Possible Reasons of Bias in Estimating the Cost of Living Index by the CPI

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Abstract

The consumer price index (CPI) tends to overestimate the cost of living because it uses a fixed basket of goods and services and does not take into account substitution of items when their relative prices change. On the other hand, to measure “pure” price changes statistical agencies attempt to adjust for quality improvements by disregarding price changes that are caused by improvements in quality. However, the consumer may not use or want the quality improvement. When adjusting for such quality improvements they will not impact the CPI, while the actual cost of living has increased. Hence, the CPI will tend to underestimate cost of living when adjusting for quality changes that are not used by the consumers. The paper discusses the CPI bias caused by inseparable property of item characteristics and suggests using direct price comparison method instead of comparing quality adjusted prices. Estimates of the upper-level substitution bias and the quality adjustment bias for Georgia are also provided in the paper.

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² The views expressed in the paper are those of the author and do not necessarily reflect views of the National Statistics Office of Georgia.
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Introduction

The Consumer Price Index (CPI) is used in many countries as a primary guide to measure the Cost of Living Index (COLI). From the conceptual point of view, the CPI differs from the COLI, as it represents the cost of goods index (COGI). The CPI measures a price change for the fixed basket of goods and services, while the COLI is a measure of a change in the minimum expenditure needed to maintain the same standard of living. The main difference in the frameworks of the COGI and the COLI is, that the later allows substitution of items when their relative prices change, while the COGI keeps the basket of goods and services unchanged. The difference arising between the indices is called the bias, when the CPI is supposed to measure the COLI. The following components are considered as main constituents of the total bias: Upper-level substitution bias; elementary aggregate bias; outlet substitution bias; new products bias; quality adjustment bias and the bias caused by inseparable property of the item characteristics. The paper overviews biases pertinent to the CPI of Georgia, provides the point estimate for the upper-level substitution bias and the range for the quality change bias. The paper discusses plausibility of using direct price comparison method instead of comparing quality adjusted prices for goods and services with perfectly inelastic demand.

The structure of the paper is as follows: The first section provides a brief overview of the COLI and the CPI frameworks, while the second part summarizes the CPI construction in Georgia. The third part of the paper provides possible reasons of bias in the CPI and the forth one presents the CPI bias estimates for Georgia. Summary of the key points discussed in the paper is provided in the conclusion.
1. **A brief overview of the COLI framework and the CPI as the COGI**

The theory of measuring the cost of living index initially was developed by Konus (1924) in the first quarter of 20th century, which was further expended with significant contributions of Pollak (1989) and Diewert (1976). Detailed analysis of conceptual framework for constructing the COLI and the COGI is provided by Schultze et al. (2002). According to the CPI manual (ILO et al. 2004) “a COLI measures the change in the *minimum* cost of maintaining a given level of utility, or welfare, that results from changes in the prices of the goods and services consumed.”

An ideal COLI for an individual is a ratio of minimum expenditures in the current and the reference periods needed for maintaining the same utility level. According to the economic theory approach the COLI is bounded by the Laspeyres and the Paasche indices (see Annex 1) from the upper and the lower sides respectively (CPI manual, ILO et al., 2004, ch.17). The bounds are justified as the framework of the index allows substitution of goods and services used by a consumer, when their relative prices change. The Fisher (1922) index calculated as a geometric average of the two indices - Laspeyres and Paasche proves to be a good approximation to the COLI.

The COLI may be considered as conditional or unconditional. The conditional COLI measures the index change caused by prices only, while the unconditional COLI accounts for all the comprehensive reasons affecting the COLI, such as environmental factors. Weather conditions may be considered as an example of environmental factors. Increased expenditures are needed to keep the same temperature in the dwelling as it was in the previous period when the winter is much colder compared to the previous one, while the prices may stay unchanged. Changes in public good provisioning are also considered as environmental factors as well, though they are not directly related to environment (see Pollak (1989) and Schultze et al. (2002)). Changes of tastes of individuals, which may be a significant reason of the COLI change, stays beyond the conditional COLI framework as well.

