How (not) to treat seasonal products

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Structure of the presentation

1. Notion of seasonality
2. Alternative methods
3. Axiomatic properties
4. A new proposal
5. Discussion
1. Notion of seasonality

- The treatment of seasonal products is perhaps the **single most important unresolved issue** in consumer price indices.
- The usual **like-with-like comparison is ruled out** by the fact of seasonal unavailability of the products to be tracked.
- In fact, it is by the very nature of the problem at hand **impossible to define the ultimately right answer** to this question.
- The literature suggests several alternative methods to deal with this difficulty; as it was to be expected, **all methods have some undesirable properties** and there can, hence, be no uniquely defined 'best' (or 'worst') method.
1. Notion of seasonality

- **Seasonal commodities** *(CPI Manual, 2004, para. 22.1)*
  - *Strongly seasonal commodities* are 'not available in the marketplace during certain seasons of the year';
  - *Weakly seasonal commodities* are 'available throughout the year, but there are regular fluctuations in prices or quantities that are synchronized with the season or the time of the year'.

- **Seasonal adjustment** *(ESS Guidelines, 2015, Basic Definitions)*
  - 'Seasonal adjustment filters out usual seasonal fluctuations ... from a time series.
  - Usual seasonal fluctuations mean those movements which recur with similar intensity in the same season each year ...
  - Movements due to exceptionally strong or weak seasonal influences ... will continue to be visible in the seasonally adjusted series.'
1. Notion of seasonality

- Seasonal commodities $\neq$ Seasonal adjustment

- *Weak seasonality* $\approx$ *Usual seasonal fluctuations*

- *Weakly seasonal commodities* = no problems 😊
  - In particular, not getting rid of seasonal fluctuations in the indices!

- *Strongly seasonal commodities* = no satisfactory way

- NB: Rolling year indices $\neq$ seasonally adjusted (Ch. 22!)
2. Alternative methods

- **Digression:** In the frequency domain, the variance of the observed time series can be decomposed in terms of the spectrum.
  - The spectrum specifies the importance of the components that occur at the frequency $\omega$.
- Time series are frequently filtered to remove unwanted characteristics, such as trends and seasonal components.
  - The gain of the filter is an expression for the spectrum of the filtered series in terms of the spectrum of the original series and the filter weights.
  - The phase function determines how a cycle at a particular frequency is shifted forward or backward in time.
2. Alternative methods

• Next, the **Turvey-Diewert artificial data set** is introduced (CPI Manual, 2004, Ch. 22).

• This data set consists of **five seasonal commodities** (1. apples, 2. peaches, 3. grapes, 4. strawberries, and 5. oranges) for **four years by months** so that there are 240 observations in all.

• At certain times of the year, **peaches and strawberries** (commodities 2 and 4) are unavailable.
2. Alternative methods

• **Monthly fixed base Laspeyres(-type) indices** (1970 Dec = 100)

• The *year-over-year* index compares the prices of one month with the *prices of the same month* in the fixed base year 1970.

\[
p^{mt} = \frac{\sum_{i \in I} p^{mt}_i \cdot q_i^{m(1970)}}{\sum_{i \in I} p_i^{m(1970)} \cdot q_i^{m(1970)}}
\]
Monthly fixed base Laspeyres(-type) indices (1970 Dec = 100)
2. Alternative methods

• **Year-over-year** index:
  • Essentially equivalent to the **annual rate** as a filter.
  • Methodologically close to the **over-the-year technique** in chained indices; 'should be avoided' (2008 SNA, para. 15.50).
  • **Seasonal pattern spurious** and infra-annual changes not meaningful; e.g. phase shift of +3 months at $\omega = \pi / 6$ (increases the further away the index is from the base year).
  • In the aggregation, index **combined with other non-seasonal indices**, and annual (inflation) rate calculated once again – yielding an even longer and more complex filter.
  • **Annual aggregation** approximately satisfied.
2. Alternative methods

- The **rolling year index** compares the **prices of the last 12 months** with the corresponding prices in the 12 months of the fixed base year 1970.

\[
(2) \quad p_{mt} = \frac{\sum_{l=1}^{m} \sum_{i \in S_l} p_i^{l(t)} \cdot q_i^{l(1970)} + \sum_{l=m+1}^{12} \sum_{i \in S_l} p_i^{l(t-1)} \cdot q_i^{l(1970)}}{\sum_{l=1}^{12} \sum_{i \in S_l} p_i^{l(1970)} \cdot q_i^{l(1970)}}
\]
2. Alternative methods

• **Rolling year** index:
  • Ties in with the **annual average** as a filter.
  • **Over-smoothes and lags** the time series; squared gain quickly drops to close to zero, phase shift of +5.5 months.
  • Not 'seasonally adjusted', also **cycle and higher frequencies dampened**; i.e. only trend but no turning points visible.
  • **Seasonal adjustment**, using much more appropriate techniques rooted in time series, should be left to users.
  • December **monthly** index equals **theoretically 'satisfactory'** **annual** index.
2. Alternative methods

• The *maximum overlap* index is applied only to the subset of commodities that is present in both months of the comparison.

