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Presentation of results of international comparisons

Some considerations about the scaling procedures

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The present note refers mainly to the procedure of **scaling of original aggregated results** (first of all, PPPs) used by Eurostat for the presentation. However general considerations given here are valid, in principle, for each multilateral comparison.

I. Scaling procedure actually applied by Eurostat

A square matrix of transitive PPPs „Country A / Country B“ (for each pair of countries) is obtained within the EKS-procedure for each heading. Due to the transitivity of EKS-PPPs It is possible to use just one row or one column only for practical purposes, i.e. EKS-PPPs to any country selected as base (this operation does not change relations between countries).

However, it is not always desirable to use a concrete country as base due to bureaucratic (political) and also practical reasons (e.g. reference PPPs for missing PPPs for some countries). Therefore Eurostat uses an additional operation. The original EKS-PPPs are transformed in a standardized form to the base „Whole region (EU15) = 1“, i.e. an imaginary "average" currency is selected as a numéraire, so called, **PPS** = Purchasing Power Standard.

The PPP between the currency of the given country **J** and a standard of reference (PPS) is calculated as the non-weighted geometric average of all PPPs for given country **J** with all „N“ EU countries (at the moment, N=15): PPP „J/Country 1“, PPP „J/Country 2“, ... , PPP „J/Country N“ (incl. PPP „J/J“ = 1). More concretely, the PPP to PPS is calculated by following formula:

$$(1) \quad \text{PPP „J/PPS“} = \prod_{i=1}^N (\text{PPP}^{j/i})^{1/N}; \quad j=1, 2, \dots, N$$

where

PPP „J/PPS“ - purchasing power parity between currency of country J and PPS¹,

PPP^{j/i} - purchasing power parity between currencies of countries J and I,

N - no. of EU countries (at the moment - 15).

The product of all PPPs „J/PPS“ obtained by formula (1) is equal to **1**.

In practice, Eurostat uses a slightly modified procedure. However, the modification concerns only the sequence of calculation but not the results as such. The following steps are carried out:

- a) a row of EKS-PPPs for selected base country (usually Germany, i.e. PPP „Country J/DM“) is the starting point;
- b) calculation of an unweighted geometric mean from PPPs for all EU countries „Country J/ DM“: PPPs „BF/DM“, „FF/DM“, „Lit/DM“, etc.)“ in this row (incl. PPP „DM/DM“ = 1), i.e. a PPP „Average imaginary currency (PPS)/DM“
- c) calculation of the standardized PPPs by formula:

¹ These PPP „J/PPS“ are named „unscaled PPP“ in the Eurostat terminology but it seems it is better to use the term „standartized“ PPPs.

$$(2) \quad \text{PPP „J/PPS“} = \text{PPP „J/DM“} : \text{PPP „PPS/DM“}.$$

Obviously, the results calculated by formula (1) are identical to the results calculated by formula (2).

The presentation of PPPs in such standardized form is difficult for understanding of user who wants to see a more familiar numéraire. Therefore the Eurostat scales the original PPPs (EU15=1) to EURO in a special way: the **total** (the sum) EU15 expenditure **for each heading** (analytical category) **in EURO** terms is the same as the **total** (the sum) EU15 expenditure **in PPP** terms.²

It can be assumed that the scaling in the form „Sum in EURO = Sum in PPS“ emanates from former Eurostat comparisons (1975, 1980, 1985) which used the **additive** multilateral aggregation methods (Gerardi method for 1975 and Geary-Khamis method for 1980 and 1985). It should be mentioned that scaling was used here at the GDP level only. For additive methods such scaling was logical.

From 1990 Eurostat/OECD use the non-additive EKS-method. The non-additive EKS-PPPs are calculated for each heading separately. In accordance with formal similarity of procedures the Eurostat uses the scaling procedure also separately for each heading. Nevertheless the mechanical application of the scaling procedure used within additive methods to the non-additive results leads to some distortions.

