An Empirical Illustration of Index Construction using Israeli Data on Vegetables

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Abstract

The paper illustrates how various commonly used index number formulae perform using monthly data on seasonal commodities (seven types of vegetable) that was collected by the Israeli Consumer Price Index program. The paper calculates standard Laspeyres, Paasche and Fisher indexes (fixed base and chained) for the years 1998-2002 and then compares these indexes with the types of index used by statistical agencies that make use of lagged expenditure data instead of current expenditure data. Rolling year annual Mudgett-Stone indexes are also calculated. Month to month Lowe, Young and Geometric Young indexes that make use of lagged annual baskets are calculated and found to have small amounts of substitution bias. The monthly chained Fisher index is found to have substantial downward chain drift bias but the month to month Rolling Year GEKS index performs well.

Key Words

Laspeyres, Paasche, Fisher, Lowe, Young and Mudgett-Stone price indexes, Consumer Price Indexes, seasonal commodities, substitution bias, chain drift.

Journal of Economic Literature Classification Numbers

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1. Introduction

In this paper, we will illustrate how various index number formulae commonly used perform using a data set that was constructed by the Israeli Consumer Price Index program. The price data consist of average monthly prices for 7 types of vegetable consumed by households in Israel and cover the 6 years 1997-2002. The Israeli CPI program also has a continuous Household Expenditure Survey and so estimates of monthly household expenditure on the 7 vegetable groups are also available. The 7 vegetable groups are as follows:

- Group 1: Cabbages;
- Group 2: Cauliflower;
- Group 3: Cucumbers;
- Group 4: Potatoes;
- Group 5: Carrots;
- Group 6: Lettuce and
- Group 7: Eggplants.

The prices are in Shekels per kilogram. Vegetables represent about 2 percent of total expenditures in the Israeli CPI. There are tremendous seasonal fluctuations in the prices of fruits and vegetables, which makes the choice of index number formula important. The data are listed in section 5 below and are taken from Diewert, Artsev and Finkel (2009).

Year over year indexes for the same month should eliminate most seasonality in the data and so this type of index will be discussed in section 2 below. In this section, we calculate standard Laspeyres (1871), Paasche (1874) and Fisher (1922) fixed base and chained indexes using the Israeli January data on vegetables. We will compare these weighted indexes with the unweighted (or more accurately, equally weighted) Carli (1764), Jevons (1865), Harmonic or Coggeshall (1887) and Carruthers, Sellwood, Ward (1980) and Dalen (1992) indexes using the January data.[2] In section 2, we will also calculate indexes that make use of delayed expenditure data and current prices; i.e., we will use the January expenditure data for 1997 to calculate Lowe (1823) indexes for the January year over year indexes for the subsequent 5 years, 1998-2002. Young (1812), Harmonic Young and Approximate Fisher indexes will also be calculated and compared to our target indexes (which are Fisher fixed base or chained indexes).

In section 3, we calculate annual indexes that treat each commodity in each month as a separate commodity (so that instead of 7 commodities, we have 7x12 = 84 commodities). This type of index was first advocated by Mudgett (1955) and Stone (1956). We first calculate standard Laspeyres, Paasche and Fisher indexes (fixed base and chained) for the years 1998-2002 and then compare these indexes with the types of index that make use of lagged expenditure data instead of current expenditure data. In particular, we compare the

[2] In order to minimize the length of this paper, we provide an analysis only of the January data. The results for the other year over year monthly indexes are similar.

Mudgett Stone annual Fisher indexes with their Lowe, Young, Geometric Young, Harmonic Young and Approximate Fisher index counterparts.

In section 4, we calculate Rolling Year Mudgett Stone indexes. The annual indexes which were originally advocated by Mudgett and Stone compared the price and quantity data for 12 calendar months with the price and quantity data for a base year consisting of 12 calendar months. Diewert (1983) (1998) (1999) observed that this type of index could be extended to compare a noncalendar year (a string of 12 consecutive months) with the corresponding months in the base year and the resulting index is called a Rolling Year Mudgett Stone index. Thus each month, a new Rolling Year MS index can be calculated. The resulting sequence of monthly indexes can be regarded as a *seasonally adjusted price index* that is centered in the middle of the current rolling year. This type of index is an easy to explain alternative to traditional moving average based methods for seasonal adjustment that require difficult decisions on the part of the operator of the adjustment process as to the exact nature of the moving average process. On the other hand, the Rolling Year Mudgett Stone methodology requires only a decision on the functional form for the index number formula. In section 4 below, we use the Israeli vegetable data to calculate Rolling Year Mudgett Stone fixed base and chained Laspeyres, Paasche and Fisher indexes over the sample period.

Up to this point, all of the indexes compare months of data in a current calendar or rolling year to the corresponding data in a base year so the indexes are *year over year comparisons*. These indexes do not provide policy makers and the public with a clear indication on the course of *short term price inflation*. Thus in section 5, we turn our attention to measures of month to month inflation. Thus in Table 11, we calculate fixed base and chained Laspeyres, Paasche and Fisher month to month indexes using our data set for the 72 months in our sample. These indexes correspond to the maximum overlap month to month indexes which were recommended in the ILO (2004). It will be seen that a *chain drift problem* emerges as a severe problem using our Israeli data set; i.e., the chained Fisher, Laspeyres and Paasche indexes fall well below their fixed base counterparts by the end of the sample period. Frisch (1936) identified this problem[3] and Szulc (1987) demonstrated its importance empirically. Szulc also introduced the term "price bouncing" into the index number literature to describe situations where prices fall due to sales and then bounce back to their presale levels and he observed how chained price indexes would fall below their fixed base counterparts under these conditions.[4] Peter Hill (1988) (1993) provided some useful advice on when to use chained versus fixed base indexes. He advocated chaining if prices (and quantities) had smooth trends and the use of fixed base indexes if there was price bouncing behavior. If there are smooth trends, then generally, chained Laspeyres and Paasche indexes will be closer to each other than their fixed base counterparts whereas if there are erratic moves in prices without clear trends, then fixed base Laspeyres and Paasche indexes will be closer to each other than their chained counterparts. In the former case, chained indexes are appropriate while in the latter case, fixed base indexes are more appropriate. For our

[3] "The divergency which exists between a chain index and the corresponding direct index (when the latter does not satisfy the circular test) will often take the form of a systematic drifting." Ragnar Frisch (1936; 8).
[4] He used data on Canadian soft drink prices to demonstrate the problem with chaining.

Israeli vegetable data, the price bouncing behavior is more prevalent than smooth trends and so the chained indexes exhibit a considerable amount of chain drift.

Ivancic, Diewert and Fox (2011) and de Haan and van der Grient (2011) suggested a useful compromise between the use of fixed base and chained indexes when there is price bouncing behavior: namely the use of Rolling Year GEKS indexes. Thus in section 6 below, we compare these RYGEKS month to month indexes with the fixed base and chained Fisher indexes that were calculated in section 5. We find that the RYGEKS indexes are quite close to the fixed base Fisher indexes.

Finally, in section 7, we compare the RYGEKS indexes with their Lowe, Young and Geometric Young counterparts using the Israeli vegetable data over the 5 years starting in January 1998 and ending in December 2002. We find that these "practical" indexes that use lagged expenditure data are subject to a certain amount of substitution bias.

2. January Year over Year Indexes

We start off by calculating fixed base Fisher, Laspeyres and Paasche indexes (P_F, P_L and P_P in Table 1 below) for the January Israeli vegetable prices, starting in January 1998 and ending in January 2002.[5] The underlying January data can be found in Diewert, Artsev and Finkel (2009).[6] We also calculate the fixed base unweighted Jevons, Carli, Harmonic and Carruthers, Sellwood, Ward and Dalen indexes (P_J, P_C, P_H and P_{CSWD}) in Table 1 below.[7] The average differences between P_L, P_P, P_J, P_C, P_H and P_{CSWD} and our preferred Fisher index P_F over the last 4 years, 1999-2002, are also listed in the last row of Table 1.

Table 1: January 1998-2002 Year over Year Fisher, Laspeyres, Paasche, Jevons, Carli, Harmonic and CSWD Fixed Base Indexes

Year/Month	P_F	P_L	P_P	P_J	P_C	P_H	P_{CSWD}
1998-1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1999-1	1.16520	1.16256	1.16784	1.19047	1.19832	1.18251	1.19039
2000-1	1.23080	1.23215	1.22946	1.21715	1.22521	1.20922	1.21719
2001-1	1.05633	1.05802	1.05465	1.11823	1.12475	1.11179	1.11825
2002-1	1.45403	1.46611	1.44205	1.44735	1.46268	1.43322	1.44788
Mean Diff.	0	0.00312	–0.00309	0.01671	0.02615	0.00760	0.01684

It can be seen that P_F, P_L and P_P are all fairly close with the Laspeyres index averaging 0.3 percentage points above its Fisher counterparts over the last 4 years and the Paasche index averaging about 0.3 percentage points below the Fisher indexes. The upward biases in the unweighted indexes are larger, with the Harmonic and Carli indexes exceeding their Fisher counterparts by 0.8 and 2.6 percentage points per year on average and the Jevons and CSWD indexes above their Fisher counterparts by 1.7 percentage points per

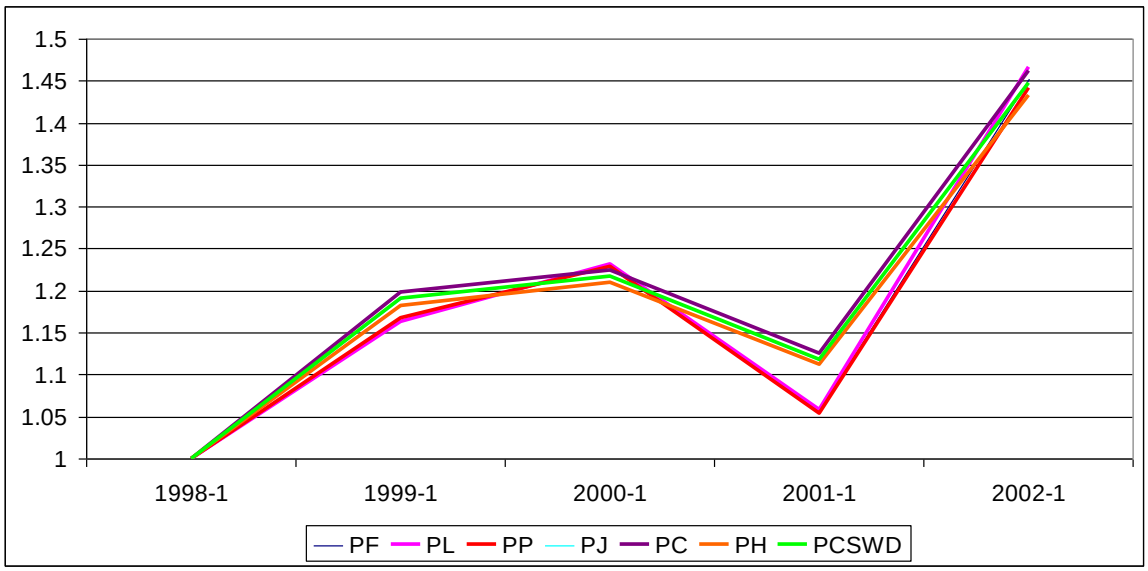
[5] We dropped the January 1997 indexes in Tables 1 and 2 because in Tables 3 and 4 below, we will compare the Fisher fixed base and chained indexes over the Januaries in 1998-2002 with indexes that use the expenditure weights of January 1997 instead of current expenditure weights.

