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THE MEASUREMENT OF INFLATION AND CHANGES IN THE COST OF LIVING

Paper submitted by the ECE secretariat <sup>1</sup>

**Introduction**

1. The purpose of this paper is to examine whether price indexes designed to measure inflation and indexes designed to measure changes in the cost of living are interchangeable or whether they are meant to measure different things. A cost of living, or CoL, index does not measure the change in the value of a fixed basket of goods and services so that its meaning as a measure of price change is not self evident. Conversely, it is well known that many price indexes provide biased estimates of changes in the cost of living.

2. A recent EU Council Regulation on the convergence of inflation in EU member countries requires inflation in different countries to be measured by means of comparable consumer price indexes. The intention is to compare rates of price inflation even though in some countries consumer price indexes

are specifically intended to measure changes in the cost of living rather than consumer inflation. This paper examines the relationship between inflation indexes and cost of living indexes in some detail to try to clarify whether their objectives are really different or not.

3. The fact that many price indexes, such as Laspeyres type price indexes,

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tend to be biased as estimates of changes in the cost of living has recently received much publicity as a result of the 'Boskin Report' <sup>1</sup>. It is argued in this paper that indexes which are biased in this way also provide equally poor (or biased) estimates of inflation in the sense that they are measuring changes in the value of inappropriate baskets of goods and services that are not typical of the span of time covered by the index. When more appropriate baskets are used, the price indexes can provide unbiased estimates of changes in the cost of living.

4. This paper is about the meaning and interpretation of price indexes in common use. It does not propose any new formulae. It reaches the conclusion that the search for good measures of inflation leads to the same kind of indexes as provide good estimates of changes in the cost of living. There is no conflict between these objectives and a single index may meet both needs, at least when the set of goods and services available to consumers remains unchanged. However, the continual enlargement of the set as a result of the development of new kinds of products should, in principle, tend to reduce the cost of living whereas it may have no systematic effect on the rate of inflation as measured by changes over time in the prices of those goods and services which are available in both periods.

#### **Inflation indexes and cost of living indexes**

5. In economic literature, there seems to be little discussion of the exact meaning of inflation. A typical text book definition is as follows: "The inflation rate is the percentage rate of increase of the level of prices during a given period." <sup>2</sup> Another widely quoted definition of inflation is that "inflation is a process of continuously rising prices, or equivalently, of a continuously falling value of money." <sup>3</sup> Defining inflation in terms of an increase in the "level of prices" suggests measuring inflation by the increase in the total monetary value of a specified, fixed set, or 'basket', of goods and services which may be bought and sold on the market. An index of inflation is interpreted in this way in this paper. It is not affected by substitutions that consumers may make in response to changes in relative prices.

6. Defining inflation in terms of "rising prices" suggests measuring it as some kind of average of the movements in individual monetary prices rather than the change in the total value of a basket of goods and services. The two approaches are equivalent if the individual price movements are weighted by the appropriate values. Either way, there is evidently no unique measure of inflation as the rate of increase in the price level depends on which basket of goods and services is selected, which in turn depends to some extent on the interests of the analyst or policy maker.

7. The economic theory of price indexes adopts a rather different approach, however, in which neither the concept of the price level nor price movements figure explicitly. The theory in fact relates to the cost-of-living, or CoL, index "which may be considered as an idealization of indices like the consumer price index and others of that type." <sup>4</sup> Pollak defines the CoL index succinctly as: "the ratio of the minimum expenditures required to attain a particular indifference curve under two price regimes." <sup>5</sup> A key

feature of the strict economic definition of a CoL index is that only a **single** indifference curve is used. A CoL index typically compares expenditures between a point on an indifference curve that the consumer actually occupies and a second point on the same curve that the consumer would take up if prices were different. When the price regimes are those occurring in two different time periods, one of which is chosen as the base period, a CoL index can be interpreted as measuring the amount by which expenditures must change between the two periods in order to enable the consumer to stay on the base period indifference curve. A CoL index does not attempt to compare two situations in which the indifference maps are different, even for a single consumer.

