Answers to Questions Arising from the RPI Consultation

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Abstract

The National Statistician in the UK opened a public consultation inviting views on a range of options for the way the Retail Prices Index (RPI) in the UK is calculated. The consultation closed on 30 November 2012 and a number of questions were raised on whether the Carli index should be replaced by other elementary indexes. This note responds to some of the questions that were asked.

Key Words

Consumer price indexes, fixed basket indexes, test approaches to index numbers, elementary indexes, the Retail Prices Index (RPI), the Harmonized Index of Consumer Prices (HICP), the Carli, Jevons and Carruthers, Sellwood, Ward and Dalen indexes.

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1. Introduction

The National Statistician in the UK opened a public consultation inviting views on a range of options for the way the Retail Prices Index (RPI) in the UK is calculated; see the ONS (2012a). The consultation closed on 30 November 2012 and a number of questions were raised on whether the Carli index should be replaced by other elementary indexes; see the ONS (2012b).

The basic problem is that the UK has two consumer price indexes:

- The CPI, which is basically the Harmonized Index of Consumer Prices (HICP) which is the Eurostat mandated Consumer Price Index which is constructed according to a common methodology across countries in the European Union and
- The Retail Prices Index (RPI) which is the national index of consumer inflation that has been used for many years in the UK.

The problem is that the two indexes have recently exhibited different rates of inflation, with the RPI giving the higher measures. The difference in the two indexes has been traced (mainly) to the use of different elementary index number formulae that are used to aggregate item prices at the lowest levels of aggregation where no quantity or expenditure weights are available to weight the various item prices. In particular, at the elementary index level of aggregation (the lowest level of aggregation where information on expenditure weights is lacking), the RPI uses the Carli formula for about 27% of the overall index, whereas the CPI or HICP does not use the Carli formula at all.[2] It is this use of the Carli index that has led the RPI to grow more quickly in recent years than the CPI. Alternatives to the Carli index are the Jevons, Dutot and Carruthers, Sellwood, Ward and Dalén elementary indexes.[3] One of the options to "harmonize" the CPI with the RPI put forth by the ONS is to replace the use of the Carli formula in the RPI with one or more of the alternative indexes. This option has raised some objections during the consultation process.[4]

The purpose of this note is to provide possible answers to some of the questions or objections raised to the possibility of eliminating the use of the Carli formula in the RPI. The *three questions* for which an answer will be attempted are as follows:

- The UK RPI is based on a Cost of Goods Index (COGI) approach to index number theory and given that a COGI[5] is the target index, how can the Jevons index be justified in this context?
- Why is the failure of the Carli index to satisfy the Time Reversal Test so important?
- Could the Dutot index replace the Carli index in the RPI?

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[2] See the ONS (2012b; 8).
[3] For definitions and discussions of these indexes see the ILO Consumer Price Index Manual or Diewert (2012).
[4] The ONS will publish the results of the consultation process towards the end of February, 2013.
[5] COGIs are discussed in Schultze and Mackie (2002; 38-93).

Answers to these three questions will be developed in sections 2-4 below.

2. How Can a Jevons Index be Justified if a COGI is the Target Index?

In order to answer this question, it is necessary to introduce some notation and review the theory behind the COGI index. It turns out that the necessary theory is essentially in section 3.2 of Diewert (2012) and so we repeat some of that material here.

Consider making an index number comparison of prices that pertain to two periods. Thus two accounting periods, t = 0,1, are specified for which micro price and quantity data for N commodities pertaining to transactions by a consumer (or a well defined group of consumers) are potentially available. Denote the price and quantity of commodity n in period t by p_n^t and q_n^t respectively for n = 1,2,…,N and t = 0,1. If there are *multiple* transactions for say commodity n within period t, then it is natural to interpret q_n^t as the *total* amount of commodity n transacted within period t. In order to conserve the value of transactions, it is necessary that p_n^t be defined as a *unit value*; i.e., p_n^t must be equal to the value of transactions for commodity n during period t divided by the total quantity transacted, q_n^t. For t = 0,1, define *the value of transactions in period t* as:

$$(1)\ V^t \equiv \sum_{n=1}^{N} p_n^t\, q_n^t \equiv p^t \cdot q^t$$

where $p^t \equiv (p_1^t,\ldots, p_N^t)$ is the period t price vector, $q^t \equiv (q_1^t,\ldots, q_N^t)$ is the period t quantity vector and $p^t \cdot q^t$ denotes the inner product of these two vectors.