[6] These data are available upon request.

[7] P_{CSWD} is equal to the geometric mean of P_C and P_H. P_{CSWD} should numerically approximate P_J very closely.

year. We cannot expect the unweighted indexes to closely approximate their weighted counterparts but it is interesting that there is a reasonably high degree of approximation for the January data. The indexes in Table 1 are plotted in Chart 1 below.

Chart 1: January 1998-2002 Year over Year Fisher, Laspeyres, Paasche, Jevons, Carli, Harmonic and CSWD Fixed Base Indexes



The Fisher, Laspeyres and Paasche indexes cannot be distinguished separately in the above Chart; they all lie well below the other indexes. The Jevons index cannot be identified on the above Chart either since it is so close to P_{CSWD}.

The differences between the various indexes can be seen more clearly when we calculate their chained counterparts. They are listed in Table 2 below.

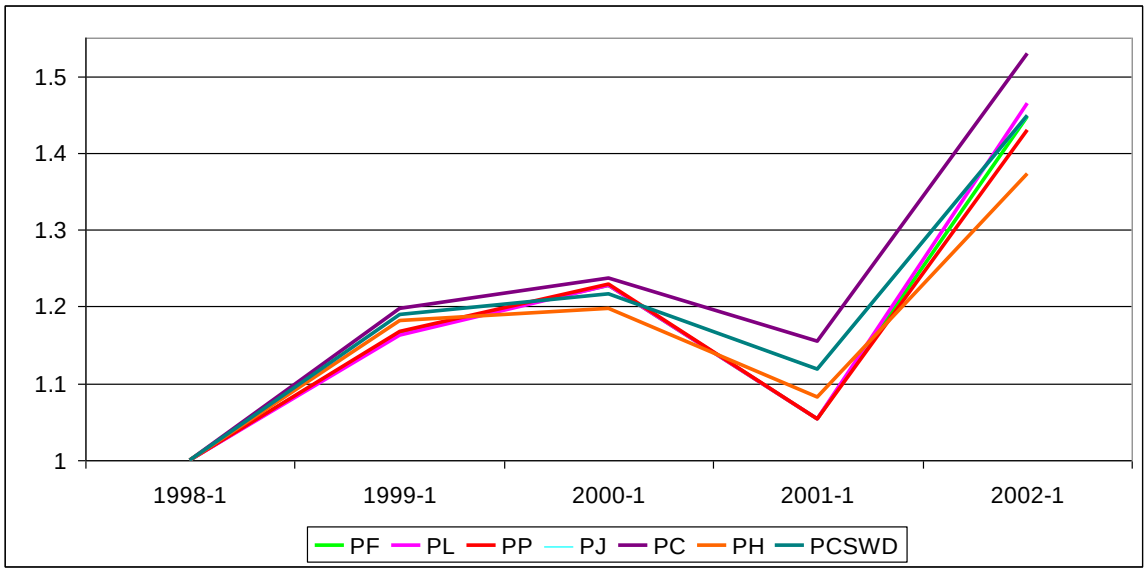
Table 2: January 1998-2002 Year over Year Fisher, Laspeyres, Paasche, Jevons, Carli, Harmonic and CSWD Chained Indexes

Year/Month	P_{Fch}	P_{LCh}	P_{PCh}	P_{JCh}	P_{CCh}	P_{HCh}	P_{CSWDCh}
1998-1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1999-1	1.16520	1.16256	1.16784	1.19047	1.19832	1.18251	1.19039
2000-1	1.22820	1.22702	1.22938	1.21715	1.23762	1.19714	1.21721
2001-1	1.05434	1.05437	1.05431	1.11823	1.15514	1.08221	1.11808
2002-1	1.44726	1.46533	1.42941	1.44735	1.52964	1.37298	1.44920
Mean Diff.	0	0.00357	–0.00351	0.01955	0.05643	–0.01504	0.01997

It can be seen that the chained Fisher, Laspeyres and Paasche indexes are still very close to each other but it can also be seen that chaining has increased the spread between the Laspeyres and Paasche indexes as compared to the spread in these indexes when they were calculated using January 1998 as the fixed base. This increase in spread is an

indication that there may be a chain drift problem with the use of chained indexes in this situation. The various chained indexes are plotted in Chart 2 below.

Chart 2: January 1998-2002 Year over Year Fisher, Laspeyres, Paasche, Jevons, Carli, Harmonic and CSWD Chained Indexes



The Fisher, Laspeyres and Paasche chained indexes are tightly clustered and can only be distinguished on the Chart over the last two years. P_{JCh} and P_{CSWDCh} cannot be distinguished separately at all on the Chart. It is evident that the chained Carli index is well above the other indexes and has an upward bias as compared to our target chained Fisher index.

In Table 3 below, we compare the year over year fixed base Fisher index for January 1998 to January 2002 with various indexes that use the quantity or expenditure information for January 1997 in place of current quantity or expenditure information. The year over year fixed base Lowe, Young, Geometric Young, Harmonic Young and Approximate Fisher indexes for month m of year y are defined as follows, where $p_n^{y,m}$, $q_n^{y,m}$ and $s_n^{y,m}$ denote the price, quantity and expenditure share for commodity n in month m of year y, where m = 1 and y = 1998, 1999,..., 2002:

$$(1)\ P_{Lo}(p^{1998,m},p^{y,m},q^{1997,m}) \equiv \sum_{n=1}^{7} p_n^{y,m} q_n^{1997,m} / \sum_{n=1}^{7} p_n^{1998,m} q_n^{1997,m} \equiv p^{y,m}\cdot q^{1997,m} / p^{1998,m}\cdot q^{1997,m};$$

$$(2)\ P_{Yo}(p^{1998,m},p^{y,m},s^{1997,m}) \equiv \sum_{n=1}^{7} s_n^{1997,m} (p_n^{y,m}/p_n^{1998,m});$$

$$(3)\ \ln P_{GY}(p^{1998,m},p^{y,m},s^{1997,m}) \equiv \sum_{n=1}^{7} s_n^{1997,m} \ln(p_n^{y,m}/p_n^{1998,m});$$

$$(4)\ P_{HY}(p^{1998,m},p^{y,m},s^{1997,m}) \equiv [\sum_{n=1}^{7} s_n^{1997,m} (p_n^{y,m}/p_n^{1998,m})^{-1}]^{-1};$$

$$(5)\ P_{AF}(p^{1998,m},p^{y,m},s^{1997,m}) \equiv [P_{Yo}(p^{1998,m},p^{y,m},s^{1997,m}) P_{HY}(p^{1998,m},p^{y,m},s^{1997,m})]^{1/2}$$

where lnx denotes the natural logarithm of x. Thus the fixed base January year over year *Lowe index* $P_{Lo}(p^{1998,1},p^{y,1},q^{1997,1})$ uses the January 1997 quantity vector, $q^{1997,1}$, as a fixed

basket which is priced out at the January price vector of year y, $p^{y,1}$, giving a total basket cost of $p^{y,1} \cdot q^{1997,1}$ and then this cost is compared to the cost of purchasing the same basket at the prices of January 1998 which is $p^{1998,1} \cdot q^{1997,1}$ and thus the resulting Lowe fixed base index for January of year y is $p^{y,1} \cdot q^{1997,1} / p^{1998,1} \cdot q^{1997,1}$. The fixed base *Young index* $P_{Yo}(p^{1998,1},p^{y,1},s^{1997,1})$ for January of year y is the share weighted average $\sum_{n=1}^{7} s_n^{1997,1} (p_n^{y,1}/p_n^{1998,1})$ of the price relatives $p_n^{y,1}/p_n^{1998,1}$ that compare the price of commodity n in January of year y with the corresponding price of commodity n in January of 1998, where the expenditure shares are the January 1997 expenditure shares $s_n^{1997,1}$ for n = 1,2,...,7.[8] The year over year *Geometric Young index* $P_{GY}(p^{1998,1},p^{y,1},s^{1997,m})$ for January of year y is a weighted geometric average of the price relatives $p_n^{y,1}/p_n^{1998,1}$ where the weights are again the January 1997 expenditure shares $s_n^{1997,1}$. The year over year *Harmonic Young index* $P_{HY}(p^{1998,m},p^{y,m},s^{1997,m})$ for January of year y is a weighted harmonic average of the price relatives $p_n^{y,1}/p_n^{1998,1}$ where the weights are again the January 1997 expenditure shares $s_n^{1997,1}$. Finally, the year over year fixed base *Approximate Fisher index* $P_{AF}(p^{1998,1},p^{y,1},s^{1997,1})$ for January of year y is the geometric mean of the Young index and the Harmonic Young index and it is a weighted counterpart to the unweighted Carruthers, Sellwood, Ward and Dalen elementary index.[9] All of these indexes have the weakness that the base quantity or expenditure share vector does not match up with either price vector. However, the Lowe and Young indexes are used by statistical agencies in place of Laspeyres indexes because they can be implemented in real time, assuming that household expenditure surveys can be used to generate household expenditure shares with a lag of a year or two. Thus the indexes defined by (1)-(5) above have the useful property that they are "practical" i.e., they can be implemented using current price data and lagged expenditure information. We can now ask how well these indexes approximate our target fixed base Fisher indexes using our Israeli data set; see Table 3 below.

Table 3: January 1998-2002 Year over Year Fisher, Lowe, Young, Geometric Young, Harmonic Young and Approximate Fisher Fixed Base Indexes

Year/Month	P_F	P_{Lo}	P_{Yo}	P_{GY}	P_{HY}	P_{AF}
1998-1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1999-1	1.16520	1.16977	1.16852	1.16310	1.15778	1.16314
2000-1	1.23080	1.23963	1.24926	1.24305	1.23689	1.24306
2001-1	1.05633	1.05616	1.05835	1.05245	1.04702	1.05267
2002-1	1.45403	1.49199	1.50830	1.48647	1.46639	1.48720
Mean Diff.	0	0.01280	0.01952	0.00968	0.00043	0.00993

Table 3 shows that on average, all 5 of the indexes defined by (1)-(5) above lie above our target fixed base Fisher indexes that are listed in the P_F columns. The mean difference (over the last 4 observations) is about 1.3, 2.0, 1.0, 0.04 and 1.0 percentage points for P_{Lo}, P_{Yo}, P_{GY}, P_{HY} and P_{AF} respectively. Thus the average "bias" for the Lowe and Young indexes is significant.

[8] Note that the Young index can be regarded as a weighted Carli index.

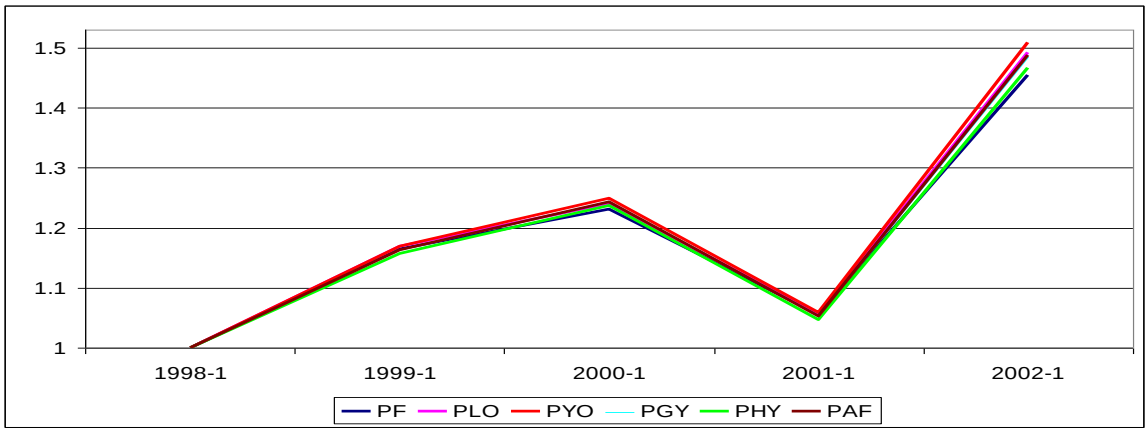
[9] It can be shown that the Approximate Fisher index will approximate the Geometric Young index to the second order around an equal price point.