8. Suppose the first period is chosen as the base period. In order to calculate the CoL index it is necessary to work out the **hypothetical quantities** of goods and services that would be purchased at the prices of the second period, assuming the consumer minimizes the cost of staying on the first period indifference curve. If the second period indifference curve were to be used as the base instead, a different CoL index would be obtained (except in certain special cases mentioned below). These two indifference curves are not the only possibilities, of course, and any number of CoL indexes could be associated with two price regimes. For example, the indifference curve could be one from some third time period. A CoL index between 1995 and 1996 might be calculated using a indifference curve from 1990. In this case, the CoL index would involve two sets of hypothetical quantities, namely the quantities that the consumer would buy to attain the 1990 indifference curve if the prices were those of 1995 and the quantities that would be bought if the prices were those of 1996. In principle, just as any number of price level indexes might be defined between two periods by choosing different baskets of goods and services, any number of CoL indexes may be defined, even for a single consumer, by choosing different indifference curves.

9. The CoL indexes associated with different indifference curves for the same consumer do not necessarily lead to different numerical values. It is well known that if the consumer's preference ordering is 'homothetic' the CoL index is independent of the indifference curve chosen as the base. Provided the preference ordering remains unchanged between two periods, the CoL indexes based on the first and second periods will coincide when preferences take this particular functional form. <sup>6</sup>

10. Two important properties a CoL index should be noted. One is that the set of quantities of goods and services associated with the prices of the period other than the base period are hypothetical and unobservable. The second property is that these hypothetical quantities are generally not the same as the actual quantities in the base period. In marked contrast to price indexes which measure the change in the value of a fixed basket of goods and services, a CoL index compares the values of two **different** baskets of goods and services. <sup>7</sup> The differences between the two baskets are systematic and predictable because, in seeking to minimize the expenditures needed to stay on the same indifference curve, the consumer would be obliged to substitute, at least at the margin, goods and services which have become relatively cheaper for those which have become relatively dearer. This is

the well known 'substitution effect'.

### **CoL indexes and Laspeyres and Paasche price indexes**

11. A CoL index cannot be calculated directly, because no statistical office can work out the necessary hypothetical quantities in practice. However, a CoL index may still be used as a norm or standard by which to evaluate the properties or behaviour of price indexes that can actually be calculated, such as Laspeyres and Paasche price indexes. Assume initially there are no changes in the set of goods and services available in the two periods. It can then be shown that the Laspeyres index provides an upper bound to the CoL index based on the indifference curve of the first period. If expenditures were increased in proportion to the Laspeyres price index, the consumer must have the purchasing power to buy the base period basket of goods and services, but this basket would no longer be optimal if relative prices have changed. The consumer would be able to reach a higher indifference curve by making substitutions in response to the changes in relative prices so that a smaller increase in expenditure would be sufficient to stay on the same indifference curve.

12. Conversely, by analogous reasoning, the Paasche index can be shown to provide a lower bound to the CoL index based on the second period. If the consumer's preferences were homothetic and remained unchanged between the two periods, the two CoL indexes would coincide so that the Laspeyres and Paasche indexes would provide upper and lower bounds to the same CoL index. Moreover, if the preferences were not merely homothetic but the utility function were a homogeneous quadratic, the CoL index would be equal to the geometric average of the Laspeyres and Paasche indexes, namely the Fisher index.<sup>8</sup>

13. Diewert has taken the analysis one step further by showing that certain types of index number may be expected to provide a reasonable **approximation** to the CoL index under fairly general conditions, even though they may not equal it exactly. Such indexes are described by Diewert as 'superlative', the Fisher index being one of them.<sup>9</sup> Thus, it may be quite feasible to make an acceptable estimate of CoL index indirectly even though it cannot be calculated directly.

14. These results have had a profound influence on attitudes towards consumer price indexes. There is a widespread perception that Laspeyres indexes have an upward bias because they may be expected to exceed the CoL index, while Paasche indexes are perceived to have a downward bias. The bias is often described as 'substitution bias' because Laspeyres and Paasche indexes use fixed quantity weights and do not permit substitutions in response to changes in relative prices. The Fisher index, on the other hand, is not presumed to be biased either way.

### **Changes in tastes**

15. The generalizations made above are fairly robust when tastes do not change and the same goods and services are available in both time periods. The situation is more complicated, however, when either of these two

conditions is not satisfied.