Of course, the separate scaling for each heading currently used by Eurostat does not change volume indices and PPPs between countries but the scaling procedure used for each heading separately changes the structure of PPPs (relations between PPPs) within one country).

An example from the EU'96 comparison for rents (see Eurostat paper „Rents“ presented on the respective PPP meeting - July '98) is given below. A calculation of PPP structures for two basic headings "Actual rent in tenant flats" and "Actual rent in tenant houses" was carried out on the basis of data from the mentioned paper (see Table 1).

It can be seen that the structures of PPPs in the original terms and in scaled terms respectively are different (sometimes substantially) although, formally, the base is the same: „Whole region (EU15) =1“. Moreover, several results are even very surprising. As price data for the "Tenant house" were not available for Luxembourg (**LUX**), Portugal (**PORT**) and Sweden (**SW**), the reference PPPs from the "Tenant flats" were used, i.e. the relations between PPPs for these basic headings should be 1 for these countries but after the scaling to ECU the relation between these two PPPs are changed (difference about 20%). Obviously the original (unscaled) PPPs reflect the reality much better than PPPs scaled to ECU in the Eurostat way. But the results of the EU comparison are presented mainly in scaled form and the users analyse, firstly (or, at all, only), the scaled PPPs.³

² The OECD scales the original PPPs to USD in an analogous way.

³ Both sets of PPPs (standardized=unscaled and scaled) were presented in the Eurostat paper „Rents“ but only the scaled PPPs are presented in the official publications at the level of analytical categories. Therefore the users have no possibility to analyse the „true“ original structure of PPPs.

The reason for the changes in the PPP structure after scaling can be explained in the example given below.

Let us assume PPPs (to PPS) in a standardized form (EU15=1 or the product of exchange rates to EURO is equal to the product of the PPPs) and respective nominal values in the national currencies (NV) for two headings (I and II).

The scaling of PPPs for heading I to EURO is achieved in the following way:

The total EU15 expenditure for this heading in EURO terms must be the same as total EU15 expenditure in PPP terms, i.e. we must find (calculate) a coefficient K^I that holds this equivalence:

$$(3) \quad K^I * \sum_j^{15} NV_j^I / PPP_j^I = \sum_j^{15} NV_j^I / XR_j,$$

where

NV_j^I , PPP_j^I - nominal value and PPP to PPS for heading I of country „J“,
 XR_j - exchange rate (EURO=1) - for country „J“.

The term (3) can be transformed:

$$(4) \quad K^I = \frac{\sum_j^{15} NV_j^I / XR_j}{\sum_j^{15} NV_j^I / PPP_j^I} = \frac{\sum_j^{15} (NV_j^I / PPP_j^I) * (PPP_j^I / XR_j)}{\sum_j^{15} NV_j^I / PPP_j^I} =$$

$$= \frac{\sum_j^{15} PLI_j^I * RV_j^I}{\sum_j^{15} RV_j^I} = \sum_j^{15} PLI_j^I * ShRV_j^I,$$

where

RV_j^I - real value (in PPP terms) and PLI_j^I - a specific comparative price level index (PPP/XR) for heading „I“ of country „J“;

$ShRV_j^I$ - share of country J in Total Real value for the given heading I
 $(\sum_j^{15} ShRV_j^I = 1)$.

The term (4) shows that the coefficient for the scaling (and the respective scaled PPPs) depends on two components: countries' price level indices and countries' shares in Total Real value for the given heading. Accordingly, the scaling coefficient (4) depends on the size of countries, i.e. big (according to Real Values) countries have relatively more impact on the scaling coefficient.