It can be seen that the Geometric Young indexes are very close to the Approximate Fisher indexes as we would expect using the numerical analysis results explained in Chapter 4. It can also be seen that for all years after 1998, we have the following inequalities between the Young, Geometric Young and Harmonic Young indexes:

(6) $P_{HY}(p^{1998,m},p^{y,m},s^{1997,m}) < P_{GY}(p^{1998,1},p^{y,1},s^{1997,m}) < P_{Yo}(p^{1998,1},p^{y,1},s^{1997,1})$.[10]

The indexes in the above Table are plotted in Chart 3 below.

Chart 3: January 1998-2002 Year over Year Fisher, Lowe, Young, Geometric Young, Harmonic Young and Approximate Fisher Fixed Base Indexes



The Geometric Young and the Approximate Fisher indexes cannot be separately distinguished in the above Chart. Note that from a *relative bias* point of view, all 6 of the indexes appear to be quite close.

We now turn our attention to the chained counterparts to the fixed base indexes listed in Table 1. The chained Fisher index was explained in earlier chapters and needs no further explanation. However, for the remaining 5 indexes, some further explanation is in order. Basically, the idea is that as each year passes, we compute the current chain link using the basic formulae given by (1)-(5) above, except we update the quantity or expenditure vector by one year. Consider the case of the chained year over year Lowe index for January. For 1998, the index is set equal to 1; i.e., we have $P_{Lo}^{1998} \equiv 1$. The January 1999 index level is determined using formula (1) with y = 1999 and m = 1; i.e., we have $P_{Lo}^{1999} \equiv P_{Lo}(p^{1998,1},p^{1999,1},q^{1997,1}) = p^{1999,1}\cdot q^{1997,1}/p^{1998,1}\cdot q^{1997,1}$. Now update the quantity vector by one year to $q^{1998,1}$. The new Lowe index level for 2000, P_{Lo}^{2000}, is the 1999 level, P_{Lo}^{1999}, times the chain link going from 1999 to 2000, which is $P_{Lo}(p^{1999,1},p^{2000,1},q^{1998,1}) = p^{2000,1}\cdot q^{1998,1}/p^{1999,1}\cdot q^{1998,1}$. Now update the quantity vector by one year to $q^{1999,1}$. The new Lowe index level for 2001, P_{Lo}^{2001}, is the 2000 level, P_{Lo}^{2000}, times the chain link going from 2000 to 2001, which is $P_{Lo}(p^{2000,1},p^{2001,1},q^{1999,1}) = p^{2001,1}\cdot q^{1999,1}/p^{2000,1}\cdot q^{1999,1}$. And so

[10] The strict inequalities follow from Schlömilch's inequality provided that the two price vectors are not proportional; see Hardy, Littlewood and Polya (1934; 26).

on. The other chained indexes are calculated in a similar fashion except that we update the expenditure share vector each year instead of updating a quantity vector. The resulting chained indexes are listed in Table 4 below.

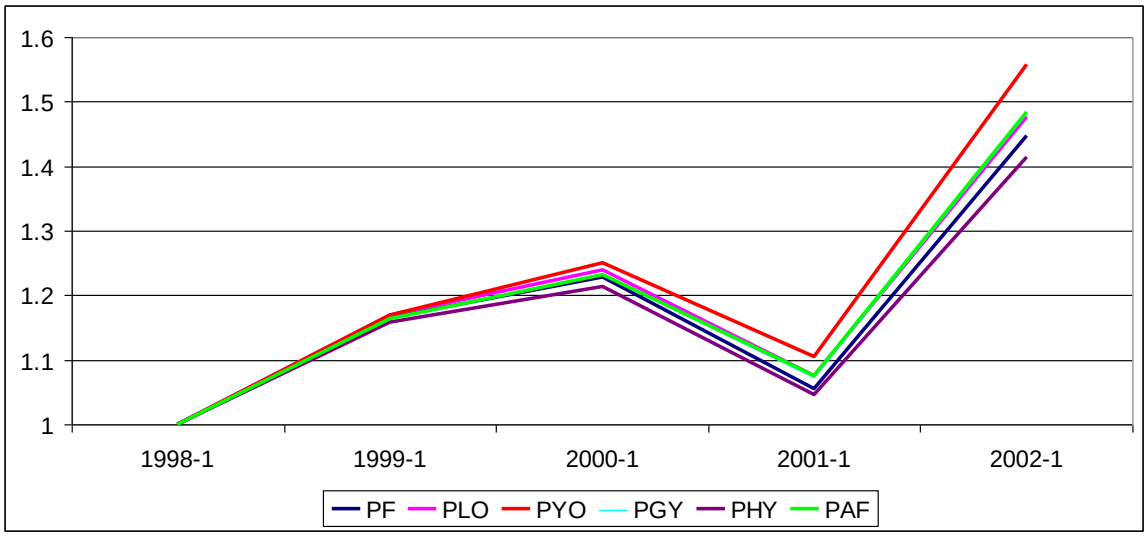
Table 4: January 1998-2002 Year over Year Fisher, Lowe, Young, Geometric Young, Harmonic Young and Approximate Fisher Chained Indexes

Year/Month	P_{FCh}	P_{LoCh}	P_{YoCh}	P_{GYCh}	P_{HYCh}	P_{AFCh}
1998-1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1999-1	1.16520	1.16977	1.16852	1.16310	1.15778	1.16314
2000-1	1.22820	1.23979	1.25008	1.23182	1.21434	1.23208
2001-1	1.05434	1.07577	1.10482	1.07451	1.04597	1.07500
2002-1	1.44726	1.47593	1.55762	1.48189	1.41401	1.48408
Mean Diff.	0	0.01657	0.04651	0.01408	−0.01572	0.01482

It is evident that chaining has increased the dispersion between the 6 indexes. The Harmonic Young index now lies below our target year over year chained Fisher indexes for January but the other 4 lie above our target Fisher indexes. The mean difference (over the last 4 observations) is about 1.7, 4.7, 1.4, −1.6 and 1.5 percentage points for P_{Lo}, P_{Yo}, P_{GY}, P_{HY} and P_{AF} respectively. Thus the average "bias" for the Lowe and Young indexes is now *larger* than it was for the fixed base indexes listed in Table 3 above. This increase in dispersion of our 6 indexes is due to the fact that vegetable prices exhibit a large amount of bouncing behavior as opposed to smooth trends.[11] The indexes listed in Table 4 above are plotted in Chart 4 below.

Chart 4: January 1998-2002 Year over Year Fisher, Lowe, Young, Geometric Young, Harmonic Young and Approximate Fisher Chained Indexes

[11] This increase in dispersion of indexes due to chaining will by no means *always* occur. If the commodity group is say basic clothing or electronic products where there is a strong downward trend in prices, chaining will typically reduce the spread between Laspeyres and Paasche indexes and will reduce the spread between the indexes defined by (1)-(5) above. Under these circumstances, it will generally be better to use chained indexes. Basically, it is best to link observations which have similar relative price structures.



As usual, the chained Geometric Young and Approximate Fisher indexes cannot be distinguished on the Chart. It is also the case that the chained P_{HYCh}, P_{GYCh} and P_{YCh} satisfy the inequalities in (6). The large upward bias in the chained Young indexes is particularly noticeable while the chained Geometric Young, the chained Approximate Fisher and the chained Lowe indexes all have an upward bias that is fairly similar.

Comparable charts for the year over year monthly indexes for the other months of the year could be produced but will be omitted here. A good summary of the average tendencies over all of these monthly indexes will be obtained by looking at the annual Mudgett Stone indexes considered in the following section.

3. Mudgett Stone Annual Indexes

Annual Mudgett Stone indexes simply treat each commodity in each season as a separate commodity and then normal index number theory can be applied to the prices and quantities that are associated with this expanded commodity space. Thus when this methodology is applied to the Israeli vegetable data, the number of commodities jumps from the monthly number of 7 to the annual number 84.

Table 5 below lists the resulting fixed base and chained Fisher, Laspeyres and Paasche annual Mudgett Stone indexes for the years 1998-2002. These indexes were not calculated for the year 1997 because later in this section, we will compare the fixed base Fisher indexes with various fixed base indexes that use either the annual quantity basket for 1997 or the annual expenditure shares for 1997 as weights for prices in subsequent years;[12] see Table 6 below.

Table 5: Annual Mudgett Stone Fixed Base and Chained Fisher, Laspeyres and Paasche Price Indexes for Seven Kinds of Vegetables, 1998-2002

[12] Thus when computing these "practical" indexes, we do not use the price data for the first year and thus these practical indexes will start at 1998 instead of 1997.

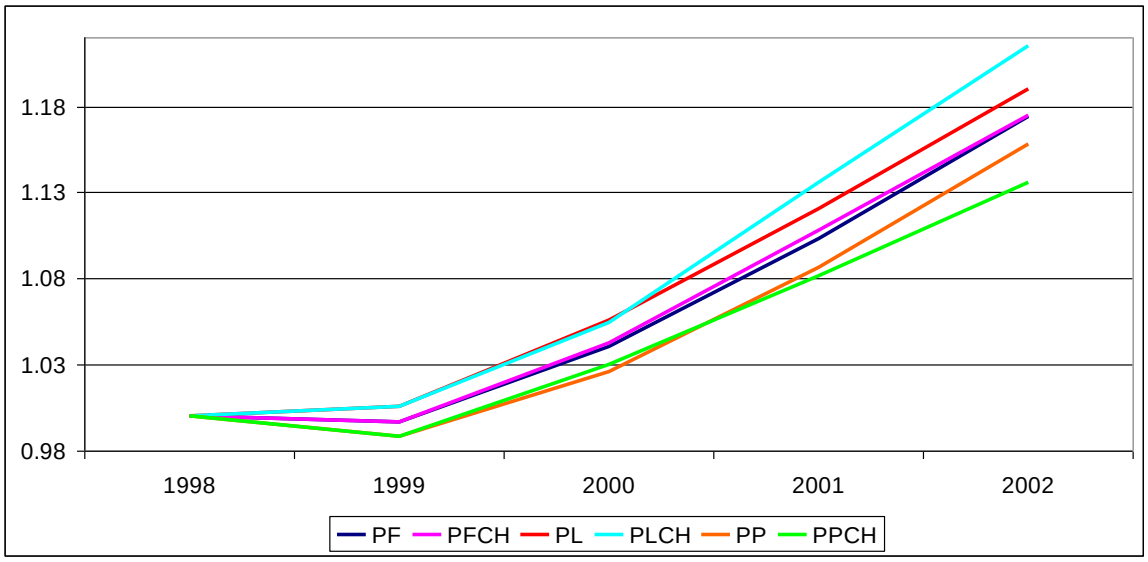
Year	P_F	P_{FCh}	P_L	P_{LCh}	P_P	P_{PCh}
1998	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1999	0.99700	0.99700	1.00554	1.00554	0.98854	0.98854
2000	1.04048	1.04237	1.05554	1.05458	1.02564	1.03030
2001	1.10316	1.10825	1.12030	1.13578	1.08627	1.08138
2002	1.17376	1.17504	1.18982	1.21546	1.15791	1.13596
Mean Diff.	0	0.00206	0.01420	0.02424	–0.01401	–0.01956

The fixed base annual Fisher indexes were on average 0.2 percentage points below their chained counterparts for the years 1999-2002, which is not a large difference, given the volatility of the underlying data. The fixed base and chained Laspeyres indexes were on average 1.4 and 2.4 percentage points above their fixed base Fisher counterparts and the fixed base and chained Paasche indexes were on average 1.4 and 2.0 percentage points below their fixed base Fisher counterparts.[13] Thus the Laspeyres and Paasche indexes are subject to some substitution bias with the Laspeyres indexes overstating inflation and the Paasche indexes understating it. Note also that the Laspeyres-Paasche spread is larger for the chained indexes than for the fixed base indexes. This indicates that the chained Fisher indexes may be subject to a small amount of chain drift.