The COGI approach to index number theory is essentially a *fixed basket approach*. In this approach, we are given a basket of commodities that is represented by the positive quantity vector q. Given the price vectors for periods 0 and 1, p^0 and p^1 respectively, we can calculate the cost of purchasing this same basket in the two periods, $p^0 \cdot q$ and $p^1 \cdot q$. Then the ratio of these costs is a very reasonable indicator of pure price change over the two periods under consideration, provided that the basket vector q is “representative”. Thus the *Lowe* (1823) *price index*, P_{Lo}, is defined as follows:

$$(1)\ P_{Lo}(p^0,p^1,q) \equiv p^1 \cdot q/p^0 \cdot q\ .$$

There are two natural choices for the reference basket: the period 0 commodity vector q^0 or the period 1 commodity vector q^1. These two choices lead to the Laspeyres (1871) price index P_L defined by (2) and the Paasche (1874) price index P_P defined by (3):[6]

$$(2)\ P_L(p^0,p^1,q^0,q^1) \equiv p^1 \cdot q^0/p^0 \cdot q^0 = \sum_{n=1}^{N} s_n^0(p_n^1/p_n^0)\ ;$$

$$(3)\ P_P(p^0,p^1,q^0,q^1) \equiv p^1 \cdot q^1/p^0 \cdot q^1 = [\sum_{n=1}^{N} s_n^1(p_n^1/p_n^0)^{-1}]^{-1}$$

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[6] Note that $P_L(p^0,p^1,q^0,q^1)$ does not actually depend on q^1 and $P_P(p^0,p^1,q^0,q^1)$ does not actually depend on q^0. However, it does no harm to include these vectors and the notation indicates that we are in the realm of bilateral index number theory.

where the period t expenditure share on commodity n, s_n^t, is defined as $p_n^t q_n^t/p^t \cdot q^t$ for n = 1,…,N and t = 0,1. Equations (2) show that the Laspeyres price index P_L can be written as a base period expenditure share weighted average of the N price ratios (or price relatives), p_n^1/p_n^0.[7] The last equation in (3) shows that the Paasche price index P_P can be written as a period 1 (or current period) expenditure share weighted *harmonic* average of the N price ratios.[8]

The COGI approach to index number theory has singled out the Laspeyres index (that uses the period 0 basket) as the target index number formula. But it can be seen that the Paasche index is just as plausible as an index of price change between the two periods as the Laspeyres index. Thus the COGI approach should also endorse the Paasche index as an equally reasonable target index! Now it can be argued that the Laspeyres index P_L is the only *practical* index of the two theoretical indexes, P_L and P_P, that can be evaluated on a timely basis and there is merit in this argument. However, at present, we are not discussing practical matters: we are discussing what should be the *theoretical target index* for a COGI and from this perspective, there is no compelling argument to prefer the Laspeyres index over the Paasche.

Thus we have two equally good target indexes are equally plausible but in general, they will give different answers. This suggests that if we require a single estimate for the price change between the two periods, then we should take some sort of evenly weighted average of the two indexes as our final estimate of price change between periods 0 and 1; i.e., if we have two equally plausible estimators for the same inflation target index, then statistical best practice suggests taking a *symmetric mean* of the two estimators. The geometric mean of the two indexes is the *Fisher* (1922) *ideal index*, P_F, defined as

$$(4)\ P_F(p^0,p^1,q^0,q^1) \equiv [P_L(p^0,p^1,q^0,q^1)\ P_P(p^0,p^1,q^0,q^1)]^{1/2}\ .$$

What is the "best" symmetric average of P_L and P_P to use as a point estimate for the theoretical COGI index? It is very desirable for a price index formula that depends on the price and quantity vectors pertaining to the two periods under consideration to satisfy the *time reversal test*. We say that the index number formula $P(p^0,p^1,q^0,q^1)$ satisfies this test if

$$(5)\ \ P(p^1,p^0,q^1,q^0) = 1/P(p^0,p^1,q^0,q^1)\ \ ;$$

i.e., if we interchange the period 0 and period 1 price and quantity data and evaluate the index, then this new index $P(p^1,p^0,q^1,q^0)$ is equal to the reciprocal of the original index $P(p^0,p^1,q^0,q^1)$.

