Group of Experts on Consumer Price Indices

Reference paper 2 for the 'Higher-level Indices Workshop'
United Nations Economic Commission for Europe
26 – 28 May 2014, Geneva, Switzerland

Why 'Lowe' when 'Young' and 'Laspeyres' are available?

Arnold Akem and Andriy Opryshko
Australian Bureau of Statistics

Abstract

This paper is a reference document to support the 'Higher-level indices workshop' conducted at the United Nations Economic Commission for Europe's (UNECE) Meeting of the Expert Group of Consumer Price Indices from 26 – 28 May 2014 in Geneva, Switzerland. The paper firstly describes a "family" of Laspeyres-type index methods that have similar attributes. This article defines the various Laspeyres-type indexes available to index compilers with a focus on the Laspeyres, Lowe and Young methods. The paper then focuses on the rationale for using the Lowe method. These approaches are demonstrated by producing various indexes for a CPI component utilising published data by the Australian Bureau of Statistics (ABS). This explores the advantages and disadvantages of using different methods and demonstrates the differences that may arise on application. The paper concludes with a validation to currently employ Lowe index methodology by the ABS and the National Statistical Offices internationally to publish reliable and timely statistics.

Note: This paper can be found on the UNECE website at:
http://www.unece.org/stats/documents/2014.05.cpi.html

Acknowledgement: Thanks to the ABS staff of the Prices Branch (Terry Bradley, Leigh Merrington and David Stewart) for their contributions and review of this paper.

1. Introduction

The purpose of this paper is to outline the differences between Laspeyres, Lowe and Young index methods as well as the rationale for the ABS to use the Lowe method to compile its Consumer Price Index (CPI) at the upper levels.

When constructing a price index, a number of practical issues needs to be confronted, some of them being: availability of data, cost-benefit of using a particular index formula methodology over another, ease of use and interpretability of the index results.

The ABS CPI is an economic indicator that measures changes over time in the general level of prices of a fixed basket of goods and services acquired by Australian household. Most CPIs compiled internationally adopt a fixed basket method, with the basket being fixed in some earlier period as it is a suitable method both from a theoretical and practical view point.

It is often suggested that the preferred index compilation method for a CPI would be a 'superlative' index such as the Fisher index, which uses price and quantity or expenditure data from both the reference and current periods. However, it is difficult in practice to obtain up to date quantities for the current period. Currently, the ABS is exploring the possibility of using transactions data to compile the CPI which is likely to be of some assistance in this area.

The upper-level indexes of the ABS CPI are calculated as weighted arithmetic averages of the percentage price changes for a defined 'basket' of goods and services. Expenditure shares are derived from the latest Household Expenditure Survey (HES) and are only updated when the new HES data becomes available. Currently this is every 6 years.

This article explores advantages and disadvantages of applying different index methods and uses real data from the CPI to construct a price index to demonstrate the differences that may arise on application. It also argues the Lowe index is the most feasible method, which allows for a timely indication of inflation in the economy.

2. Index methods

Although index formulae from first principles are based on quantities, in reality quantities for goods and services might not be observable or meaningful. As a result, the Laspeyres, Young and Lowe index formulae are algebraically expressed as weighted averages of expenditure shares and price relatives.

A Laspeyres price index formula is used in price statistics to measure the difference in the cost of a fixed basket of goods and services between the price reference period and the current period. A price reference period is the period with which current period prices are compared (OECD Glossary of Statistical terms 2003). The Laspeyres price index P_L can thus be written as an arithmetic average of the price relatives, p_i^t/p_i^0, weighted by the price reference period expenditure shares (See Appendix). Note that the price change is measured using fixed expenditure shares from the price reference period (period 0) (ILO Manual 2004; Armknecht and Silver 2012). This leads to an overstatement of inflation as the fixed expenditure shares do not change in response to a change in consumer behaviour.

The Lowe index formula also uses a fixed basket approach but it does not have price and expenditure share data from the same period, unlike the Laspeyres index. As a result, the Lowe index weights are calculated using expenditure shares from an earlier period (period b) than the price reference period (period 0). Also, due to the time required to collect and process expenditure data, such as HES, it is usually implemented in a different period to the price reference period. Consequently, the weights used in the Lowe formula are price updated (ILO

Manual 2004). Price updating revalue expenditure shares in the weight reference period at prices of the price reference period, which keeps the basket up to date and in line with the consumer expenditure trends.