The coefficient K^{II} for the heading „II“ can be obtained in the same way:

$$\sum_j^{15} PLI_j^{II} * RV_j^{II}$$

$$(5) \quad K^{II} = \frac{\sum_j^{15} PLI_j^{II} * ShRV_j^{II}}{\sum_j^{15} RV_j^{II}}$$

It is clear, the scaling coefficient K^I and the scaling coefficient K^{II} are not equal in general. Therefore the scaling procedure applied by Eurostat changes the structure of PPPs (fortunately, the scaling do not change volume indices and PPPs between countries).

There is non-consistency between the presentation of PPPs at the BH level and the PPPs at the aggregated levels. The aggregated PPPs are presented in a weighted form described above and the BH-PPPs are presented in an unweighted form: the product of the EU15 PPPs ("NC / PPS") within a BH is equal to the product of the EU15 exchange rates (NC to EURO).

Additionally, the scaling procedure with the country weights leads to an interesting paradoxical (at the 1st look) collateral effect. The initial expenditure data from the countries are corrected very oft in the following form: the total nominal value of expenditure for a heading is changed for all or some countries but the shares of BHs within this Heading are unchanged (the BH-PPPs are, as rule, also unchanged). This situation produces the following effect:

- Aggregated PPPs between the countries depends on the structure of expenditure but not on the values itself => therefore the PPPs between countries are not changed if the structures are unchanged. However the numerical values of the aggregated PPPs are presented in the scaling form (Sum EU15 in PPS = Sum EU15 in Euro - therefore PLI is always equal to 100 for EU15). It means if nominal values were changed then the scaled PPP are changed also even without any changes in price data. This lead to a formal equal changes for all EU 15 (PPP of all EU countries are multiplied by an equal coefficient because nominal expenditure data were changed) during the presentation of the PPPs in a scaled form.

So, the situation is paradoxical: there were not any changes in price data, the respective original inter-country PPPs were unchanged but the scaled PPPs and respectively the price level indices were changed for all countries in the same proportion, i.e. all countries were simultaneously more expensive (or cheaper) without any price changes. In which way this could occur!?

This paradox can be explained much easier in a mathematical form.

Let us assume that PPP_j^{Or} – is the original (non-scaled) PPP of country „J“ in a standardized form (e.g. the product of exchange rates to Euro is equal to the product of the PPPs). The scaled PPPs are obtained by the dividing of original PPPs by a scaling coefficient K (see formulas 4 and 5):

$$(6) \quad PPP_j^{Sc} = PPP_j^{Or} / K = = PPP_j^{Or} / \sum_j^{15} PLI_j^{Or} * ShRV_j.$$

So, the price level index (PLI = PPP / XR) for a Heading calculated on the basis of scaled PPP can be presented in the following form:

$$(7) \text{PLI}_j^{\text{Sc}} = \text{PPP}_j^{\text{Sc}} / \text{XR}_j = (\text{PPP}_j^{\text{Or}} / \text{XR}_j) / \sum_j^{15} \text{PLI}_j^{\text{Or}} * \text{ShRV}_j$$

$$= \text{PLI}_j^{\text{Or}} / \sum_j^{15} \text{PLI}_j^{\text{Or}} * \text{ShRV}_j ,$$

i.e. the original PLI divided by an average weighted original PLI for EU15 (the shares within the Real Value-Total are used as the weights).

The term (7) explains the paradoxical situation: the changes of PLI without any changes in price data and respectively in the original PPPs. This effect is due to changes of nominal values of expenditure for the countries – if the nominal expenditure change for different countries not in the same proportion then the average PLI change and all respective country PLIs measured relatively this average change also but in the same proportion for all countries (coefficient K).

This effect exists also for PLI measured for a group of the countries, e.g. EURO12 as an area. The PLI for a group of countries is calculated by the formula:

$$(8) \text{PLI} = \frac{\text{Sum [NV(J) / XR "NC(J)/Euro"]}}{\text{Sum [NV(J) / PPP "NC(J)/PPS"]}}$$

The term (8) shows that the group PLI depends not only on scaled PPPs but also on nominal values of the countries. These nominal values can be changed in a non-equal proportion for Euro11 countries. Therefore the group PLI for Euro12 can be changed even if the country's PPP are the same. In reality the situation is even more complicated because the scaling changes the numerical values of PPPs and respectively the group PLI.