Diewert (1997; 138) showed that the Fisher ideal price index defined by (4) above is the *only* index that is a homogeneous symmetric mean of the Laspeyres and Paasche price indexes, P_L and P_P, and satisfies the time reversal test (5) above. Thus following the

[7] This result is due to Walsh (1901; 428 and 539).
[8] This expenditure share and price ratio representation of the Paasche index is described by Walsh (1901; 428) and derived explicitly by Fisher (1911; 365).

COGI approach to its logical conclusion leads to the Fisher index (4) as being "best" from the perspective of this approach. *Thus an appropriate theoretical target index that is consistent with the COGI approach is the Fisher Ideal index*.[9]

Now suppose that we want to implement the COGI approach at the elementary level; i.e., we now suppose that price information on the N commodities is available for the two periods but quantity information is not available. How should we proceed?

Consider the first formula for the Laspeyres price index in (2): $P_L(p^0,p^1,q^0,q^1) \equiv p^1 \cdot q^0/p^0 \cdot q^0$. We have no information on the quantity vector q^0 but we might argue that a "reasonable" strategy that treats all commodities in a *symmetric manner* would be to assume that q^0 equals 1_N, a vector of ones. The resulting index turns out to be the *Dutot index*, P_D, defined as follows:

(6) $P_D(p^0,p^1) \equiv p^1 \cdot 1_N/p^0 \cdot 1_N = [\sum_{n=1}^N p_n^1]/[\sum_{n=1}^N p_n^0]$.

Similarly, if we replaced q^1 in the Paasche formula (3) by the vector of ones, 1_N, the resulting index would again collapse to the Dutot index defined by (6).

This strategy of replacing q^0 and q^1 by 1_N is fine as long *as the items being aggregated are very similar*. But in general, items will not be very similar and the resulting index defined by (6) will not give satisfactory results.[10]

However, instead of using the basket representations of the Laspeyres and Paasche formulae (the first expressions on the right hand sides of (2) and (3)), we can use the price relative and expenditure share representations of these indexes (the second expressions on the right hand sides of (2) and (3)). Thus instead of replacing q^0 and q^1 by 1_N, we replace the expenditure share vectors s^0 and s^1 by the *vector of equal shares*,[11] $(1/N)1_N \equiv (1/N,...,1/N)$. Making these replacements, formulae (2) and (3) become the following *Carli* and *Harmonic price indexes*, P_C and P_H:

(7) $P_C(p^0,p^1) \equiv \sum_{n=1}^N (1/N)(p_n^1/p_n^0)$;
(8) $P_H(p^0,p^1) \equiv [\sum_{n=1}^N (1/N)(p_n^1/p_n^0)^{-1}]^{-1}$.

Thus the conversion of the Laspeyres index (2) into the Carli index (7) when quantity information is not available supports the contention of COGI supporters that the Carli elementary index is consistent with the COGI approach. However, COGI supporters should also note that the conversion of the Paasche index (3) into the Harmonic index (8) when quantity information is not available supports the use of the Harmonic index under

[9] Once current period expenditure information becomes available, then the Paasche index can be evaluated and hence the Fisher index can also be evaluated. Thus retrospectively, the Fisher target index can be constructed (or at least an approximation to it can be constructed).
[10] In the general (heterogeneous) case, the Dutot index is not invariant to changes in the units of measurement, a fatal flaw.
[11] For a more sophisticated argument for the equal shares assumption in the face of a lack of knowledge, see Levell (2012).

these circumstances.[12] Thus COGI supporters have a choice of two equally good indexes to use at the elementary level when quantity information is not available.

