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Compositional Data Analysis.

- Appropriate for proportions e.g. $n d_x$ or $n d_x^i$
- Relative, not absolute, information
- The Simplex is the appropriate space for analysis

Key Reference:
Aitchison, J. (1986)
*The Statistical Analysis of Compositional Data.*
London: Chapman and Hall.
Lee-Carter Single Decrement Model Structure.

\[ \ln m_{x,t} = a_x + b_x k_t + \varepsilon_{x,t} \]

\[ k_t = k_{t-1} + d + e_t \]
Multiple Decrement Model Structure.

Model Structure

K x D

N

K x D

N

Rank-1 SVD

X X

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Deaths by Group of Causes: France, female

Group
- Cardio–Vascular
- Neoplasms
- Other diseases
- Respiratory
- Injury & Poison
- Digestive
- Infectious

Proportion
Deaths by Group of Causes: France, female

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Geometric Mean $d(x)$

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Cardio-Vascular | Neoplasms | Other diseases | Respiratory | Injury & Poison | Digestive | Infectious

Age Effects (scaled)

Rank 1: 82%
Rank 2: 7%
Rank 3: 3%


Cardio-Vascular | Neoplasms | Other diseases | Respiratory | Injury & Poison | Digestive | Infective

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Type
- Data
- Est
- Fcst
Life Expectancy at Birth: female

Data
- Obs

Model
- CoDa Rank 1
- CoDa Rank 3
- Lee–C Rank 1

Decrement
- Single
- Multiple

Life Expectancy at Birth: female

Fitted

Forecast

Years
### Centred data: clr transform

<table>
<thead>
<tr>
<th></th>
<th>Cardio−Vascular</th>
<th>Neoplasms</th>
<th>Other diseases</th>
<th>Injury &amp; Poison</th>
<th>Respiratory</th>
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Fitted Centred data: clr transform

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Age distribution:
- 0 to 25
- 25 to 50
- 50 to 75
- 75 to 100

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Compositional residuals: clr transform

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Age

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