16. Suppose, for example, that as a result of new information consumers realize that consumption of a certain good may have beneficial, or alternatively harmful, effects that were not known previously, as has happened for many kinds of food, drink and tobacco. Consumer preferences may be expected to change as a result. Such changes in tastes must affect consumers' welfare but, as already emphasized, a CoL index does not attempt to measure changes in the cost of living **over time** unless tastes remain the same. A CoL index involves a static comparison between two situations using only a **single** indifference curve. Changes in tastes cannot enter into the calculation of an individual CoL index because the index must be based on one or other of the indifference curves, either before or after the change in tastes occurred. It must utilize preferences at only a single point of time.

17. The consequence of a change in tastes is to accentuate the differences between the two separate CoL indexes based on the preference orderings in the first and the second periods, so that the CoL index based on the indifference curve of the second period may diverge significantly from that based on the curve of the first period.<sup>10</sup> Moreover, it is not possible to generalize a priori about which of the two CoL indexes will tend to be the larger, as this depends on which way tastes have changed.

18. Changes in tastes may also disturb the usual relationships between Laspeyres and Paasche indexes. For example, a fall in demand for a product due to change in tastes may lead to a fall in both the price and quantity bought, whereas price and quantity relatives are usually negatively correlated.<sup>11</sup> In these circumstances, generalizations about the relationships between the various indexes become increasingly hazardous.

#### **Changes in consumption possibilities over time**

19. Over the longer term, the appearance of completely new kinds of goods and services may have a more important impact on consumer welfare than changes in tastes by making it possible for consumers to enjoy benefits of a kind that were previously not merely unattainable but unknown. In this context, the major new goods consist of items such as:

electrical power and electrical and electronic goods of all kinds from the electric light bulb and telephone developed in the nineteenth century to radio, television, videos, CDs, PCS, etc., etc. developed in the present century;  
anaesthetics, antiseptics, antibiotics and other drugs;  
refrigerators and freezers and, hence, frozen and prepared foods;  
the automobile and jet air travel;

These major inventions have completely transformed the entire way of life of the average consumer over the longer term.

20. Enlarging the set of goods and services from which consumers may choose must make them better off, other things being equal. It tends to lower the cost of living significantly over the longer term. A CoL index, however, is

concerned with a static comparison using the preferences and set of goods and services available at a single point of time. When new goods appear which did not exist in the first period, they cannot be included in the CoL index based on that period, but the CoL index based on the second period is obliged to take them into account when they are actually being consumed in the second period.

21. One of the theoretical strengths of a CoL index is that new goods and services are covered even though there are no prices for them in the first period. In order to accommodate new goods in the CoL index for the second period, it is necessary to impose zero constraints for them when determining the hypothetical quantities of the goods and services that the consumer would have had to consume in the first period to attain the indifference curve of the second period.

22. Consider, initially, a CoL index based on the second period when the same  $n$  goods and services are available in both periods. Then, offer the consumer the opportunity to purchase one extra new good in the second period while keeping total expenditures unchanged. Assuming some of the new good is actually purchased, the consumer prefers the situation with  $n + 1$  goods to that with  $n$  goods as he could have simply continued to confine his purchases to the  $n$  goods if he wished. This implies that a higher indifference curve can be reached with the same total expenditure which in turn implies that the total value at first year prices of the  $n$  goods and services available in the first year would also have to be higher to enable the consumer to stay on this higher indifference curve. It follows that the CoL index with the  $n + 1$  goods would register a smaller increase than that with  $n$  goods. Thus, other things being equal, the consumer experiences a smaller increase in his cost of living when the new good is on offer than when there is no increase in the number of goods and services available, provided only that the consumer actually chooses to buy some of the new goods.

23. There is another way of analysing this situation. Even if the new good had been on offer in the first period, none of it would have been bought if the price of had been high enough. The lowest price that would reduce demand for a good to zero is described in economics as the 'demand reservation price'.<sup>12</sup> The value of the CoL index would be the same with or without imposing zero constraints on the hypothetical quantities of new goods in the first period if prices equal to, or greater than, the demand reservation prices were to be **imputed** for them in the first period. The hypothetical quantities consumed in the first period would have been zero anyway at those prices. The fact that the prices of new goods may be imagined as falling from their demand reservation prices to their actual prices in the second period gives some further insight into why enlarging the set of consumption possibilities tends to lower a CoL index. Finally, it may be noted that if the consumer does not choose to buy any of the new good when it is introduced in the second period because the price at which it is offered for sale is too high (i.e., above its demand reservation price) then obviously he is not better off because of the introduction of the new good.