The various indexes listed in the above Table are plotted on the following Chart.

Chart 5: Annual Mudgett Stone Fixed Base and Chained Fisher, Laspeyres and Paasche Price Indexes for Seven Kinds of Vegetables, 1998-2002

[13] More accurate measures of "bias" in the various formulae can be obtained by comparing the average geometric rates of growth for each series over the sample period. These rates of growth for the indexes listed in Table 5 are as follows: P_F: 1.04087; P_{FCh}: 1.04115; P_L: 1.04441; P_{LCh}: 1.04999; P_P: 1.03733; P_{PCh}: 1.03238. Thus the Laspeyres fixed base and chained indexes grow on average 0.35 and 0.91 percentage points more rapidly than their fixed base Fisher index counterparts while the Paasche fixed base and chained indexes grow on average 0.35 and 0.85 percentage points less rapidly than their fixed base Fisher index counterparts. In subsequent Tables, we will continue to report mean differences of the various indexes relative to our preferred alternative because it is easier to generate these mean differences as opposed to taking differences between average geometric rates of growth.



It can be seen that the fixed base and chained Fisher indexes are quite close and the two Laspeyres indexes are well above and the two Paasche indexes are well below the two Fisher indexes. It can also be seen that there are no traces of seasonal fluctuations in the above indexes; all of them are very smooth.

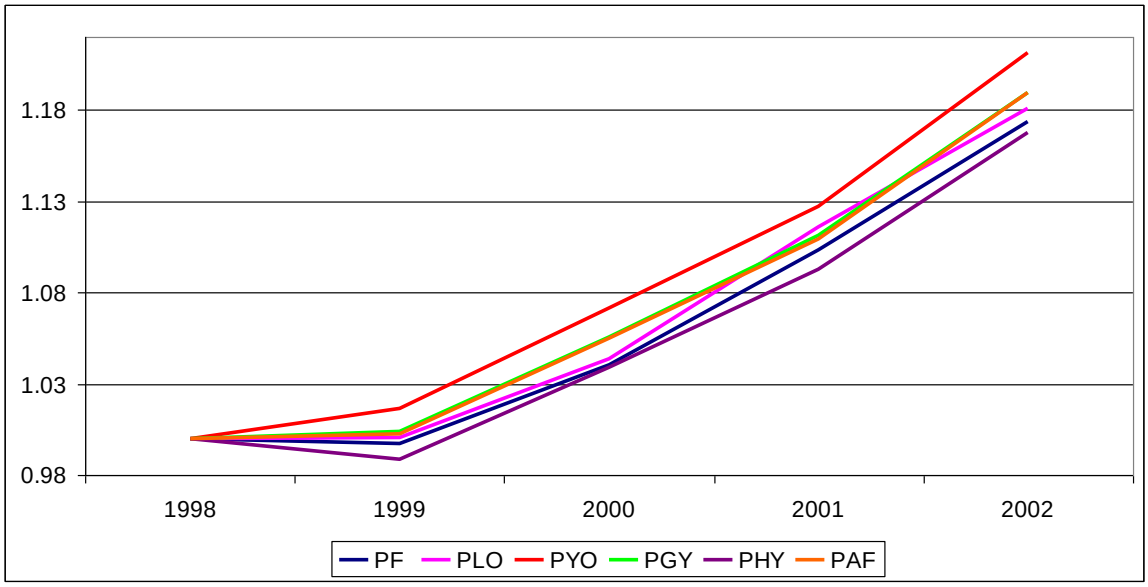
Recall our earlier discussion of the year over year fixed base Lowe, Young, Geometric Young, Harmonic Young and Approximate Fisher indexes which were defined for month m by equations (1)-(5) above. It is straightforward to modify these definitions to define the Mudgett Stone annual counterparts to these indexes. Basically, the resulting annual Lowe indexes for each year 1998-2002 will use the annual basket for the 84 commodities that pertains to 1997 and the other annual indexes will use the 1997 expenditure shares for the 84 commodities as weights. The resulting annual indexes, P_{Lo}, P_{Yo}, P_{GY}, P_{HY} and P_{AF}, are listed below in Table 6 along with our preferred index, the annual Mudgett Stone Fixed Base Fisher index, P_F.

Table 6: Annual Mudgett Stone Fixed Base Fisher, Lowe, Young, Geometric Young, Harmonic Young and Approximate Fisher Price Indexes

Year	P_F	P_{Lo}	P_{Yo}	P_{GY}	P_{HY}	P_{AF}
1998	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1999	0.99700	1.00054	1.01642	1.00374	0.98895	1.00259
2000	1.04048	1.04351	1.07148	1.05547	1.03868	1.05495
2001	1.10316	1.11565	1.12702	1.11126	1.09246	1.10960
2002	1.17376	1.18091	1.21143	1.18945	1.16786	1.18945
Mean Diff.	0	0.00655	0.02799	0.01138	–0.00661	0.01055

It can be seen that all of the "practical" indexes are on average well above the corresponding fixed base Fisher indexes, with the exception of the Harmonic Young indexes, which average 0.66 percentage points below the corresponding Fisher indexes over the years 1999-2002. Thus all of these alternative indexes suffer from fairly substantial substitution biases. The indexes in Table 6 are plotted in Chart 6 below.

Chart 6: Annual Mudgett Stone Fixed Base Fisher, Lowe, Young, Geometric Young, Harmonic Young and Approximate Fisher Price Indexes



From the above Chart, it can be seen that the fixed base Young index is well above our preferred fixed base Fisher index and that the Lowe index ends up not too far above the Fisher index.

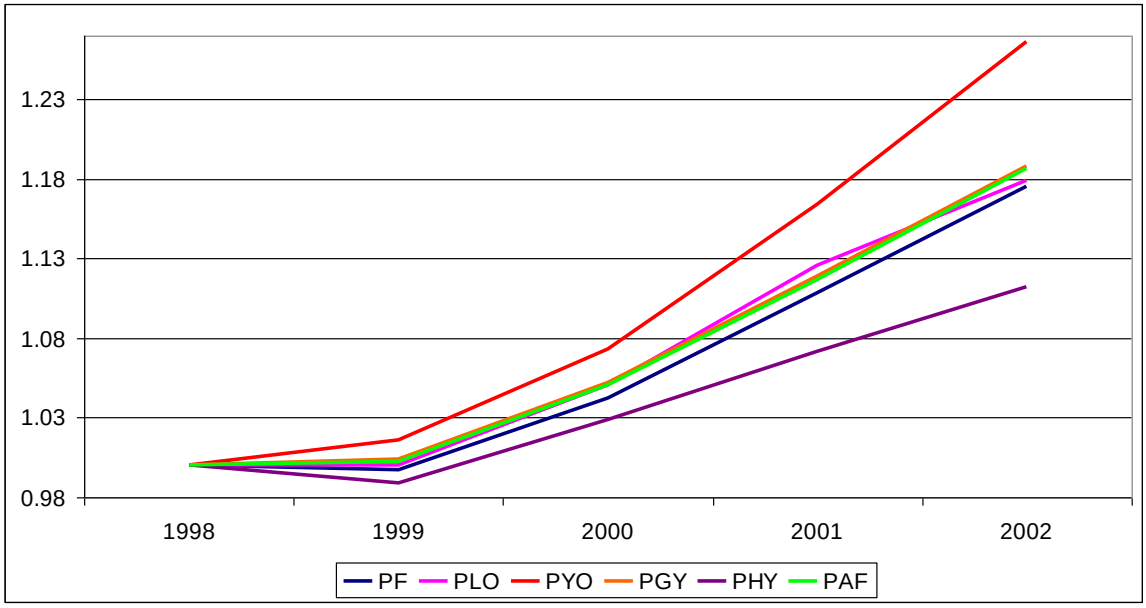
In the previous section, we explained how chained versions of the Lowe, Young, Geometric Young, Harmonic Young and Approximate Fisher indexes could be constructed for the year over year monthly indexes. The same logic can be applied to annual indexes and so chained versions of the annual Mudgett Stone indexes listed in Table 6 above are listed in Table 7 below.

Table 7: Annual Mudgett Stone Chained Fisher, Lowe, Young, Geometric Young, Harmonic Young and Approximate Fisher Price Indexes

Year	P_{FCh}	P_{LoCh}	P_{YoCh}	P_{GYCh}	P_{HYCh}	P_{AFCh}
1998	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1999	0.99700	1.00054	1.01642	1.00374	0.98895	1.00259
2000	1.04237	1.05029	1.07340	1.05189	1.02858	1.05075
2001	1.10825	1.12606	1.16425	1.11890	1.07175	1.11704
2002	1.17504	1.17882	1.26595	1.18780	1.11222	1.18660
Mean Diff.	0	0.00826	0.04934	0.00992	–0.03029	0.00858

As was the case with the fixed base indexes, all of the practical chained indexes lie above their chained Fisher index counterparts with the exception of the chained Harmonic Young indexes which lie below their Fisher counterparts. The indexes in Table 7 are plotted in Chart 7 below.

Chart 7: Annual Mudgett Stone Chained Fisher, Lowe, Young, Geometric Young, Harmonic Young and Approximate Fisher Price Indexes



The large upward biases in the chained Young indexes and the large downward biases in the chained Harmonic Young indexes are readily apparent. As usual, the Geometric Young indexes are very close to their Approximate Fisher counterparts. The chained Lowe index ends up being very close to the chained Fisher index for 2002 but for other observations, the Lowe indexes are well above their Fisher counterparts.

4. Rolling Year Mudgett Stone Indexes

In this section, we will calculate Rolling Year Mudgett Stone fixed base and chained indexes using the Laspeyres, Paasche and Fisher formulae and the monthly price and quantity data for the years 1998-2002. As a check on our computations, the Rolling Year indexes for December 1998, 1999, 2000, 2001 and 2002 should coincide with their annual counterparts listed in the previous section; i.e., when the rolling year becomes a calendar year, the resulting indexes become the usual Mudgett Stone annual indexes defined in the previous section.