Sometimes the organizers of international comparison use within the scaling procedure comparison as a numeraire the currency of a country which does not participate in the given comparison. The willingness to have an usual currency (e.g. USD) as numeraire is understandable but this can lead to many deep irritations.

For example, the African results for 1993 were presented in a scaled form with USD as numeraire although the USA did not participate in this comparison, ie. so-called nominal USD were used. In effect, these results are not comparable with data in real USD from earlier phases of ICP. It means the 1993 scaled African results in nominal USD can be used for the internal comparisons in real terms between African countries but not between any African country and USA. Unfortunately, there was a clear indication on this circumstance in the respective Eurostat publication concerning the 1993 African comparison and this led to many irritations by the users. Some users had even the opinion that the results of the 1993 African comparison are fully false because they are very low and non-fully inconsistent with the results from former rounds of the ICP. Obviously, it is not so but the applicability of these scaled results is limited by the African region only. There is no a direct possibility to evaluate the 1993 African results in the real USD due to a lack of any link: a comparison between any African country and USA (or at least an OECD country) should be done for this purpose.

The examples given above show clearly that it is difficult to obtain the results of a multilateral comparison but it is also not easy to present the obtained results. The different scaling procedures produce the different visible results and this can lead to different analytical conclusions and the interpretations of the results.

This problem has a practical importance in several aspects:

- Establishing of the Classifications

For example, the new COICOP for PPP purpose was discussed during the respective Eurostat PPP meeting (13-14.07.98). One question was the combination of rents in flats and houses in one basic heading: **CH** was against with the argumentation that PPPs for CH are very different (2.59 and 4.21 in scaled form, ratio = 1.62), also the unscaled PPPs are not very close to each other (0.19 and 0.25, ratio=1.36) but here the difference is not so high.

- Measuring of the similarities of the country's price structures

The measuring of the similarities of the country's price structures is done oft on the basis of sets of PPPs from international comparisons. It was demonstrated above, that the present scaling procedures change the original relations between the PPPs and this can lead, probably, to some distortions of original price structures.

Therefore some possible modifications of the Eurostat scaling procedures should be discussed.

II. Some proposals for the modifications of scaling procedures

Probably, it is desirable to continue the scaling to EURO as an appropriate standard of reference for users. However this should be done in an other form than before, ie. without the distortion of the original PPP structure.

The simplest way is the **approach used in the Quaranta-tables**:

**The product of the PPPs must be equal to
the product of the exchange rates (to EURO)**

It means that the standardized PPPs (as ratios to the unweighted geometric mean) must be multiplied by a general scaling coefficient which is an unweighted geometric mean for the exchange rates (to EURO) for 15 EU-Members:

$$(9) \quad K^{\text{Gen.}} = (\prod_j^{15} XR_j)^{1/15};$$

In this case the scaling coefficient (9) will be the same for all headings (analytical categories) and, in effect, the PPP structure will be held in original form but simultaneously the PPPs are presented in a form more appropriate for users (scaled to EURO / ECU - see Table 2). It seems that such approach is consistent with the general concept of the EKS method: the results are independent from the sizes of participating countries (big and small ones are „equal“).

Should it be really necessary to keep the equivalence in PPS and EURO for GDP (probably, this necessity may have some practical reasons for structural funds) then there **is another way** to scale the original PPPs to EURO, to hold the equivalence in PPS and EURO for GDP with simultaneous preservation of the structure of the original PPPs:

The scaling (the total EU15 expenditure in EURO terms is the same as total EU15 expenditure in PPP terms) is carried out for GDP only. Obtained coefficient K^{GDP} must then be applied to all other headings (analytical categories).

