

# Handling of outliers at SFSO

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25. - 27. September 2006

# Introduction

- ▶ Until 1990, robust estimation methods had rarely been applied in public statistics due to technical limits and the complexity of some methods.
- ▶ Outliers were normally treated manually.
- ▶ Theoretical and technical development allowed the implementation of new estimators for detection and treatment of outliers.

## Robust methods applied at SFSO

SFSO developed and/or implemented procedures for several estimators adapted to sampling weights.

- ▶ Trimming one-step estimator for univariate outlier detection and treatment.
- ▶ Woodruff method for estimating the confidence interval of the Median.
- ▶ One-step ratio estimator for outlier detection and treatment (re-weighting).
- ▶  $L_1$ -regression for outlier detection and imputation.
- ▶ Transformed rank correlations estimates used for the definition of a robust Mahalanobis distance. This distance function is used for multivariate outlier detection and for nearest neighbor imputation.

## Surveys where robust methods were applied

- ▶ Survey on housing rents.
- ▶ Earning structure survey.
- ▶ Enterprise census.
- ▶ Environment protection expenditures.
- ▶ Survey on production and value added.
- ▶ Survey on energy consumption.
- ▶ Retail trade statistics.
- ▶ Hospital statistics.
- ▶ Survey on income and expenditure (Household budget survey).

## Experience with robust methods in surveys

- ▶ Robust methods help to limit the influence of outliers.
- ▶ Robust methods must be adapted to sampling.
- ▶ One-step estimators are good approximations of M-estimators.
- ▶ Choice of tuning constants is often difficult (possible bias).
- ▶ The degree of robustification normally must be discussed with subject matter specialists.
- ▶ Outliers must often be checked individually to decide how they should be treated.
- ▶ Robustification should be limited to the most extreme observations; be aware of 'over-robustification'.

- ▶ A few large weights may dominate the estimation. → Analyse the sampling weights (median-dominance).
- ▶ The total weight (robustness weight  $\times$  sampling weight) may become much smaller than 1. → Lower limit for the robustness weights.
- ▶ The definition of an outlier depends on models which must be checked carefully. The model should be adequate for the bulk of the data. → Different models in different sub-populations.



## Annexe: One-step ratio estimator adapted to sampling

- ▶ Adaptation of a ratio estimator.
- ▶ Initial robust estimate of the slope  $\rightarrow$  residuals.
- ▶ Downweight the observations with extreme residuals with a robustness weight (decision based on tuning constant).
- ▶ Robust re-estimation of the slope with sampling weights  $\rightarrow$  one-step estimator of the slope.
- ▶ The one-step estimator can be used as initial estimation of the slope for the next iteration step. Convergence  $\rightarrow$  weighted M-estimator.
- ▶ Mean of the robustness weights should not be much below 1.

Cf. (Hulliger 1995), (Hulliger 1999), (Peters, Renfer, and Hulliger), (Salamin 2005) and (Bendel, Scherer, Salamin, and Glden 2006)

## Annexe: $L_1$ -regression

$L_1$ -regression: least absolute deviation regression (LAD-regression).

Minimize the absolute values of the residuals  $r$  of the linear regression model.

Outlier detection:

- ▶ Lower and upper limits for outlier detection (boxplot):  
 $r_{0.25} \pm 1.5 \times (r_{0.75} - r_{0.25})$ , with  $r_p$  the  $p$ -th quantile of  $r$
- ▶  $y_i > c\hat{\beta}_{L_1}$  flagged as outlier

Imputation:  $\hat{y}_i = x_i^T \hat{\beta}_{L_1}$

Cf. (Oetliker 2002), (Renfer 2006)

## Annexe: Transformed rank correlations (TRC)

- ▶ Robust estimate of center  $m$  and covariance matrix  $S$  based on bivariate Spearman rank correlations.
- ▶ Detection of outliers with Mahalanobis distance.
- ▶ Adaptation to sampling and missingness

Cf. (EUREDIT Project 2004a), (EUREDIT Project 2004b), (Béguin and Hulliger 2004), (Kilchmann 2006).

## Annexe: Woodruff method

Back-transforming a confidence interval on the probability with the inverse of the empirical distribution function.

Cf. (Peters and Hulliger 1996), (Graf 2002).

## Annexe: Median-dominance

- ▶ Minimal part of the largest weights accounting for more than 50% of the total weights.
- ▶ The lower  $\text{dom}_{0.5}$  is the more unbalanced are the weights.
- ▶ E.g. if  $\text{dom}_{0.5} = 30\%$  then the empirical breakdown point of the weighted median is 30% instead of the 50% of an unweighted median.
- ▶ Winsorization of the weights may resolve the problem.
- ▶ The use of the unweighted median in the one-step ratio estimator may be an alternative.

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