24. Demand reservation prices also appear to make it possible to include new

goods in a Paasche price index by using the reservation prices as if they were the missing first period prices, thereby enabling price relatives to be calculated for new goods. Of course, the resulting Paasche index would be less than the Paasche index which simply excluded the new goods. It would also provide a lower bound to the CoL index based on the second period.

25. It is quite unrealistic realistic, however, to suppose that statistical offices are in a position to make acceptable estimates of reservation prices. More fundamentally, however, even if they could be estimated, hypothetical imputed prices ought not to be used in a price index intended to measure inflation as distinct from changes in the cost of living. No transactions ever take place at such prices. Inflation is a phenomenon that can only be **measured** by recording the prices of goods and services actually bought and sold in monetary transactions.

26. Suppose in the above example that the prices of all the  $n$  goods and services available in both periods remain unchanged between the first and second periods so that any price or CoL index based on these  $n$  goods would have to be unity. When the  $n + 1$  th good is introduced and bought in the second period, the CoL index must fall below unity, whatever the price of the new good, simply because of enlargement of the consumer's choice. However, the price level cannot be described as falling when the price of every good actually bought in both periods remains the same. While this example may not be very realistic, it illustrates the possibility that a CoL index may have a downward bias as a measure of inflation when widening consumption possibilities exert a downward pressure on the CoL index.

27. The prices of many new goods fall **after** they have first been introduced on the market. Such reductions in prices ought to be captured by both price indexes and CoL indexes and should not lead to differences between them. For this reason, some new goods, such as new types of electronic equipment, may tend to reduce inflation as well the cost of living in practice. The possible divergence between a CoL index and an inflation index referred to above is attributable to the once and for all effect on consumer welfare which occurs when a consumer first has the opportunity to derive utility in new ways by purchasing a completely new kind of good. This welfare gain affects the consumer's standard of living and cost of living, but the mere introduction of a good for which there is no previous price cannot be treated as if it were equivalent to a fall in an actual price and assumed to reduce the rate of inflation.

#### **CoL and inflation indexes compared**

28. It is assumed for convenience throughout the following sections that price and quantity relatives are negatively correlated, a phenomenon which is widely observed in practice and is to be expected when the economic agents concerned are price takers. For this reason, it is assumed that Laspeyres indexes are always greater than Paasche, although it is recognised that exceptions may occur very occasionally.

29. An inflation index has been defined earlier as a price index that measures the change between two periods in the total value of a fixed basket

of goods and services. On the other hand, as also noted earlier, a CoL index measures the change in value between two baskets of goods and services whose quantities are typically not the same but are just sufficient to keep the consumer on the same indifference curve.

30. An inflation index may appear to lack theoretical backing compared with a CoL index because the CoL index has strong links with economic theories of consumer behaviour and welfare. However, the definitions of both kinds of index invoke the same kind of criterion. A CoL index is defined as the ratio of the minimum expenditures needed to attain a particular indifference curve while a price index is similarly defined as the ratio of the expenditures needed to purchase a particular basket of goods and services. Neither objective, having sufficient resources to maintain a constant level of utility as against having sufficient resources to purchase a constant basket of goods and services, seems intrinsically superior to the other from a theoretical or scientific point of view. Both types of index are arbitrary to the extent that they depend on a choice of base, although the CoL index has the advantage that it is likely to be much less sensitive to the choice of base. For example, when preferences are homothetic, the CoL indexes based on the first and second periods coincide, whereas the Laspeyres and Paasche indexes may diverge considerably if relative prices change substantially.

#### **'Bias' in CPIs as measures of changes in the cost of living**

31. Consumer price indexes (CPIs) in many countries consist of time series of Laspeyres indexes on a fixed base. They may be expected to record larger increases than the corresponding CoL indexes because of the substitution effect, especially the further away the observation period is from the fixed base period. This is often interpreted as showing that such CPIs are subject to an upward "bias", which is true when the official purpose of the CPI is to measure changes in the cost of living. Discussions of such potential bias have received a lot of attention recently, especially in the United States as a result of the 'Boskin' report<sup>13</sup>.