Some additional explanation on how the indexes get started is required. In Table 8 below, the first entry is for December 1998. For this entry, the data for the 12 months ending in December 1998 are compared with the corresponding data in the base year, which is also 1998.[14] Thus all of the indexes will equal one for this first period, since the same price and quantity data are used for both years in the index number comparison. Now consider the entry for January, 1999. For these index number comparisons, we drop the data for January 1998 and replace it with the price and quantity data for January 1999. Thus the data for the new rolling year consists of the January 1999 price and quantity data and the

[14] There are 84 prices and quantities in each price and quantity vector.

February to December price and quantity data for 1998. The new rolling year p and q vectors are compared to the base year p and q vectors, which are just the p and q vectors pertaining to the data for calendar year 1998. Thus only 7 prices out of the 84 prices in the two p vectors will be different and only 7 quantities out of the 84 quantities in the two q vectors will be different when we make the index number comparisons that correspond to the 1999-1 entry in Table 8. Now consider the entry for February 1999. The base period p and q vectors remain the same but now the current period p and q vectors drop the 7 February 1998 prices and quantities from the 1991-1 comparison vectors and replace them with the 7 February 1999 prices and quantities. Thus for these 1999:2 index number comparisons, only 14 prices out of the 84 prices in the two p vectors will be different and only 14 quantities out of the 84 quantities in the two q vectors will be different when we make the index number comparisons that correspond to the 1999:2 entry in Table 8. This process of dropping the data pertaining to the same month in the last year and adding the data for the current month for the rolling year p and q vectors continues until we reach the end of the sample period. The updated rolling year p and q vectors are compared to the corresponding (fixed) p and q vectors for the base year using the Laspeyres, Paasche and Fisher formulae. The resulting indexes are listed as P_L, P_P and P_F in Table 8 below. As a check on our computations, these indexes for 1999:12, 2000:12, 2001:12 and 2002:12 should coincide with the entries for the years 1999-2002 for the P_L, P_P and P_F indexes listed in Table 5 (which they do). Thus these Rolling Year indexes can be viewed as an extension of the Mudgett Stone methodology (which applied to calendar year comparisons of prices and quantities) to comparisons of the last 12 months of price and quantity data to the price and quantity data pertaining to a base year. The resulting series can be viewed as a seasonally adjusted price series that is centered in the middle of the current rolling year.

It is also necessary to explain how the chained indexes listed in Table 8 below were constructed. The chained index numbers listed in Table 8 are exactly equal to their fixed base counterparts for the first 24 months in the Table; i.e., the fixed base and chained indexes coincide (for each formula) for December 1998 through to November 2000. However, when we reach December 2000, the fixed base indexes compare the price and quantity data pertaining to 2000 with the corresponding data in 1998 but the chained indexes use chain links; i.e., in order to calculate the Laspeyres index P_{LCh} for December 2000, we first calculate the chain link that compares the p and q vectors for 2000 with the p and q vector for 1999 and then we multiply this chain link by the index value for December 1999. On the other hand, the fixed base Laspeyres index P_L for December 2000 directly compares the p and q vectors for 2000 with the p and q vector for 1998. Now consider the chained indexes for January 2001. For the fixed base indexes, the p and q vectors for the 12 consecutive months ending in January 2001 are compared with the p and q vectors for the base year 1998. But for the chained indexes, the data for the current rolling year ending in January 2001 are compared with the rolling year ending in January 2000, which generates a year over year chain link. Then this chain link is multiplied by the January 2000 index level to give us the chained entry for January 2001. The chained indexes for February 2001 are generated in a similar fashion. First the chain link index that compares the rolling year data ending in February 2001 with the corresponding rolling year data ending in February 2000 (for a particular formula) is calculated. Then

this chain link is multiplied by the February 2000 index level to generate the February 2001 level. And so on. The resulting indexes are listed as P_{LCh}, P_{PCh} and P_{FCh} in Table 8 below. As a check on our computations, these indexes for 1999:12, 2000:12, 2001:12 and 2002:12 should coincide with the entries for the years 1999-2002 for the P_{LCh}, P_{PCh} and P_{FCh} indexes listed in Table 5.

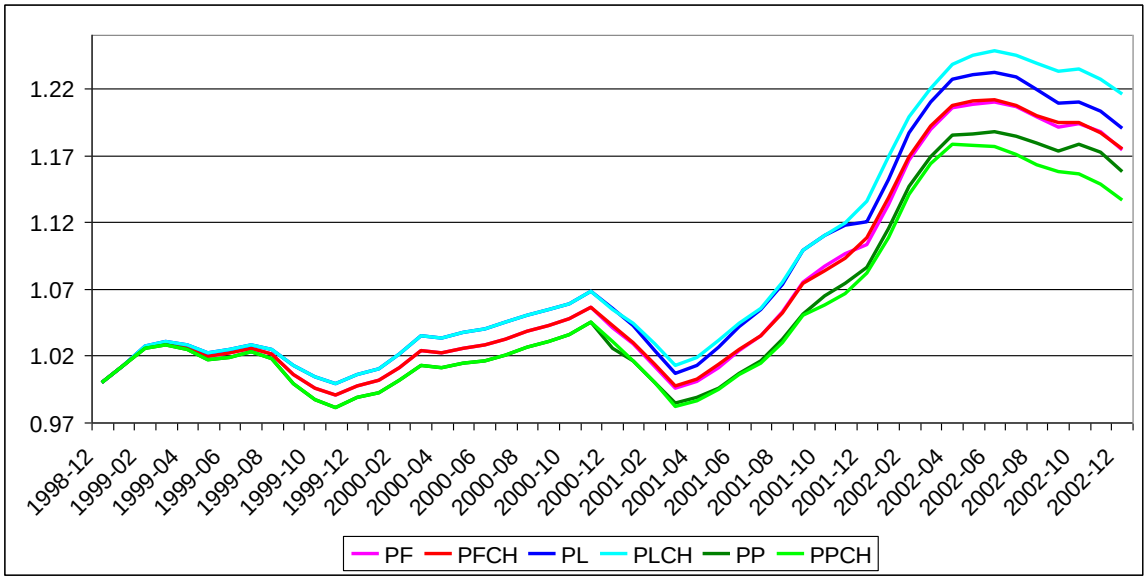
Table 8: Rolling Year Mudgett Stone Fixed Base and Chained Fisher, Laspeyres and Paasche Price Indexes for Seven Kinds of Vegetables, 1998:12-2002:12

Year/Month	P_F	P_{FCh}	P_L	P_{LCh}	P_P	P_{PCh}
1998-12	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1999-01	1.01271	1.01271	1.01264	1.01264	1.01278	1.01278
1999-02	1.02602	1.02602	1.02691	1.02691	1.02513	1.02513
1999-03	1.02955	1.02955	1.03094	1.03094	1.02817	1.02817
1999-04	1.02624	1.02624	1.02809	1.02809	1.02439	1.02439
1999-05	1.01943	1.01943	1.02203	1.02203	1.01684	1.01684
1999-06	1.02167	1.02167	1.02433	1.02433	1.01901	1.01901
1999-07	1.02536	1.02536	1.02825	1.02825	1.02249	1.02249
1999-08	1.02108	1.02108	1.02446	1.02446	1.01771	1.01771
1999-09	1.00573	1.00573	1.01278	1.01278	0.99873	0.99873
1999-10	0.99595	0.99595	1.00451	1.00451	0.98747	0.98747
1999-11	0.99027	0.99027	0.99916	0.99916	0.98146	0.98146
1999-12	0.99700	0.99700	1.00554	1.00554	0.98854	0.98854
2000-01	1.00137	1.00137	1.01046	1.01046	0.99236	0.99236
2000-02	1.01108	1.01108	1.02092	1.02092	1.00133	1.00133
2000-03	1.02353	1.02353	1.03444	1.03444	1.01273	1.01273
2000-04	1.02238	1.02238	1.03353	1.03353	1.01135	1.01135
2000-05	1.02582	1.02582	1.03764	1.03764	1.01414	1.01414
2000-06	1.02760	1.02760	1.03965	1.03965	1.01570	1.01570
2000-07	1.03257	1.03257	1.04488	1.04488	1.02040	1.02040
2000-08	1.03814	1.03814	1.05032	1.05032	1.02610	1.02610
2000-09	1.04220	1.04220	1.05408	1.05408	1.03045	1.03045
2000-10	1.04739	1.04739	1.05889	1.05889	1.03601	1.03601
2000-11	1.05633	1.05633	1.06792	1.06792	1.04487	1.04487
2000-12	1.04048	1.04237	1.05554	1.05458	1.02564	1.03030
2001-01	1.02915	1.03001	1.04271	1.04458	1.01576	1.01564
2001-02	1.01193	1.01389	1.02373	1.02851	1.00026	0.99949
2001-03	0.99556	0.99750	1.00668	1.01301	0.98456	0.98222
2001-04	1.00050	1.00237	1.01242	1.01856	0.98873	0.98644
2001-05	1.01105	1.01327	1.02642	1.03173	0.99591	0.99514
2001-06	1.02402	1.02444	1.04174	1.04379	1.00660	1.00545
2001-07	1.03492	1.03480	1.05408	1.05546	1.01610	1.01455
2001-08	1.05274	1.05190	1.07345	1.07450	1.03244	1.02977
2001-09	1.07455	1.07406	1.09889	1.09893	1.05075	1.04976
2001-10	1.08679	1.08363	1.10958	1.10970	1.06447	1.05818
2001-11	1.09585	1.09261	1.11793	1.11927	1.07421	1.06659
2001-12	1.10316	1.10825	1.12030	1.13578	1.08627	1.08138
2002-01	1.13312	1.13793	1.15183	1.16848	1.11471	1.10817
2002-02	1.16648	1.16911	1.18649	1.19867	1.14681	1.14028
2002-03	1.18927	1.19171	1.20974	1.22007	1.16914	1.16401

2002-04	1.20565	1.20754	1.22649	1.23742	1.18515	1.17839
2002-05	1.20762	1.21032	1.23022	1.24456	1.18542	1.17703
2002-06	1.20976	1.21147	1.23223	1.24782	1.18770	1.17618
2002-07	1.20610	1.20703	1.22828	1.24433	1.18433	1.17084
2002-08	1.19879	1.19975	1.21926	1.23825	1.17868	1.16245
2002-09	1.19102	1.19444	1.20906	1.23278	1.17326	1.15728
2002-10	1.19377	1.19458	1.20934	1.23482	1.17840	1.15564
2002-11	1.18731	1.18669	1.20259	1.22659	1.17224	1.14808
2002-12	1.17376	1.17504	1.18982	1.21546	1.15791	1.13596
Mean Diff.	0	0.00085	0.0159	0.0229	–0.01566	–0.02069

Due to fact that the spread between the *chained* Paasche and Laspeyres indexes is greater than the spread between the *fixed base* Paasche and Laspeyres indexes, our preferred index is the fixed base Fisher index. The average difference between each index and the fixed base Fisher index is listed in the last row of Table 8. This mean difference is calculated over the 37 observations starting at 1999:12 and ending at 2002:12 (since the fixed base and chained indexes coincide for each formula for the earlier observations). From the above Table, it can be seen that the differences between the Rolling Year fixed base and chained Fisher indexes are very small. The fixed base and chained Laspeyres indexes were on average (over the last 37 observations) 1.6 and 2.3 percentage points above their fixed base Fisher counterparts while the fixed base and chained Paasche indexes were on average 1.6 and 2.1 percentage points below their fixed base Fisher counterparts. Thus the Rolling Year Laspeyres and Paasche indexes exhibit a fair amount of substitution bias. The indexes in Table 8 are plotted in Chart 8 below.