When two equally good indexes are available to measure price change over the same group of commodities, a better index can be obtained by taking a symmetric average of the two indexes. If we take the geometric mean of the Carli and Harmonic indexes, we obtain the Carruthers, Sellwood, Ward (1980) and Dalén (1992) index, P_{CSWD}, defined as follows:

$$(9)\ P_{CSWD}(p^0,p^1) \equiv [P_C(p^0,p^1)\ P_H(p^0,p^1)]^{1/2}.$$

Our reason for taking the geometric mean of P_C and P_H rather than some other symmetric mean is that taking the geometric mean leads to an index that satisfies the important *time reversal test* as applied to elementary indexes.[13]

To sum up: when quantity information is available, an appropriate COGI target index is the Fisher ideal index and when quantity information is not available, an appropriate elementary target index is P_{CSWD} defined by (9).

Up to this point, we have not justified the use of the Jevons index[14], P_J, as an elementary index that is consistent with the COGI approach when quantity or expenditure information in the commodity aggregate is not available: instead, we have justified the use of the Carruthers, Sellwood, Ward and Dalén index, P_{CSWD}. However, the use of the Jevons index can be justified in this context because it closely approximates P_{CSWD}.[15] For COGI adherents, this approximation result by itself would not justify the replacement of P_{CSWD} by P_J as an ideal elementary formula. However, the Jevons index has an additional important property that none of the other elementary index formulae in common use possess: namely it satisfies the following *transitivity* or *circularity test*:[16]

$$(10)\ P(p^0,p^1)P(p^1,p^2) = P(p^0,p^2) \qquad \text{for all strictly positive } p^0, p^1 \text{ and } p^2.$$

If this test is satisfied, then (one plus) the rate of price change going from period 0 to 1, $P(p^0,p^1)$, times (one plus) the rate of price change going from period 1 to 2, $P(p^1,p^2)$, is equal to (one plus) the rate of price change going from period 0 to 2 directly, $P(p^0,p^2)$. If there is only one commodity in the aggregate, then the price index $P(p^0,p^1)$ just becomes the single price ratio, p_1^1/p_1^0, and the circularity test (10) becomes the equation $[p_1^1/p_1^0][p_1^2/p_1^1] = p_1^2/p_1^0$, which is obviously satisfied. An important implication of an index that satisfies the circularity test is this: if the index is chained over time (which is

[12] As noted earlier, the Paasche index is not a practical alternative to the Laspeyres index if the index is to be produced in real time. However, both the Carli and Harmonic indexes can be produced on a timely basis since expenditure share information is not required in order to calculate these indexes.
[13] This test is $P(p^0,p^1)\ P(p^0,p^1) = 1$ where $P(p^0,p^1)$ is an elementary price index.
[14] The Jevons index is defined as $P_J(p^0,p^1) \equiv \prod_{n=1}^N (p_n^1/p_n^0)^{1/N}$.
[15] The original approximation results were established by Dalén (1992) and Diewert (1995) and are repeated in the ILO *Consumer Price Index Manual* and in Diewert (2012; 33-36). The two indexes, P_J and P_{CSWD}, could not be distinguished graphically in Diewert's (2012; 47) numerical example.
[16] The term circularity test is due to Fisher (1922; 413).

the case for both the RPI and CPI) and a base period price vector is repeated at some future period, then the chained index will correctly indicate that no price change has occurred.[17]

Thus although the Jevons index cannot be directly justified by the COGI approach to index number theory, it can be justified indirectly as providing a close approximation to an index that is justified by the COGI approach to index number theory. Moreover, the Jevons index possesses more desirable properties (i.e., satisfies more desirable tests) than competing elementary indexes[18] and hence seems to be a reasonable choice as an elementary index.

3. Why is the Failure of the Carli Index to Pass the Time Reversal Test so Important?

Satisfaction of the *time reversal test* for an elementary index, $P(p^0,p^1)$, can be written as follows:

(11) $P(p^0,p^1)\ P(p^1,p^0) = 1$ for all strictly positive p^0 and p^1.