32. However, a price index which uses a fixed basket of goods and services does not have to be greater than the corresponding CoL index. The direction of 'bias' depends on whether the base period precedes or follows the observation period. Suppose, for example, 1990 is selected as the base year for a series of price indexes extending from 1985 to 1995. The price indexes for the years before 1990 consist of a series of Paasche indexes which may be expected to increase more slowly than the corresponding CoL indexes, i.e., to have a downward bias as estimates of changes in the cost of living. On the other hand, those for the years after 1990 consist of Laspeyres indexes which may be expected to rise faster than the CoL indexes and have an upward bias. The direction of bias as compared with a CoL index on the same base year is different on either side of the base year. The fact that many CPIs are presumed to have an upward bias reflects the practical point that, in general, the base year for a CPI usually precedes the observation period.

### Time series of inflation indexes

33. One attraction of an inflation index is held to be that it is a 'pure price index which moves only in response to actual price changes and is not affected by substitutions in response to relative price changes or changes in the availability of goods and services.

34. However, the rate of inflation over a span of years as measured by a pure, fixed weight price index is, to some extent, at the discretion of the compiler of the index because it is affected by the choice of base year. The rate can be varied arbitrarily by moving the base year backwards or forwards. Moving the base forwards will tend to reduce the rate of inflation, while moving it backwards will tend to increase it. The rate of increase in a CoL index, on the other hand, cannot usually be manipulated to nearly the same extent because it is much less sensitive to the choice of base year. As compared with the Col index, when the first year of a sequence of years is used as the base, the rate of inflation as measured by a fixed weight price index has an upward bias which tends to get progressively larger we move from the first to the last year. Conversely, with the last year as the base, the rate of inflation has a downward bias which tends to get progressively smaller as we move away from the first to the last year. <sup>14</sup>

35. When inflation has to be measured over a specified sequence of years, such as a decade, a pragmatic solution to the problems raised above would be to take the middle year as the base year. This can be justified on the grounds that the basket of goods and services purchased in the middle year is likely to be much more **representative** of the pattern of consumption over the decade as a whole than baskets purchased in either the first or the last years.

36. Moreover, choosing a more representative basket will also tend to reduce, or even eliminate, any bias in the rate of inflation over the decade as a whole as compared with the increase in the CoL index. As the base year is gradually moved forwards from the first year towards the last year, not only is the rate of inflation over the decade reduced, as already noted, but also the upward bias. At some point along the line, the bias must switch from being positive to negative as Laspeyres indexes are gradually replaced by Paasche indexes. Choosing a middle year just before or after the switch occurs as the base year will therefore yield a measure of inflation over the decade as a whole which is almost the same as the increase in the CoL index.

### CoL indexes and equivalent inflation indexes

37. As already noted, it is not so easy to grasp intuitively exactly what a CoL index measures when it is defined as the ratio of the expenditures on two **different** baskets, one actual and the other hypothetical. When the two years being compared are far apart, the above reasoning suggests a fixed weight price index using the weights of some intermediate year may give a rough approximation to the CoL index, assuming there is a fairly smooth transition between the first and the last years in the quantities of good and services consumed. More generally, it may be inferred that there must always be some fixed weight price index whose weights are intermediate between those of the

Laspeyres and Paasche index and whose value is equal to the corresponding CoL index, whether the two years are far apart or close together. Such an index can be derived as follows.

38. First, the CoL index, like any other index, can be expressed as a weighted average, or linear combination, of the Laspeyres and Paasche indexes.

$$\text{Write } \text{CoL} = wL_p + (1 - w)P_p$$

where  $L_p$  and  $P_p$  are the Laspeyres and Paasche price indexes. When the CoL lies between the Laspeyres and Paasche indexes  $w$  is less than unity. It is easily calculated when the values of all three indexes are known. It might be thought that a fixed weighted price index whose quantity weights for the **individual** commodities consist of linear combinations, using  $w$  and  $(1 - w)$ , of the individual quantities in the Laspeyres and Paasche indexes, would equal the CoL index, but this is not the case. It is necessary first to adjust for any general increase, or decrease, in the quantities between the first and second periods. If the quantities of one period are systematically higher than the other they would tend to dominate the average weights so that the two periods would not be treated equally or symmetrically. A correction is needed whereby all the quantities in one or other period are uniformly scaled up or down to ensure that the general levels of the quantities are the same in both periods. A formal proof is given in the Annex.