The CPI is the COGI, as it measures changes of expenditures needed to purchase the fixed basket of goods and services. The CPI is considered as a good measure to approximate the conditional COLI, as the conceptual frameworks of the COGI and the conditional COLI share many common characteristics. Measuring the CPI ideally may be conducted by the Laspeyres or the Paasche price indices (see Annex 1), but in practice both of them are related to challenges of data availability. In case of Laspeyres, quantities consumed in the price reference period is missing, which is frequently set as a month and the Paasche index requires information on quantities for the calculating period, appearing after a significant time lag. Calculation of the CPI in practice is implemented based on modified Laspeyres indices such as
the Lowe index and the Young index (see Annex 1). Modification of the Laspeyres index is conducted by changing the base period of quantities using the available information about expenditures and price indices. The Lowe index uses price updated weights, assuming that the consumer keeps the same volume of quantities of goods and services unchanged in the current period, while the Young index leaves the expenditure structure fixed, which is related to the period one or two years prior to the price reference month.

The following indices are primarily used for elementary aggregate index calculation: the ratio of arithmetic average prices (the Dutot index), arithmetic average of price relatives (the Carli index) and the ratio of geometric mean prices (the Jevons index) (see Annex 1). Based on their properties the Dutot and the Jevons are considered as preferable indices, as they satisfy the key axioms of the axiomatic approach to the index number theory. The Jevons index remains favorable than the Dutot index, as the later fails to satisfy the commensurability test (see the CPI manual, ILO et al. 2004, ch.20). Within the elementary aggregate, when price relatives are calculated, maintaining the constant quality is an essential part of the index construction. Quality adjustment methods are applied to compare prices, when the characteristics of comparable goods and services differ from each other, in order to exclude the quality change component from the price change.

2. **An overview of the CPI construction methods used by the National Statistics Office of Georgia (Geostat)** and its purposes

The CPI in Georgia is used for the following purposes mainly: 1. It is a measure of the monetary inflation and it plays the key role in the inflation targeting policy implementation process, coordinated by the National Bank of Georgia. 2. The CPI is the key indicator for indexation of different kind of monetary amounts such as incomes, social assistances, contracts etc. and 3) The CPI is used as a deflator in national accounts to calculate the real volume indicators.

The CPI is calculated in 6 largest regions of the country and then aggregated to the national level. The prices are collected in urban areas with some exceptions, where rural area prices are observed for the regional CPI. The consumer basket comprises of elementary aggregates such as bread, sour cream, refrigerator and etc.

The consumer basket and weights are updated annually, based on the national accounts information as a main source for weights, while the Household Budget Survey (HBS) remains as a complementary data source. The data on monetary expenditures for t-2 year are used for

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3 Please see the Consumer Price Index Technical Manual (2013) for more detailed description of the methodology used by the Geostat.
weights formation in year t, as they represent the latest finalized information in the national accounts regarding monetary expenditures of households. The Young index is used for aggregation, which means that the weights are not price updated.

Detailed specifications, based on a market research, are fixed for the consumer basket items by the price reference period. Elementary aggregate index is calculated using the Jevons index.

Imputation of prices for missing items is based on the class mean imputation method. Each item has assigned a status of “strongly seasonal” or an “ordinary item” and “keeping” period is determined. Strongly seasonal items are kept in the database during 12 consecutive months when they are absent from the market, while ordinary items are replaced after missing for 3 consecutive months from the market. An item is replaced directly when the enumerator is informed regarding supply ceasing of the item or closing the outlet.

The following quality adjustment methods are used, when the item has to be replaced by the product with different characteristics:

• Overlap method – prices of old and new items are observed simultaneously and the difference is totally attributed to the quality change.

• Linking method – price for the new item is registered during the last month of the keeping period (3rd or 12th month) and the difference between the price of the new item and the imputed one of the missing item is considered as the quality change.

The following considerable changes were introduced in the CPI calculation methodology in 2012: annual updating of the consumer basket was introduced; the national accounts data were considered as primary source for weights formation for the consumer basket instead of the HBS data; The Dutot index was replaced by the Jevons index for the elementary index calculation; Population shares used as city weights were replaced by the data on regional expenditures. Additional care was taken on a scope of domestic approach for regional indices, while forming the regional weights on a product level, in order to identify expenditures spent within the region. The implemented methodological changes mentioned above, ensure minimizing the CPI bias from the COLI.

