

# **ROMM Methodology for Microdata Release**

**Daniel Ting and Stephen E. Fienberg  
Carnegie Mellon University**

**Pittsburgh, PA USA**

**and**

**Mario Trottini**

**Universidad de Alicante, Spain**

# Outline

- **Some existing methods**
- **Motivating principles**
- **Random Orthogonal Matrix Masking (ROMM)**
- **Numerical implementation**
- **Risk-Utility tradeoff**
- **Simulation**

# Methods for Microdata

- **Micro data in form of  $n \times k$  data matrix  $x$ .**
- **Examples of methods in literature:**
  - Adding noise
  - “Statistical obfuscation”
  - PRAM
  - Rank swapping
  - Data shuffling
  - Multiple imputation
- **Many of these are examples of matrix masking:**
  - $x \rightarrow y = Ax + B + C$ .

# Motivating Principles

- **Need to allow data analyst to make inferences about parameters of interest in model applicable to original unreleased data.**
- **One way to achieve this is to provide details of the transformation method to allow creation of a usable likelihood function for the true unreleased data, c.f. PRAM.**
- **Another strategy is to preserve essential features of the data as part of the transformation process, e.g., minimal sufficient statistics.**
  - **For multivariate normal data, MSSs are mean and covariance matrix.**

# Random Orthogonal Matrix Masking (ROMM)

1. Generate a random orthogonal matrix,  $t$ , from a distribution  $G$  defined on group of  $n \times n$  orthogonal matrices which keep  $\mathbf{1}_n$  invariant, i.e.,  $t \mathbf{1}_n = \mathbf{1}_n$  where  $\mathbf{1}_n$  is the column vector consisting of  $n$  1's.
2. Apply orthogonal operator,  $t$ , to original data  $x$  to produce perturbed microdata  $y$ :  
 $y = tx$ .
3. Release (a)  $y$ ; (b) information that  $y$  has been obtained applying orthogonal operator randomly generated from distribution  $G$ ; (c) exact distribution  $G$ .

# ROMM Properties

***Theorem 1:*** Let  $\bar{x}$  and  $\Sigma_x$  be the sample mean and sample covariance matrix of the original microdata and let  $\bar{y}$  and  $\Sigma_y$  be the corresponding quantities in the masked microdata produced by ROMM. Then  $\bar{x} = \bar{y}$  and  $\Sigma_x = \Sigma_y$ .

***Theorem 2:*** Let  $M$  be any data masking procedure that generates a random microdata,  $y$ , with the same sample mean and sample covariance matrix as the original microdata. Then  $M$  is a special case of ROMM for a suitable choice of the “parameter”  $G$ .

# Implementation

- **Co-ordinate by co-ordinate:**
  - Add small amount of noise to identity matrix and then make it orthogonal.
    1. Choose a parameter  $\lambda > 0$  corresponding to the magnitude of perturbation.
    2. Draw  $n \times n$  random matrix  $M$  with entries from  $N(0,1)$ .
    3. Put  $P = I + \lambda M$ .
    4. Apply Gram-Schmidt and normalize the columns of  $P$  to obtain an orthonormal matrix  $T$ .
  - It is easy to see that, when  $\lambda = 0$ ,  $T$  is the identity and no perturbation has occurred. When  $\lambda = 1$ ,  $T$  is a draw from the uniform distribution on orthogonal matrices.
- **Block diagonal distributions:**
  - Orthogonal matrices with eigenvalues near 1.

# Example: Boston Housing Data

- Extract of 13 randomly selected observations (from 506) on 4 (of 20) variables:

**RM** average number of rooms per dwelling

**PTRATIO** pupil-teacher ratios per town

**LSTAT** % of lower status population

**MEDV** med. value of owner-occupied housing in \$1000

Obs	RM	PTRATIO	LSTAT	MEDV	Obs	RM	PTRATIO	LSTAT	MEDV
1	6.630	18.5	6.53	26.6	8	6.315	16.6	7.60	22.3
2	5.986	19.1	14.81	21.4	9	6.023	18.4	11.72	19.4
3	5.709	14.7	15.79	19.4	10	6.251	20.2	14.19	19.9
4	5.977	14.7	12.14	23.8	11	5.757	20.2	10.11	15.0
5	6.402	14.7	11.32	22.3	12	5.304	20.2	26.64	10.4
6	6.782	15.2	6.68	32.0	13	6.425	20.2	12.03	16.1
7	6.433	19.1	9.52	24.5					

- Comparison of:
  - ROMM, co-ordinate by co-ordinate, with  $\lambda=1/3$ .
  - Comparison with bias corrected correlated additive noise method of Kim, with  $\sqrt{c}=1/2$ .
- Regression of MEDV on other variables.