39. One way to do this is to divide all the individual quantities in the second period by the Laspeyres quantity index. Provided this scaling adjustment is applied first, the price index whose quantity weights consist of linear combinations of the actual quantities in the Laspeyres price index and the scaled quantities of the Paasche index can be shown to be identical with the CoL index. In other words, the required quantity vector to be used in the price index consists of a linear combination of the vector of actual quantities in the first period and the vector of scaled quantities in the second period.

40. Scaling all the quantities of the second period has no effect on the Paasche price index, but it does affect the quantities formed as linear combinations of the individual quantities in the two periods. An alternative scaling procedure would be to multiply all the quantities in the first period by the Laspeyres quantity index and to take a linear combination of the scaled quantities in the first period and the actual quantities of the second period. The resulting price index also coincides with the CoL. It may also be noted that an equivalent fixed weight price index can be calculated for any index and not just a CoL. For example, equivalent indexes could be calculated for superlative indexes such as the Fisher and Tornqvist indexes, if desired.

41. Thus, it is not merely possible to find a fixed weight price index whose value is identical with that of the CoL index but also one whose quantity weights are related in a simple and meaningful way to those of the Laspeyres and Paasche price indexes. Notwithstanding the fact that the CoL is **defined** as the ratio of the values of two different sets of goods and

services, one actual and one hypothetical, it is possible to find an **equivalent** price index which measures the change in the value of a fixed basket of goods and services. Moreover, the relevant basket, which consists of linear combinations of the quantities in the two periods (after scaling one or other), is obviously **intermediate** between the Laspeyres and Paasche baskets. As the Laspeyres basket is gradually transformed into the Paasche basket with the passage of time, a basket very similar to the intermediate basket is likely to be purchased at some point along the line if the transition from the Laspeyres to the Paasche baskets is fairly smooth. For this reason, the use of a middle year in a long sequence of years as the base year for a fixed weight price index any lead to a measured rate of inflation over the sequence as a whole which is not very different from the increase in the CoL.

42. The equivalent fixed weight price index does not provide an alternative way of estimating the CoL index, because it is necessary to know the value of the CoL in order to be able to calculate the value of k in the first place. However, it does provide additional insights into what a CoL measures in practice. Although a CoL is intended to measure the change in the cost of maintaining a given level of utility, or standard of living, after allowing for substitutions in response to changes in relative prices, it can nevertheless also be interpreted as measuring the change in the value of a fixed basket of good and services which is a particular blend of the baskets in the two periods being compared.

#### **Symmetric indexes**

43. An index number compares two periods. Price indexes that use baskets of goods and services consisting of linear combinations of the quantities in both periods as weights utilise more relevant information than indexes that use the quantities of only one or other of the periods. Such baskets are also more representative of purchases made throughout the two periods covered by an index. Thus, the equivalent price index to a CoL does not merely measure the change in the cost of living. It measures the change in the value of a representative basket of goods and services and must therefore be treated as a good measure of inflation in its own right on which it may be difficult to improve significantly. It is obviously superior as an inflation index to the Laspeyres or Paasche indexes, at least conceptually.

44. Laspeyres and Paasche price indexes are biased compared with the CoL. They can also be interpreted as being biased on different grounds because they use quantity weights that are themselves completely biased in favour of one or other of the periods being compared. An objective measure of inflation ought to treat both periods compared symmetrically. One such measure would be a price index which uses a simple average of the quantities in both periods as weights, but only **after** scaling the quantities in one or other periods to make sure that the quantities in one of the periods do not dominate the index.<sup>15</sup> This index would be equal to a simple arithmetic average of the Laspeyres and Paasche price indexes.

45. Other more familiar examples of symmetric price indices are the Fisher and Tornqvist indexes, both of which, significantly, are also superlative

indexes. For these indexes there is no need to scale the quantities in one or other period to ensure the periods are treated symmetrically. Both the Laspeyres and the Paasche price indexes which make up the Fisher are themselves invariant to scaling.