Chart 8: Rolling Year Mudgett Stone Fixed Base and Chained Fisher, Laspeyres and Paasche Price Indexes for Seven Kinds of Vegetables, 1998:12-2002:12



It can be seen that there are no obvious seasonal fluctuations in the above indexes. It can also be seen that the Rolling Year chained Laspeyres and Paasche indexes are well above

and below our preferred target index, P_F. Finally it can also be seen that there is little difference between P_F and its chained counterpart P_{FCh}.

It would be possible to calculate Rolling Year counterparts to Tables 6 and 7 in the previous section, where we compared the fixed base Mudgett Stone calendar year Fisher indexes with various practical indexes that relied on past quantity or expenditure vectors as weights. However, nothing new would be learned from this exercise so it is omitted here.

This completes our discussion of indexes that explicitly take seasonality into account. In the following section, we ignore the seasonality problem and just calculate month to month indexes of the usual type.

5. Standard Month to Month Indexes

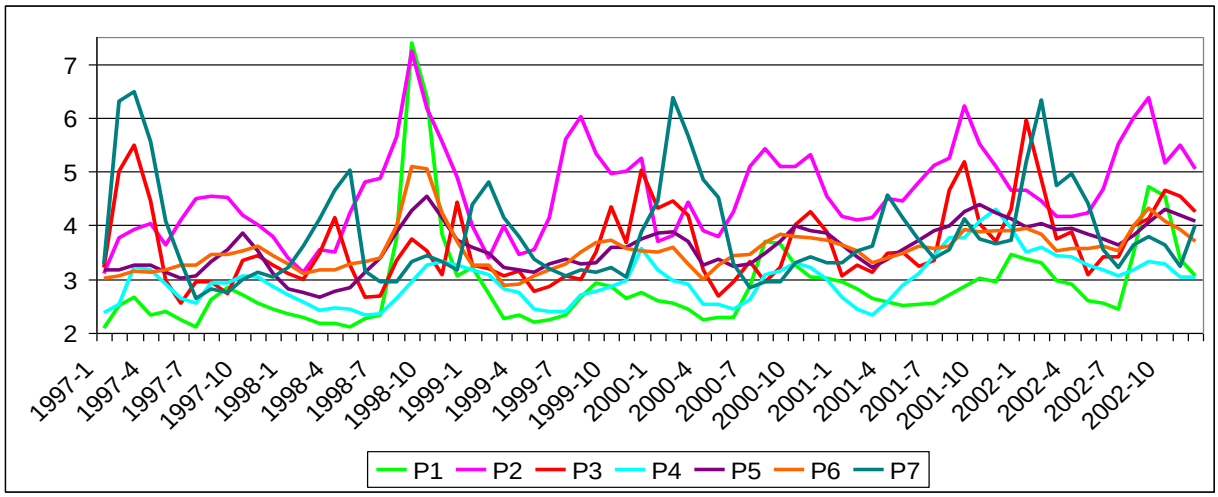
The basic price and expenditure share data over the 6 years are listed in Tables 9 and 10 below and are plotted in Charts 9 and 10.

Table 9: Monthly Prices for Seven Kinds of Vegetable, 1997:1-2002:12

Year/Month	$p_1^{y,m}$	$p_2^{y,m}$	$p_3^{y,m}$	$p_4^{y,m}$	$p_5^{y,m}$	$p_6^{y,m}$	$p_7^{y,m}$
1997-01	2.09	3.10	3.21	2.37	3.16	3.01	3.28
1997-02	2.50	3.77	5.00	2.54	3.16	3.05	6.31
1997-03	2.67	3.92	5.49	3.23	3.26	3.14	6.49
1997-04	2.34	4.04	4.46	3.17	3.26	3.12	5.55
1997-05	2.40	3.63	2.98	2.91	3.13	3.18	4.07
1997-06	2.24	4.10	2.56	2.64	3.02	3.26	3.33
1997-07	2.12	4.50	2.96	2.56	3.07	3.25	2.63
1997-08	2.61	4.54	2.96	2.93	3.33	3.46	2.82
1997-09	2.83	4.51	2.73	2.93	3.55	3.45	2.74
1997-10	2.71	4.19	3.35	3.05	3.86	3.53	2.99
1997-11	2.55	4.00	3.44	3.04	3.52	3.61	3.12
1997-12	2.45	3.80	3.27	2.86	3.11	3.44	3.03
1998-01	2.36	3.40	3.11	2.71	2.81	3.29	3.21
1998-02	2.28	3.13	2.99	2.58	2.76	3.10	3.61
1998-03	2.18	3.54	3.47	2.42	2.67	3.17	4.12
1998-04	2.18	3.51	4.14	2.46	2.77	3.18	4.64
1998-05	2.12	4.24	3.26	2.44	2.84	3.28	5.03
1998-06	2.27	4.80	2.67	2.34	3.12	3.32	3.14
1998-07	2.33	4.88	2.69	2.36	3.39	3.39	2.94
1998-08	3.76	5.65	3.35	2.65	3.88	3.99	2.95
1998-09	7.40	7.24	3.75	2.94	4.27	5.10	3.33
1998-10	6.38	6.18	3.53	3.25	4.54	5.04	3.43
1998-11	3.84	5.56	3.09	3.32	4.14	4.23	3.32
1998-12	3.05	4.89	4.43	3.26	3.72	3.70	3.16
1999-01	3.21	3.99	3.25	3.18	3.59	3.26	4.39
1999-02	2.72	3.40	3.19	3.09	3.47	3.26	4.80
1999-03	2.27	3.98	3.05	2.81	3.21	2.89	4.15
1999-04	2.34	3.46	3.15	2.76	3.18	2.91	3.80

1999-05	2.20	3.54	2.78	2.45	3.12	3.07	3.38
1999-06	2.24	4.15	2.86	2.4	3.28	3.16	3.20
1999-07	2.33	5.61	3.05	2.39	3.38	3.28	3.07
1999-08	2.67	6.02	2.99	2.71	3.29	3.50	3.17
1999-09	2.93	5.33	3.51	2.77	3.31	3.68	3.13
1999-10	2.86	4.95	4.34	2.86	3.58	3.72	3.22
1999-11	2.65	5.00	3.67	2.97	3.59	3.56	3.03
1999-12	2.76	5.24	5.02	3.57	3.74	3.52	3.88
2000-01	2.60	3.69	4.33	3.18	3.86	3.50	4.45
2000-02	2.56	3.82	4.45	2.98	3.88	3.60	6.38
2000-03	2.44	4.44	4.18	2.90	3.70	3.29	5.67
2000-04	2.24	3.89	3.16	2.53	3.25	3.00	4.85
2000-05	2.28	3.79	2.68	2.52	3.38	3.27	4.51
2000-06	2.29	4.25	2.95	2.44	3.24	3.43	3.28
2000-07	2.86	5.09	3.33	2.61	3.28	3.45	2.85
2000-08	3.71	5.42	2.96	3.09	3.49	3.68	2.94
2000-09	3.65	5.10	3.21	3.14	3.69	3.84	2.96
2000-10	3.25	5.09	4.02	3.30	3.99	3.79	3.31
2000-11	3.03	5.32	4.26	3.21	3.89	3.77	3.41
2000-12	3.02	4.54	3.87	2.99	3.85	3.73	3.31
2001-01	2.96	4.16	3.06	2.67	3.63	3.63	3.31
2001-02	2.81	4.10	3.26	2.45	3.41	3.53	3.52
2001-03	2.65	4.14	3.13	2.34	3.21	3.30	3.62
2001-04	2.57	4.49	3.47	2.58	3.36	3.39	4.57
2001-05	2.05	4.46	3.50	2.88	3.55	3.49	4.14
2001-06	2.52	4.80	3.24	3.10	3.73	3.62	3.72
2001-07	2.55	5.12	3.35	3.43	3.90	3.56	3.40
2001-08	2.71	5.25	4.64	3.76	3.99	3.61	3.54
2001-09	2.87	6.21	5.18	3.77	4.26	3.93	4.11
2001-10	3.01	5.51	4.03	4.08	4.38	3.88	3.75
2001-11	2.95	5.10	3.70	4.29	4.23	3.89	3.65
2001-12	3.46	4.66	4.29	3.94	4.12	3.91	3.72
2002-01	3.38	4.64	5.96	3.51	3.97	3.95	5.19
2002-02	3.30	4.45	4.86	3.60	4.03	3.83	6.34
2002-03	2.97	4.17	3.75	3.44	3.93	3.53	4.74
2002-04	2.91	4.17	3.87	3.42	3.94	3.57	4.95
2002-05	2.60	4.24	3.09	3.27	3.83	3.57	4.40
2002-06	2.56	4.68	3.41	3.17	3.75	3.62	3.55
2002-07	2.44	5.51	3.41	3.07	3.63	3.52	3.22
2002-08	3.49	6.00	3.99	3.16	3.82	3.98	3.63
2002-09	4.72	6.38	4.11	3.33	4.06	4.31	3.79
2002-10	4.54	5.15	4.66	3.28	4.30	4.08	3.64
2002-11	3.36	5.50	4.53	3.03	4.18	3.93	3.24
2002-12	3.07	5.04	4.25	3.03	4.08	3.69	4.01

Chart 9: Monthly Prices for Seven Kinds of Vegetable, 1997:1-2002:12



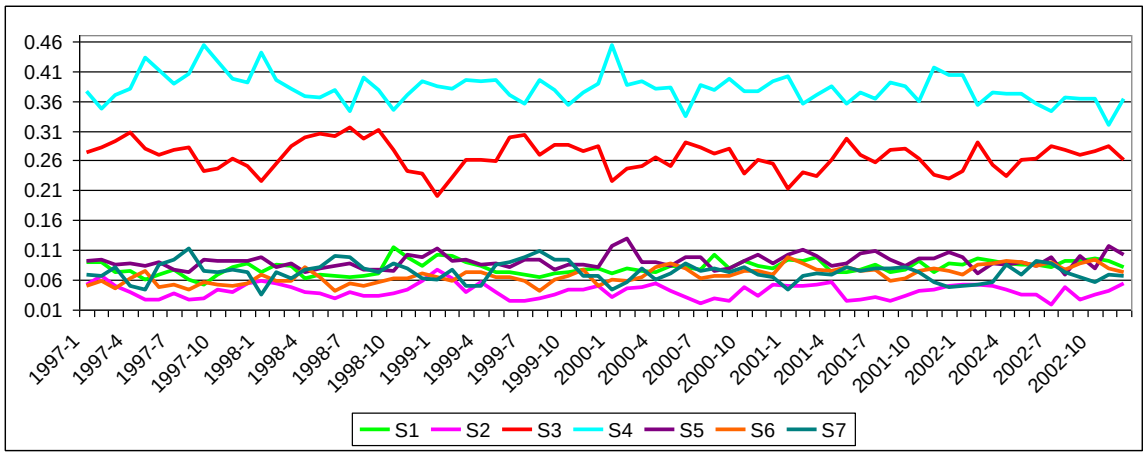
The tremendous seasonality in the price data becomes apparent upon viewing the above Chart.