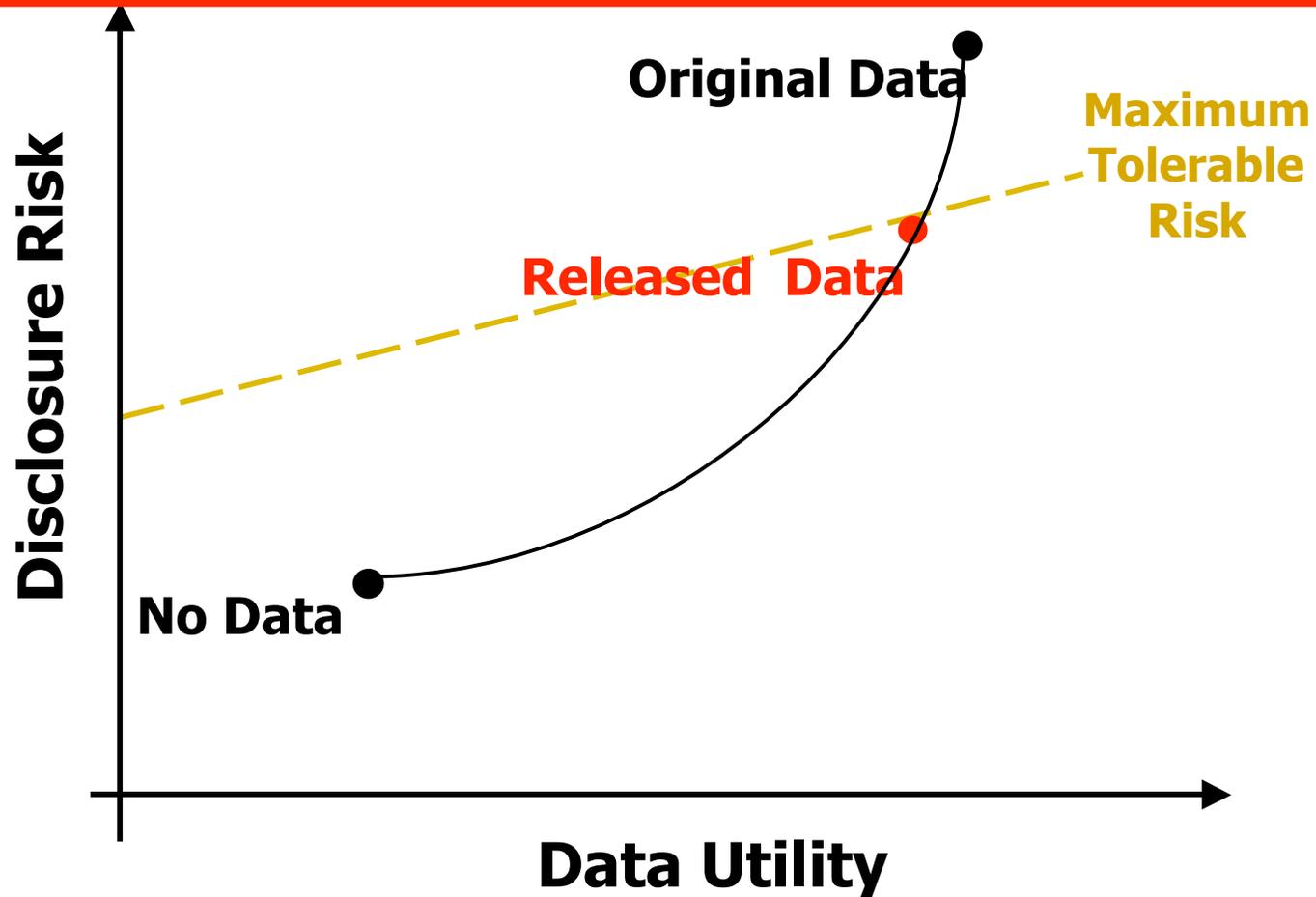
# Example Results

ROMM				Additive Noise			
RM	PTRATIO	LSTAT	MEDV	RM	PTRATIO	LSTAT	MEDV
-0.253	3.725	4.721	-7.881	-0.337	1.509	1.848	-2.173
-0.529	2.006	3.843	-9.182	0.182	2.249	-1.479	-1.238
0.404	0.738	-12.337	5.930	-0.036	2.335	-0.761	-3.027
0.045	0.880	1.249	-0.095	0.235	-0.132	-3.451	0.908
0.223	1.313	-1.086	3.740	-0.057	1.274	3.057	-3.099
-0.183	1.841	0.338	-3.079	-0.324	0.380	1.999	-3.797
-0.269	2.093	1.883	-4.906	-0.281	-1.957	3.303	0.980
0.139	-0.867	0.348	3.018	0.175	-0.046	-0.831	0.765
0.414	0.367	-1.688	0.357	0.053	-1.423	-2.049	1.165
0.212	-3.778	-5.139	4.602	0.086	-0.116	0.367	0.883
-0.437	-3.218	10.437	-2.959	-0.107	0.195	2.658	0.143
0.601	-2.851	-6.872	12.014	0.390	-0.581	-8.411	4.243
-0.457	-2.249	4.302	-2.361	-0.468	0.254	6.544	-5.830

Variiances	RM	PTRATIO	LSTAT	MEDV
ROMM	0.146	5.618	33.307	34.327
AddNoise	0.066	1.641	13.950	7.348

Original Data (ROMM)			Additive Noise		
Variable	Estimate	SE	Variable	Estimate	SE
(intercept)	-5.5641	23.6617	(Intercept)	-2.4696	25.1829
RM	7.4488	3.3663	RM	6.2655	3.5843
PTRATIO	-0.9557	0.3691	PTRATIO	-0.3930	0.4329
LSTAT	-0.1770	0.2741	LSTAT	-0.6780	0.2995

# R-U Confidentiality Map



(Duncan, et al. 2004)

# Risk Utility Tradeoff

- **A rigorous assessment of disclosure risk and utility requires:**
  - **Model for users' behaviors when the output of ROMM is released.**
  - **Assessment of agency uncertainty about this model's inputs (users' targets, prior information, estimation procedure, etc.).**
  - **Formalization of agency's perception of the consequences of data users' actions and of agency's preference structure for consequences of users' actions.**
- **We consider a simplified scenario where:**
  - **Modeling of users and agency's behaviors does not take explicitly into account some relevant aspects of the problem.**
  - **Agency has no uncertainty about the users' model inputs.**

# Technical Details

- **In the paper:**
  - **Utility**
    - **Under normality.**
    - **Under non-normality.**
  - **Disclosure Risk**

# Simulation Setup

- ***Data:*** We used same 13 values from Boston Housing Data in Example.
- ***Assumptions:***
  - Intruder's prior on the original data is uniform.
  - External information available to intruder is values of RM with i.i.d.  $N(0, (0.6)^2)$  error.
  - Agency uses coordinate-by-coordinate approach to perturb data. It releases (a) perturbed data and (b) value of parameter  $\lambda$ .
- ***Method:*** Acceptance-rejection sampling to sample from posterior.