There is an ongoing debate on whether the revaluing or price updating of weights in the Lowe index formula is necessary. However, the International Labour Organisation Manual (2004) states *"Where the weight reference period differs significantly from the price reference period, the weights should be price updated to take account of price changes between the weights reference period and price reference period. Where it is likely that price updated weights are less representative of the consumption pattern in the price reference period this procedure may be omitted."*

Finally, instead of holding expenditure shares constant in the price reference period or price updating expenditure shares to the price reference period as the Laspeyres or Lowe, the Young index formula keeps expenditure shares of the earlier period (period b) constant (ILO Manual 2004). The Young index formula, unlike the Lowe, does not price update or revalue the expenditure shares either. As a result, the weights are from some earlier period (period b).

While the Laspeyres, Young and Lowe methods are all fixed basket indexes, conceptual and mathematical differences between them are exposed when applied to real data as shown in the analysis below.

The ABS currently conducts the HES every six years. Due to the amount of time taken to process this information, there is a time lag of approximately 12 months preceding its introduction into the CPI (Armknecht and Silver 2012; CPI CSM 2011). A Laspeyres index will result from introducing the expenditure data at the same period of collection and assuming the price reference period was the same (i.e. b=0). However, this is not possible practically; hence the ABS must decide whether to price update the quantities. The ABS has chosen to use a Lowe index method (i.e. with the weights price-updated to the price reference period) to be compile the Australian CPI.

3. ABS and the Lowe index method

Price index theory provides prices statisticians with guidance to employ the best practices and formulae in compiling price indexes in order to produce reliable price measures. However, the highly desirable methods must be balanced against the practical ones - it would be highly desirable to use a superlative index formula such as the Fisher ideal for all price indexes, but timing issues and data availability preclude this (Diewert 1998). A superlative index requires price and expenditure share information from both current and reference periods (OECD Glossary of Statistical terms 2003). With current period expenditure data being difficult to obtain in a timely manner, following this path would cause undesirable delay in the publication of the quarterly CPI, making it less relevant. A timely CPI is of outmost importance for use in monetary policy, for the public and other organisations in contracts and indexation, and other areas of the ABS that require these figures as input to other statistics (e.g. National Accounts). It is for these reasons the ABS publishes the CPI within three to four weeks of the reference period (quarter). Such urgency and the practical challenge of obtaining current period quantity data make it impossible for the National Statistical Organisations (NSOs) to use the Fisher or any other superlative index formula to calculate their indexes.

The Lowe index formula as used by the ABS has some desirable features making it very attractive to NSOs and the ABS in particular. Mathematic and axiomatic properties of the Lowe method have significant advantage over the other index formulae in discussion. The Lowe index satisfies 12 axioms with time reversibility and transitivity among them (ILO Manual 2004), and presents a highly desirable behaviour in relation to price relatives. Both Laspeyres and Young do not possess the same mathematic properties to pass these tests (Armknecht and Silver 2012).

In addition, the Lowe index formula is a popular choice with other statistical organisations because it is conceptually simple, easy to explain to users and meaningful with little challenges regarding its practical applicability. Using a 'fixed basket' of goods and services allows for a simple comparison of prices in the current period to the price reference period. These are important considerations for the ABS as the fixed basket Lowe index keeps weights unchanged between any of the two periods under consideration. The Lowe index falls in the class of "pure price index" (Bishop 2013).

Furthermore, the Lowe index formula, as applied by the ABS, provides practical advantages and financial savings through the repeated use of HES weights (expenditure) to calculate its CPI. As previously mentioned, any superlative index formula would require current period expenditure share information, which is costly and impractical. As a result, application of the Lowe index formula provides a less costly option, relying on HES expenditure data between the 6-yearly reweight periods (CPI CSM 2011).