46. If the sole objective were to have a symmetric index which provides an objective measure of inflation, there might not be much to choose between an arithmetic and a geometric average of the Laspeyres and Paasche price indexes. However, taking other criteria into account, the Fisher index has to be the preferred measure because of its other decisive advantages, such as satisfying the factor and time reversal tests. There has, of course, to be some fixed weight price index which is equivalent to the Fisher, and it can be inferred that the relevant basket of goods and services would be a fairly even blend of the first and second period baskets (with  $k$  just less than a half).

47. Thus, the Fisher emerges as a good measure of inflation, while it is also a superlative index which may be expected to approximate to the CoL. The Fisher is not alone, however, as some other symmetric indexes may also perform well both as measures of inflation and proxies for the CoL. It must be concluded that there is no conflict between measuring inflation and measuring changes in the cost of living, despite the widespread impression fixed weight indexes must be biased. Fixed weight indexes may be biased, but only when they use biased weights that are unrepresentative of the whole time span covered by the index. Such indexes are equally unsatisfactory as measures of inflation. Unfortunately, many fixed weight indexes that are calculated in practice use weights that are not merely biased but seriously biased.

#### **Chain price indexes**

48. When indexes have to be compiled for a sequence of years, the repeated use of the fixed weights of the some selected base year is bound to become increasingly unsatisfactory as the sequence gets longer and the base year becomes more remote. While the use of a middle year as base may help to reduce biases over the sequence as a whole, it is not practical when the sequence gets longer with the passage of time and the middle year itself is continually shifting forwards. In any case, the weights of some past year become irrelevant when interest is mainly focussed on recent movements as distinct from comparisons with some point in the past. In order to have relevant and representative weights it is necessary to update them frequently and to measure price changes over short spans of time. In other words, a chain index should be compiled.

49. The basic arguments in favour of chaining consumer price indexes may be summarised as follows. Provided the relative quantities and prices of most of the goods and services covered by a CPI do not fluctuate much, as seems plausible for annual data on household consumption, the gap between the Laspeyres and the Paasche price indexes will tend to be reduced by rebasing annually. All measures of inflation or the cost of living tend to be more closely bunched together so that the measured rate of inflation become less sensitive to the choice of index number. Relative prices do not have time to

change much between two consecutive years and chain Laspeyres and Paasche indexes are less susceptible to bias than indexes which attempt direct comparisons between years that are far apart.

50. The overlap between the sets of goods and services available in two years tends to be greatest when they are consecutive so that it is possible to make the maximum use of the price and quantity information available in each and every year when compiling an annual chain index. Relative prices and patterns of consumption also do not change much from year to year so that each link in the chain consists of a robust index. On the other hand, index numbers which attempt to compare directly two years that are far apart are likely to have to omit much of the price and quantity information in each of the years because many of the goods and services are only to be found in one or other of the two years. The quality and reliability of such indexes inevitably suffer from the poor overlap between sets of goods and services available in the two years. There will also tend to be a big divergence between the Laspeyres and Paasche indexes which makes the comparison less meaningful. In general, direct index number comparisons should be avoided between periods in which patterns of consumption are very different. It is sometimes argued that while chain indexes clearly provide good measures of short term movements between consecutive years, they are not so good at measuring changes between years that are far apart. It is, of course, intrinsically much more difficult to make meaningful comparisons between years that are far apart than close together whatever method is adopted, but the above arguments suggest that chain indexes are likely to yield indexes that are actually superior to indexes that attempt to make a direct comparison between years that are far apart.

51. Chain indexes are completely objective in the sense that they are not dependent on the arbitrary choice of a base year. When there is no base year there is also never any necessity to make revisions to previous estimates as a result of having to update the base year periodically.

52. Finally, chaining reduces the scope for the various kinds of bias discussed in this paper. For convenience, most chain index tend to use Laspeyres type indexes because it is easier and quicker to obtain the requisite information on weights for the earlier of each pair of years, especially for the most recent years. However, the gap between the Laspeyres and the Paasche indexes tends to be very small for each year to year measure, especially for a series such as consumers' expenditures whose commodity composition tends to be stable and slow moving. It may make relatively little difference whether a Laspeyres or a Paasche chain index is used, and this is precisely the advantage gained by chaining. When the gap between the chain Laspeyres and Paasche is small for each link in the chain, either chain index may be expected to track the chain CoL fairly closely.