• (3) \( p^{mt} = p^{(m-1)t} \cdot \frac{\sum_{i \in S_m \cap S_{m-1}} p^{mt}_i \cdot q^{(m-1)t}_i}{\sum_{i \in S_m \cap S_{m-1}} p^{(m-1)t}_i \cdot q^{(m-1)t}_i} \)
Monthly fixed base Laspeyres(-type) indices (1970 Dec = 100)
2. Alternative methods

- **Maximum overlap index:**
  - Linked to the *monthly rate* as a filter.
  - **Systematically biased** because it is applied only to the subset of commodities that is present in adjacent months.
  - **Subject to chain drift** since constructed month to month.
2. Alternative methods

- The *carry forward* index uses an annual basket and the *last available price* when the commodity was available in the market place.

\[
(4) \quad p_{mt} = \frac{\sum_{i=1}^{N} \hat{p}_{i}^{mt} \cdot q_{i}^{1970}}{\sum_{i=1}^{N} \hat{p}_{i}^{12(1970)} \cdot q_{i}^{1970}}
\]
2. Alternative methods

- **Carry forward index:**
  - **Last observed price** of the season determines the index results for the whole out-of-season period without anticipating a change in the price during that period.
  - **Eliminates fluctuations** for the part of the weight of products that are not in-season, e.g. impact of seasonal sales.
2. Alternative methods

- The imputation index uses a method to **fill in the missing prices** instead of carrying forward the prices of seasonally unavailable items.

\[ (5) \quad p_{mt} = \frac{\sum_{i=1}^{N} \hat{p}_{i}^{mt} \cdot \bar{q}_{i}^{1970}}{\sum_{i=1}^{N} \hat{p}_{12(1970)}^{i} \cdot \bar{q}_{i}^{1970}} \]

- For the **European HICP**, estimated prices shall be used for seasonal products that are out-of-season.
2. Alternative methods

- **Imputation index:**
  - **Empirical similarity** with carry forward index in the Turvey-Diewert data set is artificial; the imputation is taken to be a price index that grows at a monthly rate of 0.8%. In case of imputation of zero growth, the carry forward index would result again.
  - Imputations should be based on *appropriate estimation procedures* (and not just on previously observed prices).
  - Preferably, the estimated price is adjusted by the change in observed prices on average over *all seasonal products that are in-season* in the same subdivision of COICOP (or at a more detailed level).
2. Alternative methods

- The *Rothwell* index makes use of *seasonal baskets* in the base year (and also of base year unit value prices).

\[
R = \sum_{b=0}^{\pi} \sum_{o=1}^{\omega} \frac{b_o \cdot \omega_o}{b_o + \omega_o}
\]

- Likewise, for the *European HICP*, a seasonal product that is out-of-season shall have a weight of zero.
2. Alternative methods

- **Rothwell index:**
  - Basic difference to the European HICP ('class-confined seasonal weights index') is that the *seasonal baskets in the Rothwell index differ between months within the in-season period* (and that the Rothwell index makes use of base year unit value prices).
  - **Month-to-month changes** of the index are not only resulting from price changes but also from weights shifting from one commodity to another (impact of changing weights on the monthly change of the HICP is reduced to a minimum).
  - **Annual inflation rates** (and seasonally adjusted results) are identical to those from the year-over-year index.
3. Axiomatic properties

- Assessment of the **index performance**:
  - **All present commodities are included and all prices are observed** (transactions); not satisfied by maximum overlap, carry forward and imputation indices.
  - **Index is neither smoothed nor lagged** (turning points); not satisfied by year-over-year (annual rate), rolling year (annual average) and maximum overlap (monthly rate) indices.
  - **Infra-annual pattern is not distorted** (genuine seasonality); not satisfied by year-over-year (spurious) and rolling year (over-smoothed) indices.
3. Axiomatic properties

- **Month-to-month inflation rates are unbiased;** unfeasible (without seasonal adjustment) but not even meaningful for year-over-year (spurious seasonality), rolling year (oversmoothes) and maximum overlap (systematically biased) indices (carry forward and imputation indices use estimated prices, Rothwell index biased).

