Estimation of the number of irregular foreigners in Poland using non-linear count regression models

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## Outline

## Introduction

## Basic information about the Centre for the Methodology of Population Studies

- dr hab. Marcin Szymkowiak is Deputy Director of the Statistical Office in Poznań, Poland.
- dr Maciej Beręsewicz is head of the Centre for the Methodology of Population Studies at Statistical Office in Poznań, Poland.
- This is a new unit responsible for
- estimation of population size,
- integration of administrative data,
- studying hard-to-reach populations,
- developing tools in open-source languages (such as R, Julia),
- and developing new statistical methodology.


## Acknowledgements

This study is based on the working paper Estimation of the number of irregular foreigners in Poland using non-linear count regression models by Beręsewicz \& Pawlukiewicz (2020) [arXiv:2008.09407]

Since this paper is being considerably revised, the model and the results may change.

## Motivation

- Irregular (undocumented) migration is hard to measure as the underlying population is hard-to-reach.
- Several approaches have been proposed in the literature, which are based for instance on residual, single-source capture-recapture or multiple estimation system methods.
- Majority of methods assumes access and integration of data at the unit-level.
- The proposed approach requires access to aggregated data and is based on a functional form and certain assumptions that can be verified using available data.


## Outline

The model

## The model

- The original model is based on Prof. Li-Chun Zhang's (University of Southampton, University of Oslo, Statistics Netherlands) working paper entitled Developing methods for determining the number of unauthorized foreigners in Norway (2008).
- The author proposes a model that requires only three types of variables:

1. the number of apprehended irregular foreigners (denoted as $m$ ),
2. the number of foreigners who faced criminal charges (denoted as $n$ ),
3. the number of foreigners registered in the central population register (denoted as $N$ ).

## Assumptions

- Let $M_{t}$ be the size of the population of unauthorized resident at time $t$ (e.g. end of the year) - the random variable.
- Let $N_{t}$ be the be the size of the known reference (proxy) population at the same time $t$ - the fixed, known covariate.
- The target parameter is the theoretical size of irregular residents, which is defined as the conditional expectation of $M_{t}$ given $N_{t}$ with respect to $f\left(M_{t} \mid N_{t}\right)$ denoted by

$$
\xi_{t}=\mathbb{E}\left(M_{t} \mid N_{t}\right)
$$

## Assumptions

- As Zhang (2008) notes, the theoretical size is defined as the conditional expectation of the random variable, which makes it possible to get rid of the spurious variation as long as the reference population size is held fixed.
- The purpose of introducing $N_{t}$ is two-fold:

1. it serves as an explanatory variable for the irregular size $M_{t}$,
2. it provides an interpretation of the irregular size $M_{t}$ in analogy to $N_{t}$.

- In this way, the theoretical size is a stable measure of the target variable as variation in $M_{t}$ is linked to that of $N_{t}$.


## Assumptions

- Let $m_{i t}$ be the observed number of irregular foreigners from country $i$ (this may also indicate more detailed populations e.g. sex-age group for a given country).
- Let $n_{i t}$ be the observed number of (legally staying) foreigners from country $i$.
- Let $p_{i t}$ be the probability for an irregular resident to be observed in administrative data (say Border Guards).
- Let

$$
m_{i t} \sim \operatorname{Poisson}\left(\lambda_{i t}\right)
$$

- Let $\lambda_{i t}=\mu_{i t} u_{i t}$, where $\mu_{i}=\mathbb{E}\left(M_{i t} p_{i t} \mid n_{i t}, N_{i t}\right)=\mathbb{E}\left(M_{i t} \mid N_{i t}\right) \cdot \mathbb{E}\left(p_{i t} \mid M_{i t}, n_{i t}, N_{i t}\right)$


## Assumptions

- The final model consists of the following set of equations

$$
\begin{align*}
\xi_{i t} & =\mathbb{E}\left(M_{i t} \mid N_{i t}\right)=N_{i t}^{\alpha} \\
\omega_{i} & =\mathbb{E}\left(p_{i t} \mid M_{i t}, n_{i t}, N_{i t}\right)=\mathbb{E}\left(p_{i t} \mid n_{i t}, N_{i t}\right)=\left(\frac{n_{i t}}{N_{i t}}\right)^{\beta},  \tag{1}\\
u_{i t} & \sim \operatorname{Gamma}(1, \phi),
\end{align*}
$$