53. When viewed as a measure of inflation a chain index is also bound to perform well. The bias involved in using weights based on only one or the two periods being compared must be very small when the two periods are consecutive. The weights are not likely to differ much between two consecutive years, especially for a series such as consumers' expenditure, so that either year's weights will be fairly representative of the pattern of

consumption over the two year period as a whole.

54. Of course, data permitting, the best solution of all is use to use a chain Fisher. This must provide the best practical measure of both inflation and changes in the cost of living, and can be interpreted indifferently either way.

## Endnotes

- <sup>1</sup> Advisory Commission To Study The Consumer Price Index (Boskin Report), 1996, Toward a More Accurate Measure of the Cost of Living, Final Report, Washington, December 4, 1996.
- <sup>2</sup> R. Dornbusch and S. Fischer (1994): Macroeconomics , p. 10.
- <sup>3</sup> D. Laidler and M. Parkin, 'Inflation: a survey ', Economic Journal, 85, Dec. 1975, p. 741.
- <sup>4</sup> Franklin M. Fisher and Karl Shell, 'The Economic Theory of Price Indices ', Academic Press. New York and London, 1972, p.ix.
- <sup>5</sup> Robert A. Pollak, 'The Theory of the Cost-of-Living Index ', Oxford University Press. New York and Oxford, 1989, p.6.
- <sup>6</sup> See Pollak, *op. cit.*, pp. 18-20. Preferences are homothetic when each indifference curve has the same 'shape ': that is, each curve is a uniform enlargement, or contraction, of other curves. Pollak remarks: "These results are important not because we believe that people 's indifference maps are homothetic but because we believe they are not." This "implies that the cost-of-living index depends on the choice of the base level of expenditure." Pollak, p. 20.
- <sup>7</sup> It is even possible that both set of quantities would be hypothetical, but they would still be different from each other.
- <sup>8</sup> For further explanation , and proofs, of the statements made in this paragraph, the reader is referred to Pollak pp. 10 to 20.
- <sup>9</sup> See W. E. Diewert, 'Exact and Superlative Index Numbers ', Journal of Econometrics, 4, 1976, pp. 115-145.
- <sup>10</sup> They could diverge even if preferences were homothetic both before and after the change in tastes.
- <sup>11</sup> For example, both the price and quantity of beef bought by consumers in Europe fell as a result of consumers becoming aware of the possibility of being infected by BSE (or 'mad cow ' disease).
- <sup>12</sup> See, for example, Fisher and Shell, *op. cit.*, pp. 22-26.
- <sup>13</sup> Advisory Commission To Study The Consumer Price Index (Boskin Report), 1996, Toward a More Accurate Measure of the Cost of Living, Final Report, Washington, December 4, 1996. The Boskin Report identified other sources of bias as well as that arising from the substitution effect, in particular insufficient allowance for improvements in quality, a practical deficiency which would affect any kind of index based on such deficient data..
- <sup>14</sup> The statements in this paragraph apply equally, *mutatis mutandis*, to the measurement of economic growth using fixed weight volume indexes.
- <sup>15</sup> Because of the scaling, this index is not quite the same as that proposed by Marshall which uses a simple average of the actual quantities in both periods.

**ANNEX**

**A Fixed Weighted Index that is Equivalent to a COL Index**

Laspeyres price index  $L_p = \sum (p_t q_o) / \sum (p_o q_o)$

Paasche price index  $P_p = \sum (p_t q_t) / \sum (p_o q_t)$

Laspeyres quantity index  $L_q = \sum (p_o q_t) / \sum (p_o q_o)$

The objective is to find Q's

such that 
$$\sum (p_t Q) / \sum (p_o Q) = COL$$

Solve  $wL_p + (1-w)P_p = COL \quad \text{for } w$

Write 
$$\bar{q}_t = q_t / L_q$$

Then, 
$$Q = wq_o + (1-w)\bar{q}_t$$

Proof: 
$$\sum (p_o Q) = w \sum (p_o q_o) + (1-w) \sum (p_o q_t) / L_q$$

$$= \sum (p_o q_o)$$

$$\sum (p_t Q) = w \sum (p_t q_o) + (1-w) \sum (p_t q_t) / L_q$$

$$= w \sum (p_t q_o) + (1-w) P_p \sum (p_o q_o)$$

$$\sum (p_t Q) / \sum (p_o Q) = wL_p + (1-w)P_p$$

= COL

Q.E.D.