4. Using real data from the CPI

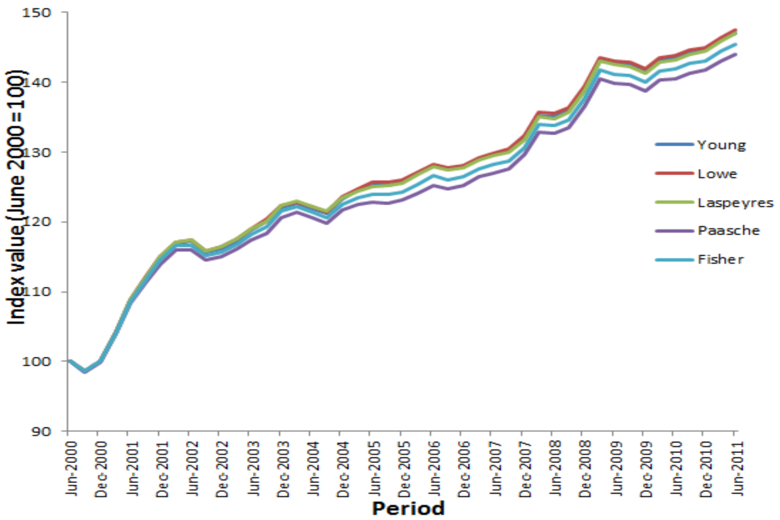
Table 1 below presents an analysis on real data with differences in annual growth rates and the degree of deviation from the ideal Fisher index over the June 2000 to June 2011 period. The difference between the Lowe index and the ideal Fisher index can be interpreted as the bias for this component of the CPI.

Table 1: Average Annual Growth Rates for arithmetic indexes on real CPI data.

	Laspeyres	Lowe	Young	Paasche	Fisher	Published Data
Growth Rate (2000-2011)	3.6%	3.6%	3.6%	3.4%	3.5%	3.6%

Using real data from a CPI component from June 2000 to June 2011 shows that Laspeyres, Lowe and Young indexes have similar growth rates over this period. The average annual growth rates confirm the upward bias of the Laspeyres and Lowe compared to the superlative Fisher method. This upward bias generally increases with the aggregation of more upper level components as each component contributes a certain level of upward bias. Note: The average annual growth rate for the Lowe is the published CPI data from June 2000 to June 2011. See Appendix for more time series graphs constructed using various index formulae on real data for this upper level component.

Graph 1: Comparison of arithmetic indexes and Fisher index on a CPI component.



5. Concluding remarks

Despite the algebraic relationship shown to exist between Laspeyres, Lowe and Young index formulae, the results using real data will vary depending on the weighting system settings, the behaviour of prices and elasticity of the goods and services within the component. The Lowe and Young index methods have similar attributes as Laspeyres and can be expressed as one, hence they are referred to as Laspeyres-type indexes. The ABS has chosen to use Lowe methodology due to practical constraints of releasing a timely CPI, ease of use, and axiomatic advantages of Lowe method over other indexes discussed. While real data calculations display similar growth rates, there are other factors impacting the choice or a change of an index formula. Taking into consideration existing debates on index formula selection, just switching an index formula requires changes to the fundamental concepts used in the CPI and to the systems involved to accommodate different data requirements and users. In the end, the ABS and NSOs internationally apply Lowe index methodology to publish reliable and timely statistics due to its practicality.

References:

- Diewert, WE 1998, "*Index Number Issues in the Consumer Price Index*", Journal of Economic Perspectives 12(1): 47–58.
- Armknecht, P and Silver, M 2012, "*Post-Laspeyres: The case for a New Formula for compiling Consumer Price Indexes"*, IMF Working Paper WP/12/105, International Monetary Fund, Washington, D.C, April.
- Bishop, G 2013, "*Geometric Indexes and Substitution Bias in the CPI*", Ottawa Group Meeting.
- Rossiter, J 2005, "*Measurement Bias in the Canadian Consumer Price Index*", Bank of Canada Working Paper No. 2005-39.
- Australian Bureau of Statistics 2011, "*Consumer Price Index: Concept Sources and Methods*", Cat. No. 6461.0.
- International Labour Office (ILO/IMF/OECD/IMECE/Eurostat/ World Bank) 2004, "*Consumer Price Index Manual: Theory and Practice*", Geneva.
- International Labour Office 2003, "*Resolution concerning consumer price indices*", Conference of Labour Statisticians.
- OECD Glossary of Statistical Terms 2003. Available at: <http://stats.oecd.org/glossary/detail.asp?ID=5669 >