- From which we can derive the following relationship for $\mu_{i t}$

$$
\begin{equation*}
\mu_{i}=N_{i}^{\alpha}\left(\frac{n_{i}}{N_{i}}\right)^{\beta} \tag{2}
\end{equation*}
$$

## The target quantity

- We are interested in the target parameter describing the number of irregular residents. Given the above model, the target parameter is defined as

$$
\begin{equation*}
\xi=\sum_{i=1}^{C} E\left(M_{i} \mid N_{i}\right)=\sum_{i=1}^{C} N_{i}^{\alpha}, \tag{3}
\end{equation*}
$$

- and its estimator is given by

$$
\begin{equation*}
\hat{\xi}=\sum_{i=1}^{C} N_{i}^{\hat{\alpha}} \tag{4}
\end{equation*}
$$

where $\hat{\alpha}$ is the estimator of $\alpha$.

## Estimation of the parameters and verification of assumptions

- The parameters are estimated using maximum likelihood (the loglik function, gradient and hessian are provided in the working paper).
- This model can be further extended to account for covariates.
- Assumptions of the model can be verified using the following linearized model

$$
\begin{equation*}
\log \left(\frac{m_{i}}{N_{i}}\right)=(\alpha-1) \log N_{i}+\beta \log \left(\frac{n_{i}}{N_{i}}\right)+\epsilon_{i} \tag{5}
\end{equation*}
$$

- We should expect a negative relationship with $\log N_{i}$ and a positive one with $\log \left(n_{i} / N_{i}\right)$.


## Outline

## Data

## Definitions

- For administrative purposes, Polish authorities (Polish Border Guard, 2020) use the term illegal stay, which is defined as a stay which does not comply with the legal provisions describing the conditions that foreigners must meet in order to enter and stay in the Republic of Poland.
- If a foreigner is found to be staying in Poland illegally, an administrative procedure is initiated whereby the person is obliged to leave the country.


## Data - Border guard data

Tab. 1: The number of irregular foreigners in Poland by place of apprehension and re-apprehension status in 2019

| Half | Same year | Within country | Airports | Ukraine | Russia | Belarus | Total |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| I | No | 3,190 | 710 | 6,879 | 106 | 785 | 11,670 |
| I | Yes | 29 | 1 | 0 | 0 | 0 | 30 |
| II | No | 3,437 | 1,016 | 8,492 | 143 | 1,052 | 14,140 |
| II | Yes | 70 | 0 | 0 | 0 | 0 | 70 |

## Data - Police data

Tab. 2: The number of foreigners in police records by registration type and residence status (registered for temporary stay or permanent residence) in 2019

| Half | Registered | Procedural | Search | Traffic | Criminal | Total |
| :--- | :--- | :--- | ---: | ---: | ---: | ---: |
| I | Yes | 1,499 | 715 | 9,286 | 10 | 11,510 |
| I | No | 4,046 | 6,522 | 2,477 | 16 | 13,061 |
| II | Yes | 2,080 | 878 | 11,988 | 6 | 14,952 |
| II | No | 4,644 | 5,979 | 2,867 | 11 | 13,501 |

## Data - registered population

Tab. 3: The number of foreigners in the PESEL register by registration type at quarter ends in 2019

| As at | No address | Temporary | Permanent | De-registered | Expired | Outside |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 31.03 | 81,202 | 242,318 | 56,476 | 16,158 | 124,368 | 332,256 |
| 30.06 | 107,545 | 249,154 | 57,656 | 16,246 | 157,476 | 383,283 |
| 30.09 | 134,483 | 246,990 | 59,228 | 16,340 | 196,209 | 441,705 |
| 31.12 | 160,868 | 252,245 | 60,440 | 16,386 | 225,690 | 496,374 |