# Resulting Data

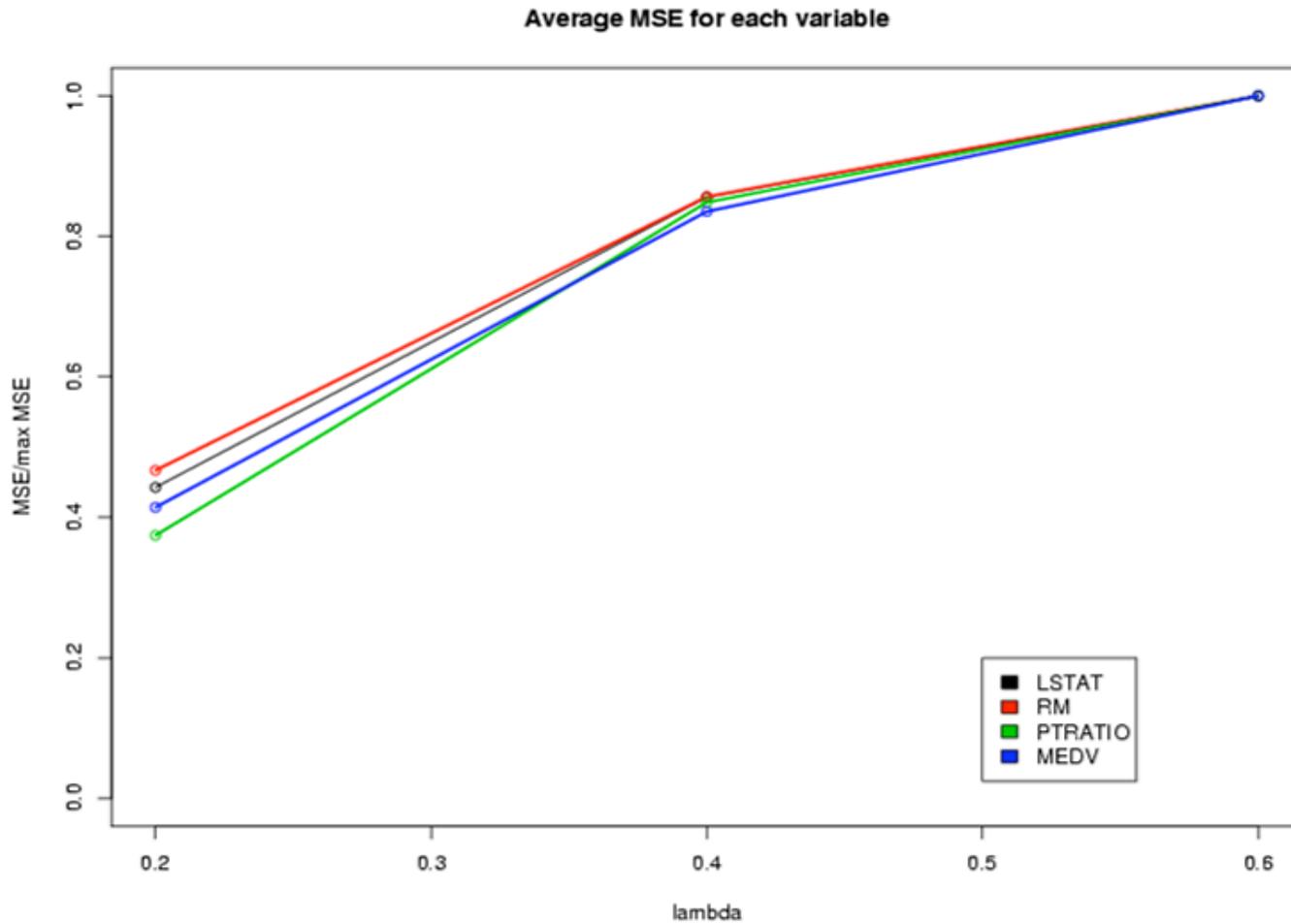
- **For  $\lambda=0.2, 0.4,$  and  $0.6,$  we generated a sequence of  $N=80$  (perturbed data, intruder data) pairs and sampled from the posterior of the original data given each pair.**
- **We then calculated point estimates for each “individual.”**
- **We then looked at the behavior of the distribution of these point estimates which allows us to evaluate risk.**

## Results (small $\lambda$ )

- For small  $\lambda$  (i.e.,  $\lambda = 0.2$ ), the mean point estimates were close to the true values.
- Average MSE of point estimates for confidential variable LSTAT was 7.99 (variance of LSTAT is 27.38).
- Compared to just using the values in the perturbed data, the MSE of the posterior point estimates were substantially reduced
  - Average difference in MSE for LSTAT was 2.18.

# Results (all $\lambda$ )

- For all  $\lambda$ , posterior point estimates shrunk towards the mean for each variable with “shrinkage” increasing with  $\lambda$ .
- In general, MSE of point estimates increased with  $\lambda$  in concave fashion as expected.
- Outliers:
  - 12<sup>th</sup> observation has an unusually high value for LSTAT and posterior point estimates for this tended to be biased towards the mean.
  - MSE of posterior point estimates for this observation was substantially *higher* than estimates from the perturbed data for  $\lambda = 0.2$  and  $0.4$ . For  $\lambda=0.6$ , the MSE was slightly lower.



Plot illustrating the disclosure protection increasing with  $\lambda$  and the marginal protection diminishing.

# Summary

- **Motivating principles**
- **Random Orthogonal Matrix Masking (ROMM)**
  - Preserves means and covariance matrix.
  - Includes “statistical obfuscation” as a special case
- **Numerical implementation**
  - Comparison with additive noise: more variability (protection) but superior regression estimates.
- **Risk-Utility tradeoff**
- **Preliminary Simulation**
  - Can use RU idea to pick a suitable value of  $\lambda$ .

# The End

# Google™ 2084

Google Search

I'm Feeling Lucky

I'm Feeling Paranoid

- Your Brain
- Your Home
- Family
- Friends
- Ex-friends
- Relatives
- Co-workers
- Ex-spouse(s)
- Enemies
- Satellite Photos of People You Want to Spy On
- Satellite Photos of People Spying on You
- ~~Medical Records~~
- Credit Reports
- Tax Records
- Phone Records
- Court Documents
- Other People's Conversations
- Books
- Movies
- TV Shows
- Music
- Pornography
- Your Past
- Your Present
- Your Future