3. **Sources of Bias in the CPI**

The CPI bias in a context of the COLI is the difference between the CPI and the true COLI. Numerous research papers can be found concerning the CPI bias when it is used as a measure of the COLI and many of them provide point estimates as well. Studies conclude, that the true COLI is overestimated by the CPI. The CPI bias for the USA estimated by Advisory
Commission to study the CPI (Boskin et al, 1996) equals 1.1 percent, while Shapiro and Wilcox (1996) consider the CPI bias between the range of 0.6-1.5 percent. According to Crawford (1998) the CPI is biased by 0.7 percent in Canada and the bias varies between 0.35-0.8 percentage points in the UK according to Cunningham (1996). It should be mentioned, that many studies are conducted around two decades ago and point estimates may not reflect the current situation, because of rapidly changing market environment. The following components may be considered as main constituents of the CPI bias when measuring the COLI:

1. **Upper-level substitution bias** – caused by substituting items by consumer, when their relative prices change; 2. **Elementary aggregate bias** – caused by formula bias or substitution of brands within the elementary aggregate; 3. **Outlet substitution bias** – arising from customers shifting to discounted outlets; 4. **Quality change bias** – caused by failing to separate quality from the price change; 5. **New products bias** – generated by introducing new items in the basket with significant delay and 6. The bias caused by inseparable property of item characteristics – arising when an item is comprised by a bundle of inseparable characteristics/features and the consumer is left with the only option to purchase the whole bundle, notwithstanding which characteristics constitute the same utility level as in previous period.

**Upper-level substitution bias** is appears in the CPI, as it uses the fixed basket of goods and service and does not allow substitution of items when their relative prices change. According to the economic theory approach, consumers tend to substitute items when their relative prices increase, in order to minimize expenditures and maintain the unchanged standard of living. The behavioral pattern used by consumers leads to attaining the same welfare level by less increased expenditures, than it is reported by the CPI change. The Laspeyres index tends to overstate an actual change in the cost of living, as it keeps the consumer basket unchanged and does not take into account consumer's behavior regarding substitution. The upper-level substitution bias has been reported by many studies and it is frequently calculated as a difference between the Laspeyres and superlative indices. Upper level substitution bias for the USA presented by the Advisory Commission (see Boskin et al. (1996)) amounts to 0.15 percent annually, while Cunningham (1996) estimates the range of bias between 0.08-0.25 percent for the UK.

**Elementary aggregate bias** is related to the formula bias or to the substitution within the elementary aggregate. Formula bias arises when the index suffers from an upward bias relative to the average price trend. The elementary aggregate index is biased when the arithmetic
average of price ratios (the Carli index) is used for the index calculation. Using geometric mean in the elementary index is considered as preferable, when the goal is to approximate the COLI under Cobb-Duglas preferences.

**Outlet substitution** bias generates when consumers tend to shift to discounted outlets. New outlets out of the CPI sample may offer reduced prices and attract significant portion of consumers. The outlets are included in the CPI sample after significant delay and the index fails to reflect the COLI decrease by not reflecting reduced prices at new outlets. Outlet substitution bias estimated by Lequiler (1997) for France is within the range of 0.05-0.15 percent.

**New products** bias is related to the belated inclusion of new items in the consumer basket, while introduction of new goods at the competitive market is an inevitable process. The bias is related to the period between the introduction of the product at the market and its inclusion in the consumer basket, which is omitted by the CPI. Examples of such kind of items may be digital cameras, TV sets with LED screens, cell phones and etc. It should be mentioned, that newly appeared cheaper brands also cause the same type of bias. Newly appeared items are characterized with significant price reductions after their introduction and causing an upward bias in the CPI when they are left outside the CPI coverage. Crawford (1998) estimates the sum of the biases generated by new products and new brands for Canada, amounting to 0.3 percent.

**Quality change** bias is related to improper adjustment of prices, when the quality change is not accurately identified and separated from the price change. Many items included in the consumer basket are subject to permanent improvement, because of rapid technological changes, creating a challenge for price comparison. Several types of methods are used to deal with quality changes, out of which three should be outlined.

- Explicit quality adjustment method is used when the information regarding the monetary amount attributable to the quality change is available. Producers of goods and services represent the best source of monetary values for the quality change and cooperation with them is quite essential for obtaining this information. Adjustments of package size is also considered as explicit quality adjustment method.
- Overlap and linking methods may be considered as the second type of method used to construct the index, when the quality of comparable items differ. Overlap method requires to observe market prices of old and the new items in the same period and assigns the
difference to the quality change. The linking method compares the actual market price of the new item with the imputed price of the old one and assigns the difference to the quality change. Similarity in these methods is, that both of them both of them attribute total difference in prices of new and old items to the quality change and neither of them disaggregate the price change into components of quality and price change. Using these methods is a source of bias, since possible price change is attributed to the quality change.