Table 10: Monthly Expenditure Shares for Seven Kinds of Vegetable, 1997:1-2002:12

Year/Month	$s_1^{y,m}$	$s_2^{y,m}$	$s_3^{y,m}$	$s_4^{y,m}$	$s_5^{y,m}$	$s_6^{y,m}$	$s_7^{y,m}$
1997-01	0.08916	0.05245	0.27273	0.37587	0.09091	0.05070	0.06818
1997-02	0.08898	0.06568	0.28178	0.34746	0.09322	0.05720	0.06568
1997-03	0.07364	0.05039	0.29264	0.37016	0.08527	0.04651	0.08140
1997-04	0.07401	0.03971	0.30686	0.38087	0.08664	0.06137	0.05054
1997-05	0.06067	0.02740	0.27984	0.43249	0.08219	0.07436	0.04305
1997-06	0.06858	0.02655	0.26991	0.41150	0.08850	0.04867	0.08628
1997-07	0.07657	0.03712	0.27842	0.38747	0.07657	0.05104	0.09281
1997-08	0.05981	0.02632	0.28230	0.40431	0.07177	0.04306	0.11244
1997-09	0.05134	0.02934	0.24205	0.45232	0.09291	0.05623	0.07579
1997-10	0.06841	0.04427	0.24547	0.42656	0.09054	0.05231	0.07243
1997-11	0.08135	0.03968	0.26389	0.39683	0.09127	0.04960	0.07738
1997-12	0.08705	0.05357	0.25000	0.39063	0.09152	0.05357	0.07366
1998-01	0.07317	0.05765	0.22616	0.44124	0.09756	0.06874	0.03548
1998-02	0.08515	0.05347	0.25545	0.39406	0.08119	0.05743	0.07327
1998-03	0.08245	0.04863	0.28330	0.38055	0.08668	0.05708	0.06131
1998-04	0.06299	0.03937	0.29921	0.36811	0.07283	0.08071	0.07677
1998-05	0.06800	0.03800	0.30400	0.36600	0.07800	0.06400	0.08200
1998-06	0.06621	0.02968	0.30137	0.37900	0.08219	0.04110	0.10046
1998-07	0.06506	0.03855	0.31566	0.34217	0.08675	0.05301	0.09880
1998-08	0.06573	0.03286	0.29577	0.39906	0.07746	0.04930	0.07981
1998-09	0.07157	0.03288	0.31141	0.37718	0.07737	0.05609	0.07350
1998-10	0.11470	0.03763	0.27778	0.34409	0.07527	0.06272	0.08781
1998-11	0.09867	0.04364	0.24288	0.37002	0.10247	0.06262	0.07970
1998-12	0.08264	0.05785	0.23760	0.39256	0.09711	0.07025	0.06198
1999-01	0.10097	0.07767	0.20000	0.38447	0.11262	0.06408	0.06019
1999-02	0.09980	0.06188	0.23154	0.37924	0.09182	0.05788	0.07784
1999-03	0.08861	0.04008	0.26160	0.39451	0.09283	0.07173	0.05063

1999-04	0.08251	0.05697	0.26130	0.39293	0.08448	0.07269	0.04912
1999-05	0.07258	0.03831	0.25806	0.39516	0.08669	0.06452	0.08468
1999-06	0.07283	0.02362	0.29921	0.37008	0.08071	0.06496	0.08858
1999-07	0.06754	0.02397	0.30283	0.35512	0.09368	0.05882	0.09804
1999-08	0.06497	0.02784	0.26914	0.39443	0.09281	0.04176	0.10905
1999-09	0.07143	0.03475	0.28571	0.37838	0.07722	0.05985	0.09266
1999-10	0.07221	0.04376	0.28665	0.35230	0.08534	0.06565	0.09409
1999-11	0.07724	0.04384	0.27557	0.37370	0.08559	0.07724	0.06681
1999-12	0.07853	0.04887	0.28447	0.38918	0.08202	0.05061	0.06632
2000-01	0.06966	0.03146	0.22472	0.45393	0.11685	0.06067	0.04270
2000-02	0.07857	0.04643	0.24643	0.38571	0.12857	0.05893	0.05536
2000-03	0.07466	0.04715	0.25147	0.39293	0.09037	0.06483	0.07859
2000-04	0.07312	0.05336	0.26482	0.37945	0.08893	0.08103	0.05929
2000-05	0.08400	0.04200	0.25000	0.38200	0.08400	0.08800	0.07000
2000-06	0.08121	0.03016	0.29002	0.33411	0.09745	0.07889	0.08817
2000-07	0.07658	0.02027	0.28153	0.38739	0.09685	0.06306	0.07432
2000-08	0.10108	0.02796	0.27097	0.37849	0.07527	0.06667	0.07957
2000-09	0.07911	0.02434	0.27992	0.39757	0.07911	0.06694	0.07302
2000-10	0.09091	0.04793	0.23802	0.37521	0.09256	0.07438	0.08099
2000-11	0.08301	0.03282	0.26062	0.37645	0.10232	0.07529	0.06950
2000-12	0.07854	0.05172	0.25479	0.39272	0.08812	0.06897	0.06513
2001-01	0.09293	0.05051	0.21212	0.40202	0.10101	0.09697	0.04444
2001-02	0.09193	0.04933	0.23991	0.35426	0.10987	0.08744	0.06726
2001-03	0.09836	0.05123	0.23361	0.36885	0.10041	0.07787	0.06967
2001-04	0.07186	0.05589	0.26148	0.38323	0.08383	0.07585	0.06786
2001-05	0.07240	0.02489	0.29638	0.35520	0.08824	0.08145	0.08145
2001-06	0.07658	0.02703	0.27027	0.37387	0.10360	0.07432	0.07432
2001-07	0.08454	0.03093	0.25773	0.36289	0.10928	0.07629	0.07835
2001-08	0.07372	0.02457	0.27788	0.39130	0.09452	0.05860	0.07940
2001-09	0.07599	0.03282	0.27979	0.38515	0.08290	0.06218	0.08117
2001-10	0.09075	0.04110	0.26370	0.35959	0.09589	0.07534	0.07363
2001-11	0.07329	0.04397	0.23616	0.41531	0.09609	0.07818	0.05700
2001-12	0.08741	0.05070	0.22902	0.40385	0.10664	0.07517	0.04720
2002-01	0.08541	0.05160	0.24199	0.40391	0.09786	0.06940	0.04982
2002-02	0.09589	0.05175	0.29072	0.35312	0.07154	0.08524	0.05175
2002-03	0.09233	0.05052	0.25261	0.37456	0.08711	0.08711	0.05575
2002-04	0.08703	0.04433	0.23317	0.37274	0.08539	0.09195	0.08539
2002-05	0.08429	0.03448	0.26054	0.37165	0.09004	0.09004	0.06897
2002-06	0.08594	0.03516	0.26367	0.35547	0.08398	0.08398	0.09180
2002-07	0.08147	0.01833	0.28310	0.34216	0.09776	0.08961	0.08758
2002-08	0.09074	0.04726	0.27788	0.36484	0.06805	0.07750	0.07372
2002-09	0.09109	0.02632	0.26923	0.36235	0.09919	0.08704	0.06478
2002-10	0.09601	0.03442	0.27536	0.36413	0.07971	0.09420	0.05616
2002-11	0.09217	0.04174	0.28348	0.32000	0.11652	0.07826	0.06783
2002-12	0.08202	0.05410	0.26003	0.36300	0.10122	0.07330	0.06632
Mean	0.08041	0.04101	0.26661	0.38096	0.09046	0.06795	0.07259

Chart 10: Monthly Expenditure Shares for Seven Kinds of Vegetable, 1997:1-2002:12



It can be seen that while the monthly expenditure shares also have some substantial fluctuations, the amount of variability in the expenditure shares is far less than the variability in the monthly prices. It can also be seen that commodity groups 3 and 4 are the most important ones (cucumbers and potatoes respectively); the other expenditure shares are generally below 11%.

In Table 11 below, month to month fixed base Fisher, Laspeyres and Paasche indexes, P_F, P_L and P_P, are calculated along with their chained counterparts, P_{FCh}, P_{LCh} and P_{PCh}.

Table 11: Monthly Fixed Base and Chained Fisher, Laspeyres and Paasche Price Indexes for Seven Kinds of Vegetables, 1997:1-2002:12

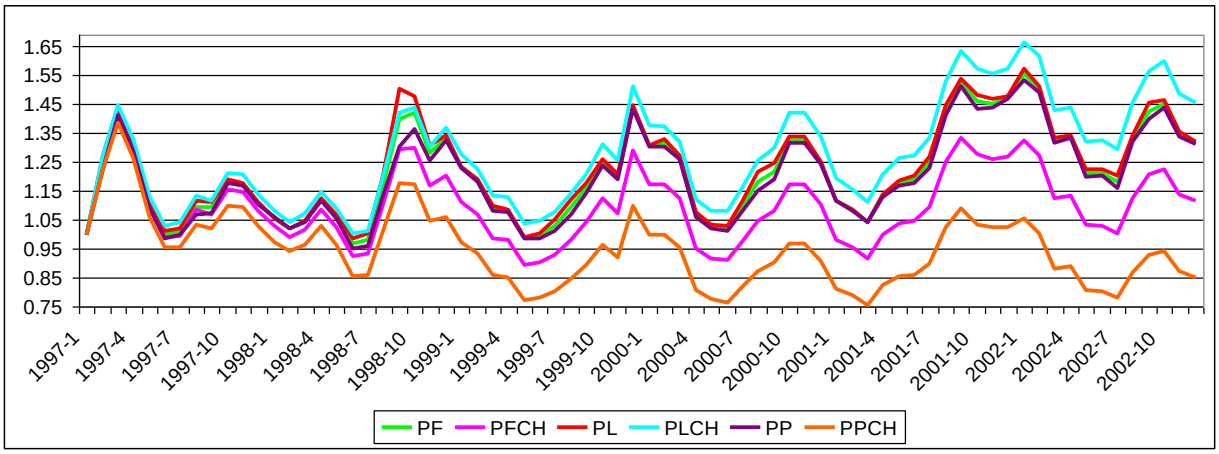
Year/Month	P_F	P_{FCh}	P_L	P_{LCh}	P_P	P_{PCh}
1997-01	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1997-02	1.24729	1.24729	1.27153	1.27153	1.22352	1.22352
1997-03	1.42582	1.41629	1.44052	1.44595	1.41128	1.38723
1997-04	1.30070	1.29643	1.31156	1.32822	1.28992	1.26539
1997-05	1.09801	1.09192	1.10671	1.12772	1.08937	1.05725
1997-06	0.99886	0.98951	1.01214	1.02746	0.98575	0.95297
1997-07	1.00906	0.99577	1.02180	1.03958	0.99648	0.95380
1997-08	1.09279	1.08313	1.11703	1.13368	1.06908	1.03484
1997-09	1.09229	1.06760	1.11086	1.11595	1.07404	1.02134
1997-10	1.18171	1.15364	1.18750	1.21144	1.17596	1.09860
1997-11	1.17184	1.14767	1.17779	1.20600	1.16593	1.09217
1997-12	1.10676	1.08210	1.11062	1.13736	1.10292	1.02952
1998-01	1.05641	1.02667	1.05521	1.08238	1.05761	0.97382
1998-02	1.01895	0.98988	1.02010	1.04099	1.01780	0.94128
1998-03	1.04343	1.01518	1.04736	1.07101	1.03951	0.96226
1998-04	1.12030	1.08721	1.12398	1.14785	1.11662	1.02977
1998-05	1.06086	1.02470	1.06764	1.09125	1.05413	0.96221
1998-06	0.96871	0.92433	0.98696	1.00119	0.95080	0.85337
1998-07	0.98023	0.93276	1.00054	1.01053	0.96033	0.86097
1998-08	1.16568	1.10189	1.20105	1.20027	1.13136	1.01158
1998-09	1.39832	1.29310	1.50103	1.42069	1.30264	1.17696
1998-10	1.41965	1.29970	1.47889	1.43953	1.36279	1.17344