- **Annual inflation rates are unbiased;** satisfied for year-over-year and Rothwell indices (again carry forward and imputation indices use estimated prices, not meaningful for rolling year and maximum overlap indices).

- **Monthly data exactly aggregate to annual averages;** approximately satisfied for year-over-year and Rothwell indices (rolling year index equals annual index in December).
## 3. Axiomatic properties

### Assessment of index performance

<table>
<thead>
<tr>
<th>Index</th>
<th>Coverage and scope met</th>
<th>Neither smoothed nor lagged</th>
<th>Infra-annual pattern genuine</th>
<th>Monthly inflation rate unbiased</th>
<th>Annual inflation rate unbiased</th>
<th>Annually aggregates exactly</th>
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<td>x</td>
<td>x</td>
<td>x</td>
<td>(x)</td>
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<td>(✓)</td>
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<tr>
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<td>(✓)</td>
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<td>✓</td>
<td>(x)</td>
<td>✓</td>
<td>x</td>
</tr>
</tbody>
</table>
3. Axiomatic properties

From the six alternative methods, two stand out:

- **Imputation indices** (with *carry forward indices* just being the *inferior* degenerate case of zero growth); and
- **Rothwell indices** (with *year-over-year indices* yielding the same annual rates and seasonally adjusted results); in the European HICP the impact on the monthly change is reduced.

Their main difference lies in the use of an **annual and seasonal basket**, respectively.

The others are at least weakly dominated, two of which are even **strictly dominated**:

- **Rolling year indices** (over-smoothes); and
- **Maximum overlap indices** (systematically biased).
4. A new proposal

- **Two new alternative methods** are proposed: literally comparing 'apples with oranges'.

- The **unit value index** compares the unit value of one month with the **unit value in the 12 months of the fixed base year 1970**.

\[
(7) \quad p^{mt} = \frac{\sum_{i \in S_m} p^{mt}_i \cdot q^{mt}_i}{\sum_{i \in S_m} q^{mt}_i} \cdot \frac{\sum_{l=1}^{12} p_i^{l(1970)} \cdot q_i^{l(1970)}}{\sum_{l=1}^{12} \sum_{i \in S_l} q_i^{l(1970)}}
\]
4. A new proposal

• The *quality-adjusted unit value index* applies an implicit quality adjustment to account for **differences in the price levels** of the commodities.

  • **Quality adjustment factors** $\gamma$ (weighted time-product dummy method): 0.99 (apples), 1.22 (peaches), 0.87 (grapes), 2.32 (strawberries) and 1.00 (oranges, numéraire).

• (8) $p^{mt} = \frac{\sum_{i \in S_m} p_i^{mt}q_i^{mt} / \sum_{i \in S_m} \gamma_i q_i^{mt}}{\sum_{l=1}^{12} \sum_{i \in S_l} p_i^{l(1970)}q_i^{l(1970)} / \sum_{l=1}^{12} \sum_{i \in S_l} \gamma_i q_i^{l(1970)}}$
5. Discussion

- The quality-adjusted unit value index is similar to the Rothwell index and the unit value index is so after seasonal adjustment, too.

- By and large, the unit value indices share the properties of the Rothwell index; the annual rate is only approximately unbiased (and annual aggregation does not even hold approximately).
5. Discussion

- The **unit value indices are simpler to calculate** than the Rothwell index since commodities do not need to be matched in the base year – this is **particularly the case with scanner data**.
- Even **evolving weak seasonality**, i.e. when commodities become available sooner or later in a year, would be technically irrelevant.
- This idea can be extended to **multilateral methods** more generally.
The Delegates are invited to:

• **take note** of the assessment of index performance (trade-off between imputation and Rothwell indices);
  • interpretable month-to-month inflation rates → imputation

• **provide views** on the two new alternative methods (unit value indices) proposed; and

• **discuss ideas** on the treatment of seasonal products with a view to the implementation in CPI production.
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Monthly fixed base Laspeyres(-type) indices, seasonally adjusted

- Year-over-year
- Rolling year
- Maximum overlap
- Carry forward
- Imputation
- Rothwell
Annual inflation rates (in %)

- Imputation
- Rothwell
- Unit value
- Quality adjusted
Monthly fixed base Laspeyres(-type) indices, seasonally adjusted