Hedonic regression method is the third and more complicated one, compared to other methods discussed above. Hedonic regressions are used to evaluate the main characteristics and components of products and tries to estimate an effect of changed characteristics on the price of the item. The method is not easy to implement as it requires construction of the model properly explaining the price change. Significant variables of the model may change quite frequently, because of rapidly changing technologies of production.

The quality change bias may have an upward or downward effect on the CPI, conversely to the substitution bias with upward effect. The quality change bias appears when the quality adjustment methods do not accurately measure the price change attributable to the quality change. Diewert and Lawrence (1999) provide an aggregated estimated range for the quality change and new goods biases for New Zealand amounting to 0.35 -0.6 percent.

Each of the method discussed above consider, that the improved quality provides proportional increase of consumer satisfaction and disregard the law of diminishing marginal utility. The CPI tends to underestimate the COLI, as it overvalues the satisfaction received by the consumer from an additional unit, or from the improved feature of the product and requires to adjust prices accordingly.

The Bias caused by inseparable property of characteristics

The core conceptual pillar of the COLI framework is that the consumer is able to substitute goods and services when their relative prices change. Substitution of items may not always be desirable or possible, because of consumer preferences (when they are described using Leontief utility function, or the consumer has a constant demand for the product), or market conditions (when there is a single producer of the product and it can be considered as a monopoly). Using the given budget constraint obtaining such an item is not always possible when the price increases along with quality. Constituent ingredients of the product in many cases are inseparable and the consumer is unable to select only those characteristics, which are needed to attain the same utility level as in the previous period. The consumer is left with the only option to get compensated and obtain the improved item with increased price. Marginal cost
of attaining the same utility level increases for the consumer, while the welfare gain is still obscure.

Internet service provisioning may be considered as an example of the above mentioned situation, when there is a single provider of the service. In case the provider company decides to increase the lowest fee of the service, but provides improved quality by increasing the speed of the internet, the consumer needs to allocate the increased budget to obtain expensive service, but the better one, even though his/her utility may not increase by such improvement.

The simplified model reflects the above mentioned logic, which requires the following assumptions: the total bundle of goods and services obtained by the consumer comprises of substitutable n-1 items and the nth item, which has no substitutes and the consumer has a constant demand on it; The price of each item \( p \) is determined by the quality change coefficient \( b \) and the price component \( \hat{p} \); The quality change coefficient \( b \) equals 1 when the quality of the item is unchanged, while \( b>1 \) when the quality is improved; The price of the item in period \( t \) is proportionally changed to reflect the quality change:

\[
p^t = \hat{p}^t b^t
\]  

Minimum total expenditure to attain the fixed utility level in period 0 equals:

\[
E^0(p^0_i, q^0_i, p^0_n, q^0_n) = \sum_{i=1}^{n-1} p^0_i q^0_i + p^0_n q^0_n
\]

after substituting \( p^t = \hat{p}^t b^t \) from (1)

\[
E^0 = \sum_{i=1}^{n-1} \hat{p}^0_i b^0_i q^0_i + \hat{p}^0_n b^0_n q^0_n
\]  

(2)

When the quality of the item \( n \) is improved, the coefficient \( b^1_n \geq 1 \) and the total expenditure to obtain the same quantities as in period 0 equals:

\[
E^1(p^1_i, q^0_i, p^1_n, q^0_n) = \sum_{i=1}^{n-1} p^1_i q^0_i + p^1_n q^0_n
\]

after substituting \( p^t = \hat{p}^t b^t \) from (1)

\[
E^1 = \sum_{i=1}^{n-1} \hat{p}^1_i b^1_i q^0_i + \hat{p}^1_n b^1_n q^0_n
\]  

(3)
If we assume that the quality of the item n is improved and all the rest variables are unchanged in period 1, expenditures in period 1 exceeds those in period 0, causing the COLI to increase, since \( b^n_1 > b^n_0 \). Budget of the consumer should be compensated to cover increased expenditures required for attaining the same utility level. It should be mentioned, that changing the welfare level of the consumer is quite obscure. According to the preferences of the consumer, utility level may stay unchanged even after the quality improvement.