The equation (11) has the following interpretation. Use the elementary index to compute (one plus) the rate of price change $P(p^0,p^1)$ that results from the period 0 price vector p^0 changing to the period 1 price vector p^1. Now suppose that the period 2 price vector reverts back to the period 0 price vector p^0 and compute (one plus) the rate of price change $P(p^1,p^0)$ that results from this change. The product of the two price changes, $P(p^0,p^1)\ P(p^1,p^0)$, should equal 1 to indicate that no overall price change has taken place between periods 0 and 1. Here is the problem with the Carli formula: not only does not satisfy (11) but it *fails* (11) with the following definite inequality:

(12) $P_C(p^0,p^1)P_C(p^1,p^0) > 1$

unless the price vector p^1 is proportional to p^0 (so that $p^1 = \lambda p^0$ for some scalar $\lambda > 0$), in which case, (11) will hold. The main implication of the inequality (12) is that the use of *the Carli index will tend to give higher measured rates of inflation* than a formula which satisfies the time reversal test (using the same data set and the same weighting). There are numerous empirical examples of this upward bias in the Carli formula, that start with the numerical results in Fisher (1922).[19] The upward bias can be substantial if monthly chained Carli indexes are used.[20]

[17] The index also needs to satisfy the *identity test*: $P(p,p) = 1$.
[18] See Diewert (1995) (2012; 36-39) on the test approach to elementary indexes.
[19] In more recent times, the empirical results in Szulc (1983) (1987) were extremely influential, leading Statistics Canada to abandon the use of the Carli formula in 1978 for the Dutot and then later for the Jevons formula. Other statistical agencies eventually followed this example.
[20] See the examples in the ONS (2012b) and the example in Appendix D in particular which deals with the price bouncing behavior that was first emphasized by Szulc (1983) (1987).

Fisher (1922; 66 and 383) was the first to establish the upward bias of the Carli index[21] and he made the following observations on its use by statistical agencies:

"In fields other than index numbers it is often the best form of average to use. But we shall see that the simple arithmetic average produces one of the very worst of index numbers. And if this book has no other effect than to lead to the total abandonment of the simple arithmetic type of index number, it will have served a useful purpose." Irving Fisher (1922; 29-30).

It took 70 to 80 years before Fisher's advice was followed by major statistical agencies around the world.[22]

4. Can the Dutot Elementary Index Replace the Carli Index?

In section 2 above, we showed that the Dutot index was consistent with the COGI approach to index number theory in the elementary index context when the basket or quantity vector in the COGI index is replaced by a vector of ones. Thus the question arises: could the use of the Carli index in the RPI be replaced by the use of the Dutot index?

The answer to the above question is yes, provided that for each sampled item in the index, an approximate expenditure weight can be found for that item class. As was indicated in Section 2 above, the Dutot index is not a satisfactory elementary index if two or more items sampled in the expenditure class under consideration have different units of measurement. However, if each expenditure class (for which expenditure weights can be obtained) has only one item in it where prices are collected, then the Dutot index will work in a satisfactory manner. However typically, satisfactory expenditure weights can be constructed for only 500 to 2000 expenditure classes and so a satisfactory pure Dutot strategy would be limited to the collection of only 500 to 2000 item prices. Thus the choice of a representative item in each expenditure class becomes very important.

It appears that a pure Dutot methodology can work well in practice, at least at the aggregate level. Evans (2012) reported that Slovenia uses only the Dutot index in constructing its national CPI. Of course, Slovenia also produces the Eurostat mandated HICP, which used only the Jevons index at the elementary level. Evans compared the two indexes and found that there was little difference. Thus it appears that a pure Dutot index methodology at the elementary level can work in a satisfactory manner, at least at the aggregate index level. However, as was indicated in Diewert (2012; 48), it is not always the case that the Dutot index will closely approximate the corresponding Jevons index if there is substantial price heterogeneity in the elementary stratum under consideration. Thus we would expect the components of the Slovenian CPI and the corresponding components of the HICP to differ much more than the overall indexes. The pure Dutot methodology appears to be a bit risky, at least to this author.

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[21] See also Szulc (1987; 12) and Dalén (1992; 139). Dalén (1994; 150-151) provides some nice intuitive explanations for the upward bias of the Carli index.

[22] Evans (2012) shows that the UK is the only European Union country that uses the Carli index in its national CPI. Of course, the Eurostat mandated HICP has ruled out the use of the Carli index from its inception.

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