Appendix

2.0 Laspeyres-type formulae, relationships and differences

Laspeyres price index, $P_L(p^0, p^t, q^0) = \frac{\sum_{i=1}^n p_i^t q_i^0}{\sum_{i=1}^n p_i^0 q_i^0} = \sum_{i=1}^n (\frac{p_0^t}{p_i^0}) s_i^o$

Young price index, $P_Y(p^0, p^t, s^b) = \frac{\sum_{i=1}^n p_i^0 q_i^b}{\sum_{i=1}^n p_i^0 q_i^b} = \sum_{i=1}^n s_i^b (p_i^t / p_i^0)$

Lowe price index, $P_{Lo}(p^0, p^t, q^b) = \frac{\sum_{i=1}^n p_i^t q_i^b}{\sum_{i=1}^n p_i^0 q_i^b} = \sum_{i=1}^n (\frac{p_0^t}{p_i^0}) s_i^{ob}$

Geometric Young price index $\mathrm{P_{GY}} = \prod_{i=1}^n (\frac{p_i^t}{p_i^0})^{s_i^b}$

Geometric Lowe price index $\mathrm{P_{GLo}} = \prod_{i=1}^n (\frac{p_i^t}{p_i^0})^{s_i^{ob}}$

$$where\ s_i^{ob} = \frac{\sum_{i=1}^n p_i^0 q_i^b}{\sum_{i=1}^n p_i^0 q_i^b} = \frac{p_i^b q_i^b (p_i^0 / p_i^b)}{\sum_{j=1}^n [p_j^b q_j^b (p_j^0 / p_j^b)]}, \qquad b < 0 < t$$

$$s_i^b = \frac{p_i^b q_i^b}{\sum_{i=1}^n p_i^b q_i^b}$$

$$s_i^0 = \frac{p_i^0 q_i^0}{\sum_{i=1}^n p_i^0 q_i^0}$$

p_i^t = current period price for item i
p_i^0 = price reference period price for item i
q_i^0 = price reference period quantity for item i
s_i^0 = price reference period expenditure shares for item i
s_i^b = period b expenditure shares for item i

A decomposition of Laspeyres-type formula into the others reveals where the relationship lies and how differences arise when applied on real data subject to consumer behaviour.

2.1 Lowe vs Young

The relationship between the Lowe and the Young indexes can be demonstrated by:
$\mathrm{P_{Lo}}(\mathrm{p^0, p^t, q^b}) - \mathrm{P_Y}(\mathrm{p^0, p^t, q^b})$

$$= \sum_{i=1}^n (p_i^t / p_i^0) s_i^{ob} - \sum_{i=1}^n s_i^b (p_i^t / p_i^0)$$

$$= \sum_{i=1}^n (s_i^{0b} - s_i^b)(p_i^t / p_i^0)$$

Let $P_{Lo}^{t/0}$ denote the Lowe index from period 0 to period t and $p_Y^{t/0}$ denote the Young index from for the same period.

$$=\sum_{i=1}^{n}\left({p_i^t}/{p_i^0} - P_Y^{t/0}\right)(s_i^{0b} - s_i^b) \text{ using } \sum_{i=1}^{n}(s_i^{0b} - s_i^b)(P_Y^{t/0})$$

$$=\sum_{i=1}^{n}\left({p_i^t}/{p_i^0} - P_Y^{t/0}\right)\left(\frac{p_i^0/P_i^b}{P_Y^{t/0}} - 1\right) s_i^b$$

$$= \mathrm{P_Y}(\mathrm{p^0}, \mathrm{p^t}, \mathrm{q^b}) + \sum_i \left(\frac{p_i^t}{p_i^0} - p_Y^{t/0}\right)\left(\frac{p_i^t}{p_i^0} - p_Y^{0/b}\right)(w_i^b/P_Y^{0/b})$$

The relationship between the Lowe and Young indexes is some covariance between price relatives above and below the Young between period b to 0 and period 0 to t. If prices increase from period b to 0 and from period 0 to t, this will result in a positive covariance and the Lowe index will exceed the Young. If these prices are in opposite directions from period b to 0 and period 0 to t, the covariance will be negative and Young index will be higher than the Lowe. The relationship between the Lowe and the Young index is dependent on price behaviour. The ILO CPI Manual (2004) summarises this as: *"The Lowe index gives more weight to those elementary indices the prices of which have increased by more than average from b to 0 and less weights to those where the prices have increased by less than average. Therefore, if there are long-term trends in the prices, so that prices which have increased relatively from b to 0 continues to do so from 0 to t, and prices which have fallen from b to 0 continues to fall, the Lowe index will exceed the Young index. This indicates a long-run tendency for the Lowe index to exceed the Young index."*