The CPI framework requires to compare the prices of goods and services with the same quality. According to the CPI methodology, in the ideal case, the quality change coefficient \( b^n_1 \) is estimated and the price \( p^n_1 \) is adjusted in order to obtain \( \hat{p}^n_1 \) and measure the “pure” price change caused by factors other than quality change. Comparing the quality adjusted prices does not reflect changes of the needed income to compensate the increased expenditures for maintaining the same utility level.

Apart from the quality change, altering the minimum package size of the \( n^{th} \) item affects the required expenditures in the same way as the quality change described above. The factors affecting the COLI other than prices are beyond the conditional COLI framework, though neither monopolistic market conditions (eliminating substitution options), nor consumer’s preferences (constant demand for the product) can be classified as environmental factors to fall within the scope of the unconditional COLI coverage.

The CPI does not reflect increase in the COLI, when it uses the quality adjustment methods for price comparison under the special conditions presented in the simplified model. Using direct price comparison method instead of comparing quality adjusted prices would be a suggested approach to avoid the difference between the CPI and the COLI. Applying the direct price comparison method should be implemented with special care and proper analysis of consumer preferences and demand function of the product.

4. The CPI bias estimates for Georgia

The section provides point estimates of the substitution bias and evaluates the range of the quality adjustment bias for Georgia. The formula bias should not be considered as a pertinent for the Georgian CPI, since the Jevons index is used for the elementary aggregate. The new products bias and the outlet substitution bias would be better to measure after scanner data is available, in order to avoid misleading conclusions. Measuring the bias caused by inseparable
property of characteristics requires comprehensive information regarding consumer preferences and product demand functions, which is a subject of further research.

4.1 Estimation of upper-level substitution bias for Georgia using the Fisher ideal index

Superlative indices are considered as the best approximations to the true COLI. The upper-level substitution bias may be evaluated by comparing the Laspeyres index with the Fisher index, which is a geometric mean of the Laspeyres and the Paasche price indices. Calculation of the Fisher index may be fulfilled retrospectively, as the Paasche index requires the data on prices and quantities for the current - calculating period.

Methodological changes implemented by the Geostat in 2012 year proves to be a step forward to minimizing the substitution bias and the period from 2012 to 2016 year is chosen to analyze the Laspeyres, the Paasche and the Fisher indices. Annual consumer basket update leads to decreasing expenditure shares of items with increased relative prices and makes the overall index tend to slow down the growth rate.

Initial step to calculate the Fisher ideal index is to compute the Laspeyres and the Paasche indices. Estimation of the true Laspeyres index requires additional effort, since the official CPI of Georgia is based on the Young index. During 2012-2016 years the consumer basket was updated 5 times, several items were added to and excluded from the basket during this period. Calculation of the Fisher index was conducted implementing the following steps: the data set of all the unique 310 elementary indices was created with the base period of December 2011; Missing items were imputed based on the upper level group index and were assigned symbolic weights close to zero; Annual average indices were calculated compared to previous year; Annual weights with appropriate base periods were used for aggregating annual average indices in order to calculate the true Laspeyres and the Paasche indices. Aggregated indices were rebased to 2013 year, as the index show upward trend since 2013.

Calculation of the Fisher index is based on the formula proposed by Boldsen Hansen (2007) to analyze the CPI of Denmark:

\[
I_{\text{Fisher}}^{0:t} = \left( \frac{\sum w_i^0 p_i^t}{p_i^0} \right)^{1/2} \left( \frac{\sum w_i^t p_i^0}{p_i^t} \right)
\]

Where \(I_{\text{Fisher}}^{0:t}\) indicates the Fisher index for period \(t\) compared to period 0; \(w_i^0\) and \(w_i^t\) represents weights of the item \(i\) in year 0 and \(t\), respectively;

\(w_0 = \frac{p_i^0 q_i^0}{\sum p_i^0 q_i^0}\) and \(w_t = \frac{p_i^t q_i^t}{\sum p_i^t q_i^t}\) indicate expenditure shares for item \(i\) in periods 0 and \(t\).

\(\frac{p_i^t}{p_i^0}\) is the ratio of annual average indices for the elementary aggregate \(i\).
Estimation results presented in Table 1 show, that the difference between the Young and the Laspeyres indices equals 0.5 percent in 2016 compared to 2013 year. Annual average difference between the indices amounts to 0.18 percent. The difference between the indices are attributable to the substitution of goods during the weights and price reference periods used for the Young index.