The problem with scaling is especially actual after the introduction of the EURO in the 12 EU countries. It can be believed that the question about the price structures (structures of PPPs) and their changes within EURO12 countries will be very important for the analysis of the impact of the introduction of a common currency. Therefore a correct presentation of PPPs will have a big practical and even political importance in the near future.

Table 1

Present Eurostat version: Sum of EU15 in PPS = Sum of EU15 in EURO

Data from EUR'96	Unscaled PPPs (EU15=1) ^{*)}			PPP's scaled to ECU (PPS=1)		
	Tenant flats	Tenant houses	Ratio (Fl.=1)	Tenant flats	Tenant houses	Ratio (Fl.=1)
DE	0.1707	0.2124	1.244	2.368	3.529	1.490
FR	0.5157	0.5436	1.054	7.173	9.032	1.259
IT	95.99	73.18	0.762	1331.3	1215.9	0.913
NL	0.1274	0.1428	1.121	1.767	2.372	1.342
BE	2.810	3.090	1.100	38.97	51.34	1.317
LX	2.976	2.976	1.000	41.28	49.45	1.198
UK	0.03348	0.03258	0.973	0.4643	0.5413	1.166
IR	0.05245	0.04612	0.879	0.7274	0.7663	1.053
DK	0.4962	0.5446	1.098	6.882	9.049	1.315
GR	14.31	16.31	1.140	198.51	270.93	1.365
ES	6.387	4.883	0.765	88.58	81.13	0.916
PO	4.7490	4.7490	1.000	65.87	78.91	1.198
OS	0.8627	0.9098	1.055	11.97	15.12	1.263
SW	0.7115	0.7115	1.000	9.87	11.82	1.198
SF	0.4782	0.4500	0.941	6.633	7.478	1.127
CH	0.1869	0.2533	1.355	2.592	4.208	1.623
IS	5.255	5.315	1.011	72.88	88.31	1.212
NO	0.6917	0.6189	0.895	9.590	10.280	1.072
PL	0.02003	0.02024	1.010	0.278	0.336	1.210

Table 2

New proposed version: Product of PPPs (PPS=1) of EU15 = Product of XR (ECU=1) of EU15

Data from EUR'96	Unscaled PPPs (EU15=1) ^{*)}			PPP's scaled to ECU (PPS=1)		
	Tenant flats	Tenant houses	Ratio (Fl.=1)	Tenant flats	Tenant houses	Ratio (Fl.=1)
DE	0.1707	0.2124	1.244	2.871	3.573	1.244
FR	0.5157	0.5436	1.054	8.675	9.144	1.054
IT	95.99	73.18	0.762	1614.7	1231.0	0.762
NL	0.1274	0.1428	1.121	2.143	2.402	1.121
BE	2.810	3.090	1.100	47.27	51.98	1.100
LX	2.976	2.976	1.000	50.06	50.06	1.000
UK	0.03348	0.03258	0.973	0.563	0.548	0.973
IR	0.05245	0.04612	0.879	0.882	0.776	0.879
DK	0.4962	0.5446	1.098	8.347	9.161	1.098
GR	14.31	16.31	1.140	240.72	274.36	1.140
ES	6.387	4.883	0.765	107.44	82.14	0.765
PO	4.7490	4.7490	1.000	79.89	79.89	1.000
OS	0.8627	0.9098	1.055	14.51	15.30	1.055
SW	0.7115	0.7115	1.000	11.97	11.97	1.000
SF	0.4782	0.4500	0.941	8.044	7.570	0.941
CH	0.1869	0.2533	1.355	3.144	4.261	1.355
IS	5.255	5.315	1.011	88.40	89.41	1.011
NO	0.6917	0.6189	0.895	11.64	10.41	0.895
PL	0.02003	0.02024	1.010	0.337	0.340	1.010

*) **LX, PO and SW** did not report original price data for "Tenant house", so the reference PPPs from "Tenant flats" were used for this basic headings.