2.2 Lowe vs Laspeyres

Lowe and Laspeyres formula can be shown to relate as follows:

$$\mathrm{P_L}(\mathrm{p^0}, \mathrm{p^t}, \mathrm{q^0}) = \frac{\sum_{i=1}^{n} p_i^t q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0} = \sum_{i=1}^{n}\left(\frac{p_0^t}{p_i^0}\right) s_i^o$$

Define $\sum_{i=1}^{n}\left(\frac{p_0^t}{p_i^0}\right) s_i^o = \sum_{i=1}^{n} r_i\, s_i^o = r^*$

Where $r_i = {p_i^t}/{p_i^0}$ and $s_i^0 = \frac{p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}$; i=1,.....n

The Laspeyres quantity index comparing quantities from year b to quantities in period 0 by period 0 prices is defined by:

$$Q_\mathrm{L}(\mathrm{q^0}, \mathrm{q^b}, \mathrm{p^0}) = \frac{\sum_{i=1}^{n} p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^0} = \sum_{i=1}^{n}\left(\frac{q_0^b}{q_i^0}\right) s_i^o$$

Define $\sum_{i=1}^{n}\left(\frac{q_0^b}{q_i^0}\right) s_i^o = \sum_{i=1}^{n} t_i\, s_i^o = t^*$

Where $t_i = {q_i^b}/{q_i^0}\,;\, i = 1, \dots, n$

A Lowe index which uses quantities from year b as weights for comparison of prices in period t and period 0 will differ from the Laspeyres index which uses quantities in period 0 as weights for comparing prices between period t and 0 by the covariance term below.

$$\mathrm{P_{Lo}}(\mathrm{p^0}, \mathrm{p^t}, \mathrm{q^b}) = \mathrm{P_L}(\mathrm{p^0}, \mathrm{p^t}, \mathrm{q^0}) + \frac{\sum_{i=1}^{n}(\mathrm{r_i} - \mathrm{r^*})(\mathrm{t_i} - \mathrm{t^*}) s_i^o}{\mathrm{Q_L}(\mathrm{q^0}, \mathrm{q^t}, \mathrm{p^0})}$$

This relationship stipulates:

- if the covariance term is zero, the Lowe price index coincides with the Laspeyres price index;
- if this covariance is negative, the Lowe index will be lower than the Laspeyres index; and

- if the covariance is positive, the Lowe index will be higher than the Laspeyres index.

From dot point 2, If the consumer doesn`t respond to the price changes by the substitution of goods and services, the Lowe index can be regarded as a superlative index. From an economical point of view this index is quite improbable – and the Lowe index, calculated for the period from 0 to t, will most probable be higher than the Laspeyres index for the same period, calculated by using the weights from the period 0.

2.3 Young vs Laspeyres

The Young and Laspeyres index formulae have the following relationship:

$$P_Y(p^0, p^t, s^b) - P_L(p^0, p^t, q^0)$$
$$= \sum_{i=1}^{n} ({p_i^t}/{p_i^0}) s_i^b - \sum_{i=1}^{n} s_i^0 ({p_i^t}/{p_i^0})$$

$$= \sum_{i=1}^{n} (s_i^b - s_i^0)({p_i^t}/{p_i^0})$$

$$= \sum_{i=1}^{n} (s_i^b - s_i^0)(r_i)$$

$$= \sum_{i=1}^{n} (s_i^b - s_i^0)(r_i) - r^* + r^* \sum_{i=1}^{n} (s_i^b - s_i^0),$$ by definition, $\sum_{i=1}^{n} s_i^b = \sum_{i=1}^{n} s_i^0 = 1$

$$Hence, P_Y(p^0, p^t, s^b) - P_L(p^0, p^t, q^0) = \sum_{i=1}^{n} (s_i^b - s_i^0)(r_i) - r^*$$

Where $\sum_{i=1}^{n} (s_i^b - s_i^0)(r_i - r^*)$ is the covariance between the difference in weights from year b and the price reference period 0, and the deviation of the relative prices from their average value.