In 2016 difference between the true Laspeyres and the Fisher indices amounts to 0.67 percent with the base period 2013 and the annual average for three years equals 0.22 percent.

Estimation results show, that the annual average substitution bias amounts to 0.22 percent for Georgia.

Table 1

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Graph 1

Annual average substitution bias for Georgia differs from the analytical results presented by Boldsen Hansen (2007) regarding the CPI of Denmark, where an annual rate of change in the CPI exceeds the superlative indices (Fisher, Walsh, Marshall-Edgworth and Tornqvist) by 0.05 percentage points.
4.2  Quality adjustment bias estimates

Quality adjustment bias in the CPI of Georgia may be caused by using the overlap and the linking methods for comparing items with changed qualities. Explicit quality adjustment is used when the package size of the item is different, but stays within the predetermined limits. The bias may be positive or negative depending on the price and quality change relationship. When the quality change of the item is higher than it is reflected in the price change of the item, an upward bias takes place, while the opposite result is achieved when the quality is increased less, compared to the price change.

To evaluate the quality change bias, the formula proposed by Diewert (1998) is used:

\[ BB_Q = \frac{(1 + i)se}{1 + e} \]

Where: \( i \) is an official inflation rate, \( s \) is a share of replaced items, \( e \) is a percentage increase in efficiency of new models which was not captured when new models were included in the index.

In order to evaluate the quality change bias, durables and semi durable commodities were identified in the consumer basket, which could be prone to quality adjustment bias. The group comprises of household appliances, automobiles, mobile phones and audio-visual, photographic and information processing equipment. The variable \( e \) is defined as a range between 30-70 percent during the period 2012-2017 years. An average inflation rate in 2017 year compared to 2012 amounts to 14 percent, while the share of changed items varies between 4-6 percent.

Estimation results show, that the quality adjustment bias based on the above mentioned analysis varies between the range of 0.05-0.11 percent.

**Conclusion**

Quality adjustment methods recommended by the CPI methodology may cause downward bias of the COLI by disregarding the diminishing rate of the marginal utility obtained from the extra unit of improved features of the product.

The CPI fails to serve as a compensating tool, as it requires using quality adjusted prices, when a consumer has a constant demand for the product and there is no option for substitution. To eliminate the bias direct price comparison method should be used instead of comparing quality adjusted prices.
Based on the estimates for Georgia the Young index overestimates the true Laspeyres index by 0.18 percentage points, calculated as an average for three years. An estimated annual average upper-level substitution bias amounts to 0.22 percent for Georgia, while the quality change bias range is estimated between 0.05-0.11 percent.

**Annex 1**

Laspeyres index - \( I_L^{0:t} = \frac{\sum_i p_i^t q_i^0}{\sum_i p_i^0 q_i^0} \)

Paasche index - \( I_P^{0:t} = \frac{\sum_i p_i^t q_i^t}{\sum_i p_i^0 q_i^t} \)

Fisher index - \( I_F = \left( I_L \ast I_P \right)^{1/2} \)

Young index \( I_{YG}^{0:t} = \frac{\sum_i \frac{p_i^t}{p_i^0} \ast \frac{q_i^b}{q_i^0}}{\sum_i \frac{p_i^0}{p_i^0} \ast \frac{q_i^b}{q_i^0}} \)

Lowe index \( I_{Lo}^{0:t} = \frac{\sum_i p_i^t q_i^b}{\sum_i p_i^0 q_i^b} \)

Dutot index \( I_D^{0:t} = \frac{\sum_i \frac{p_i^t}{n} / n}{\sum_i \frac{p_i^0}{n} / n} \)

Carli index \( I_C^{0:t} = \frac{\sum_i \frac{p_i^t}{n} / p_i^0}{n} \)

Jevons index \( I_J^{0:t} = \left( \prod_i p_i^t \right)^{1/n} / \left( \prod_i p_i^0 \right)^{1/n} \)

Where:
- \( p_i^t \) is the price of item i in period t;
- \( p_i^0 \) is the price of item i in period 0;
- \( q_i^0 \) is a quantity of item i in period 0;
- \( q_i^t \) is a quantity of item i in period t;
- \( q_i^b \) is a quantity of item i in period b;
- n is a number of items.
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