s_i^t = \frac{p_i^t q_i^t}{\sum_{i=1}^{n} p_i^t q_i^t}$$

$$s_i^0 = \frac{p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}$$

$$\widehat{s_i^t} = \frac{t}{T} s_i^T + \left(1 - \frac{t}{T}\right) s_i^0$$

p_i^t = current period price for item i

p_i^0 = price reference period price for item i

q_i^b = period b quantity for item i

q_i^0 = price reference period quantity for item i

q_i^t = current period quantity for item i

s_i^0 = price reference period expenditure shares for item i

s_i^t = current period expenditure shares for item i

$\widehat{s_i^t}$ = annual interpolated expenditure weights

T = benchmark period