The sign and magnitude of the covariance in this relationship depends on the position of period b in relation to the price reference period 0 and the elasticity of the goods and services under consideration. Considering period b prior to the weight reference – the covariance will be positive for inelastic goods and services whose prices have increased, resulting in the Young index higher than the Laspeyres index. In contrast, a negative covariance will exist for elastic goods and Laspeyres index will be higher than the Young (see graphs below).

Generally, the Lowe index formula bounds the Laspeyres-type and Fisher index formula from above. This is highlighted in literature and the ILO Manual (2004), suggesting the Lowe index is upwardly biased against even the already biased Laspeyres index formula. The relationship between the three formulae and differences on real data is shown to depend significantly on the elasticity of goods and services and the formula effect which arises from the conceptual differences and can be seen in graphs 1 and 2 below.

3.0 Other considerations: Mathematic and axiomatic requirements

Mathematic and axiomatic requirements of index formulae applied at the upper (weighted) level are very important in determining the choice of index formula once the index framework has been established. The Lowe method has a significant advantage over the other index formulae and satisfies very important and desirable mathematical properties for index numbers, though it still falls short of being a superlative ideal index. Despite the list of axioms being inevitably arbitrary, some axioms are more important than others.

The Lowe index formula, as applied by the ABS, has proven to satisfy the very desirable mathematical properties of the index number theory as demonstrated by the axiomatic approach to index number theory. The Lowe index satisfies the 12 axioms and presents very good behaviour in relation to price variables. On the other hand, the Young index satisfies only 10 out of these 12 axioms failing the very important time reversal and transitivity tests, while Laspeyres index formula fails the time reversal test.

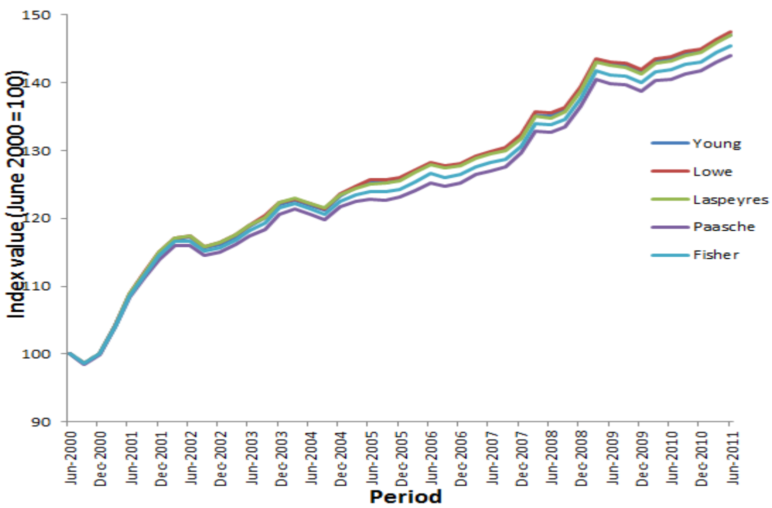
The most desirable and important mathematical axioms and their application are time reversibility and transitivity, explored below.

Time reversibility
Time reversibility of the Lowe index formula produces consistent results whether it is calculated from period 0 to period 1 or in reverse. More specifically, if all data for the two periods are interchanged, then the resulting price should be the reciprocal of the original index. This would be of a serious disadvantage for users if the ABS index formula did not satisfy this property and this index is being used for contract indexation as is the case in some situations.

Transitivity
This test requires the product of the index going from period 0 to 1 multiplied by the index going from period 1 to 2 to be equal to the direct index that compares prices in period 2 with those of period 0.This maintains and connects the series to existing data making it easy to interpret without any loss of information or discontinuity. In the real world, the quantities do change and chaining enables the quantities to be continually updated to account for the changing universe of products.

The Lowe index emerges in a very positive light from the axiomatic approach, satisfying the time reversal and transitivity tests, while the Young and Laspeyres do not.

Graph 1: Comparison of arithmetic indexes and Fisher index on a CPI component.



4.0 Recent considerations

Debate on the upward bias of the Lowe index formula has called for consideration of the geometric counterparts for use at upper levels. These proposals are under investigation by the ABS.