## Data - comparison

Tab. 4: The number of foreigners and countries by data source, sex and period before applying the condition for the model

| Classification |  | Number of foreigners |  | Number of countries |  |
| :--- | :--- | ---: | ---: | ---: | ---: |
| Source | Sex | $1^{\text {st }}$ period | $2^{\text {st }}$ period | $1^{\text {st }}$ period | $2^{\text {st }}$ period |
| PESEL | Total | 232,468 | 234,194 | 151 | 147 |
|  | Women | 137,424 | 137,880 | 145 | 140 |
|  | Men | 95,044 | 96,314 | 127 | 130 |
| Border Guard | Total | 3,187 | 3,435 | 77 | 68 |
|  | Women | 762 | 776 | 40 | 39 |
|  | Men | 2,425 | 2,659 | 72 | 67 |
| Police (all) | Total | 20,138 | 23,330 | 100 | 98 |
|  | Women | 3,017 | 3,079 | 58 | 57 |
|  | Men | 17,121 | 20,251 | 94 | 94 |

## Data - data for the model

- In our study we used Polish data from two halves of 2019 for the foreign population aged 18+.
- In addition, we derived data broken down by sex and economic age group (18-59 and 60+ for women; 18-64 and 65+ for men).
- The PESEL register contained people from 151 and 147 countries in the first and second half of the year, respectively, police data - around 100, and Border Guard records - around 70.
- The model requires that the following conditions hold: $m_{t i j}>0, n_{t i j}>0$ and $n_{t i j} / N_{t i j}<1$, so we created a new dataset that meets these requirements.
- After applying this condition, we received a total of 73 countries (including category other), of which 50 were observed in both periods and 23 only in one ( 65 in the first and 58 in the second half of 2019).


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## Results

## Assumptions



Fig. 1: The relationship between the log of the PESEL population and the log of the BG-to-PESEL counts at the end of first and third quarter of 2019

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## Assumptions



Fig. 2: The relationship between the log of police-to-PESEL counts and the log of BG-to-PESEL counts at the end of first and third quarter of 2019

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## Population size estimation results

Tab. 5: Quality of models used in the study and the estimated population $\hat{\xi}$

| Covariates for $\alpha$ | LogLik | AIC | BIC | $\hat{\xi}$ |
| ---: | :--- | :--- | :--- | ---: |
| At the end of $1^{\text {st }}$ |  |  |  |  |
| quarter 2019 |  |  |  |  |
| No covariates | -733.1 | $1,470.3$ | $1,475.5$ | $24,119.9$ |
| Ukraine | -648.7 | $1,303.5$ | $1,311.3$ | $20,835.8$ |
| Sex | -682.5 | $1,371.0$ | $1,378.8$ | $51,982.8$ |
| Ukraine \& Sex | -630.1 | $1,268.1$ | $1,278.6$ | $34,870.1$ |
| At the end of 3 $3^{\text {rd }}$ quarter of 2019 |  |  |  |  |
| No covariate | -822.2 | $1,648.3$ | $1,653.4$ | $23,582.6$ |
| Ukraine | -735.7 | $1,477.5$ | $1,485.1$ | $21,139.0$ |
| Sex | -742.2 | $1,490.3$ | $1,497.9$ | $65,011.0$ |
| Ukraine \& Sex | -689.8 | $1,387.6$ | $1,397.8$ | $49,080.1$ |

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## Discussion

## Discussion

- In the paper we propose a different approach to estimating the hard-to-reach population of irregular foreigners based on a flexible non-linear count regression model.
- The approach is an alternative to classic capture-recapture methods, which rely on one or multiple sources, and the interpretation of results is more intuitive because the irregular population is conditionally dependent on the regular population.
- The approach only requires administrative data and, as a result, the quality of our estimates depends on the availability of high-quality register-based statistics.
- Selection of data for the model should be strictly connected with the definition of the irregular population used in the study.


## Outline

Literature

Statistical Office

## Literature (selected)

- Beręsewicz, M., Gudaszewski, G., and Szymkowiak, M. (2019). Estymacja liczby cudzoziemców w Polsce z wykorzystaniem metody capture-recapture. Wiadomości Statystyczne. The Polish Statistician, 64(10), 7-35.
- Beręsewicz, M., \& Pawlukiewicz, K. (2020). Estimation of the number of irregular foreigners in Poland using non-linear count regression models. arXiv preprint arXiv:2008.09407.
- Polish Border Guard (2020). Consequences of illegal stay
- Zhang, L.-C. (2008). Developing methods for determining the number of unauthorized foreigners in Norway. Statistics Norway (SSB), Division for Statistical Methods and Standards. www. ssb. no.(accessed July 28, 2008)


## Thank you!