1998-11	1.28106	1.16679	1.30632	1.30108	1.25628	1.04636
1998-12	1.33301	1.20290	1.34128	1.36744	1.32479	1.05816
1999-01	1.23186	1.11266	1.23435	1.27602	1.22936	0.97022
1999-02	1.18474	1.06890	1.18917	1.22547	1.18033	0.93233
1999-03	1.08748	0.98601	1.09626	1.13420	1.07877	0.85718
1999-04	1.07926	0.97850	1.08321	1.12787	1.07532	0.84891
1999-05	0.98725	0.89540	0.99023	1.03598	0.98428	0.77390
1999-06	0.99319	0.90229	1.00350	1.04542	0.98298	0.77876
1999-07	1.02875	0.92875	1.04879	1.07772	1.00908	0.80036
1999-08	1.09287	0.98256	1.11908	1.14006	1.06727	0.84683
1999-09	1.15816	1.04040	1.17497	1.20847	1.14159	0.89571
1999-10	1.24876	1.12425	1.26066	1.31154	1.23698	0.96370
1999-11	1.19835	1.07313	1.20671	1.25422	1.19004	0.91818
1999-12	1.44037	1.28804	1.44663	1.50961	1.43413	1.09899
2000-01	1.30623	1.17235	1.30807	1.37596	1.30439	0.99887
2000-02	1.31501	1.17161	1.32942	1.37399	1.30075	0.99903
2000-03	1.26745	1.12399	1.27400	1.32088	1.26094	0.95645
2000-04	1.06813	0.95101	1.07595	1.12073	1.06036	0.80699
2000-05	1.02732	0.91572	1.03482	1.08140	1.01989	0.77542
2000-06	1.01890	0.90932	1.02638	1.08168	1.01147	0.76442
2000-07	1.09856	0.97657	1.11670	1.16488	1.08071	0.81870
2000-08	1.18276	1.04572	1.21502	1.25433	1.15136	0.87181
2000-09	1.21659	1.08100	1.24508	1.29632	1.18876	0.90144
2000-10	1.32657	1.17334	1.33711	1.42124	1.31611	0.96868
2000-11	1.32547	1.17232	1.33660	1.42085	1.31444	0.96726
2000-12	1.24677	1.10096	1.25104	1.33754	1.24251	0.90623
2001-01	1.11589	0.98262	1.11447	1.19291	1.11732	0.80941
2001-02	1.08409	0.95404	1.08551	1.15632	1.08267	0.78715
2001-03	1.04136	0.91541	1.04332	1.10954	1.03940	0.75525
2001-04	1.13360	0.99802	1.13836	1.20903	1.12887	0.82383
2001-05	1.17620	1.03911	1.18321	1.26354	1.16923	0.85455
2001-06	1.18903	1.04614	1.20125	1.27218	1.17693	0.86026
2001-07	1.24727	1.09482	1.26685	1.33424	1.22798	0.89836
2001-08	1.42867	1.25184	1.44416	1.53047	1.41335	1.02394
2001-09	1.52607	1.33424	1.53970	1.63240	1.51257	1.09054
2001-10	1.45729	1.27580	1.48041	1.57363	1.43454	1.03434
2001-11	1.45278	1.26086	1.46996	1.55316	1.43581	1.02356
2001-12	1.47178	1.26954	1.47752	1.57125	1.46606	1.02576
2002-01	1.55319	1.32420	1.57437	1.66298	1.53229	1.05444
2002-02	1.50107	1.27179	1.51217	1.61636	1.49004	1.00068
2002-03	1.32435	1.12246	1.33248	1.42814	1.31627	0.88221
2002-04	1.33801	1.13139	1.34227	1.43916	1.33376	0.88944
2002-05	1.21180	1.03095	1.22557	1.31865	1.19819	0.80602
2002-06	1.21206	1.02932	1.22351	1.32386	1.20072	0.80032
2002-07	1.18236	1.00412	1.20458	1.29401	1.16054	0.77917
2002-08	1.33152	1.12320	1.34295	1.45298	1.32018	0.86826
2002-09	1.42609	1.20667	1.45480	1.56472	1.39794	0.93054
2002-10	1.45043	1.22631	1.46502	1.59711	1.43598	0.94160
2002-11	1.34557	1.13857	1.35562	1.48545	1.33561	0.87270
2002-12	1.31664	1.11343	1.32076	1.45512	1.31253	0.85197

Mean Diff.	0	–0.11639	0.01382	0.06525	–0.01352	–0.26716

The chained Paasche and Fisher indexes exhibit a tremendous amount of downward chain drift. The fixed base Laspeyres indexes are on average 1.38 percentage points above their fixed base Fisher counterparts while the fixed base Paasche indexes are 1.35 percentage points below their Fisher counterpart, which indicates a certain amount of substitution bias in these fixed base P_L and P_P indexes. Turning to the chained indexes, the chained Paasche and Fisher indexes are below the fixed base Fisher indexes. The chained Fisher indexes are on average a huge 11.6 percentage points below their fixed base Fisher counterparts. Note that the spread between the chained Laspeyres and Paasche indexes is much larger than the spread between the fixed base Laspeyres and Paasche indexes, indicating that there is a chain drift problem. Obviously, households stock up on vegetables when they are relatively cheap and we have a situation that is similar to the sales phenomenon that was explained in section 5 of Chapter 6, leading to a tremendous chain drift problem. The indexes in Table 11 are plotted in Chart 11 below.

Chart 11: Monthly Fixed Base and Chained Fisher, Laspeyres and Paasche Price Indexes for Seven Kinds of Vegetables, 1997:1-2002:12



It can be seen that the fixed base Fisher, Laspeyres and Paasche indexes are all tightly clustered in a narrow band. On the other hand, the chained indexes are all rather far from this cluster with the chained Laspeyres index (the top line) well above and the chained Fisher and chained Paasche (the bottom two lines) well below the fixed base indexes.

The seasonality in the data is apparent in the above Chart. However, it can be seen that the seasonal peaks and valleys are not completely regular, which makes seasonal adjustment difficult.[15]

The problem with the fixed base Fisher month to month indexes is that the first month of the sample period plays an asymmetric role. The use of Rolling Year GEKS indexes

[15] This point was noted in Diewert, Artsev and Finkel (2009).

should eliminate this asymmetry and also eliminate the chain drift problem that affects the chained indexes listed above.

6. Rolling Year GEKS Month to Month Indexes

The Rolling Year GEKS method is explained in Ivancic, Diewert and Fox (2011) and so this explanation will not be repeated here except to note that the first 13 entries in Table 12 below were obtained by applying the GEKS multilateral index number method to the first 13 months of price and quantity data, running from January 1997 through January 1998. The RYGEKS entry for February 1998 was obtained via a two stage procedure. In stage 1, GEKS multilateral indexes were constructed for the 13 consecutive months of data starting in February of 1997 and ending in February of 1998. Then the rate of change in the resulting multilateral indexes going from January 1998 to February 1998 was used to update the January 1998 index level in the second stage. Similarly, the RYGEKS entry for March 1998 was obtained by first constructing GEKS multilateral indexes for the 13 consecutive months of data ending in March of 1998 and then the rate of change in the resulting multilateral indexes going from February 1998 to March 1998 was used to update the February 1998 index level in the second stage. And so on. The resulting RYGEKS indexes are listed in Table 12 below along with the fixed base and chained month to month Fisher indexes, P_F and P_{FCh}, that were listed in Table 11 above.

Table 12: Monthly Rolling Year GEKS, Fixed Base and Chained Fisher Price Indexes for Seven Kinds of Vegetables, 1997:1-2002:12

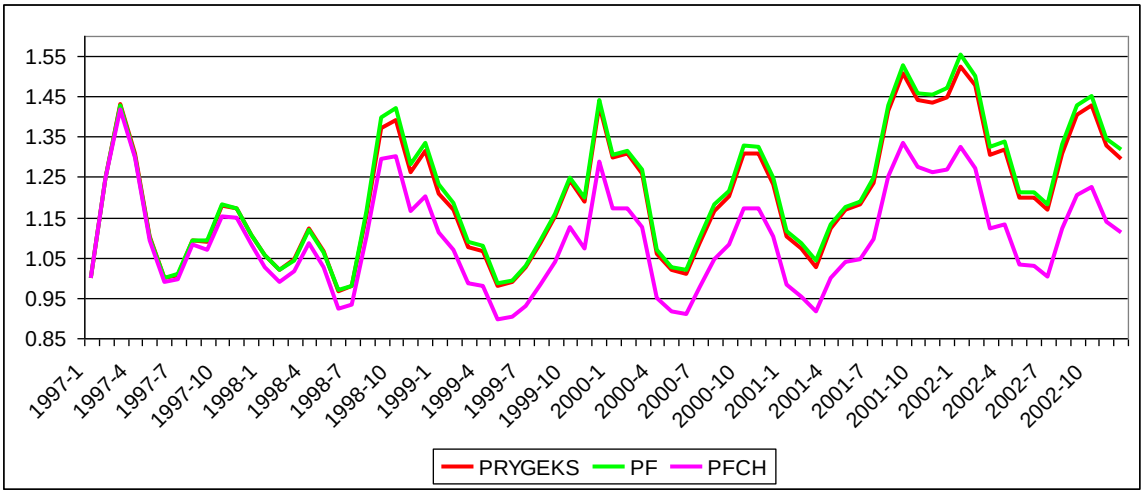
Year/Month	P_{RYGEKS}	P_F	P_{FCh}
1997-01	1.00000	1.00000	1.00000
1997-02	1.24911	1.24729	1.24729
1997-03	1.43004	1.42582	1.41629
1997-04	1.30637	1.30070	1.29643
1997-05	1.10064	1.09801	1.09192
1997-06	0.99957	0.99886	0.98951
1997-07	1.00746	1.00906	0.99577
1997-08	1.09164	1.09279	1.08313
1997-09	1.08871	1.09229	1.06760
1997-10	1.17873	1.18171	1.15364
1997-11	1.17166	1.17184	1.14767
1997-12	1.10539	1.10676	1.08210
1998-01	1.05430	1.05641	1.02667
1998-02	1.01852	1.01895	0.98988
1998-03	1.04457	1.04343	1.01518
1998-04	1.12342	1.12030	1.08721
1998-05	1.06464	1.06086	1.02470
1998-06	0.96721	0.96871	0.92433
1998-07	0.97874	0.98023	0.93276
1998-08	1.15510	1.16568	1.10189
1998-09	1.36998	1.39832	1.29310
1998-10	1.39249	1.41965	1.29970
1998-11	1.26200	1.28106	1.16679

1998-12	1.31464	1.33301	1.20290
1999-01	1.20809	1.23186	1.11266
1999-02	1.16715	1.18474	1.06890
1999-03	1.07692	1.08748	0.98601
1999-04	1.06698	1.07926	0.97850
1999-05	0.98080	0.98725	0.89540
1999-06	0.99071	0.99319	0.90229
1999-07	1.02465	1.02875	0.92875
1999-08	1.08622	1.09287	0.98256
1999-09	1.15069	1.15816	1.04040
1999-10	1.24067	1.24876	1.12425
1999-11	1.18845	1.19835	1.07313
1999-12	1.42683	1.44037	1.28804
2000-01	1.29703	1.30623	1.17235
2000-02	1.30797	1.31501	1.17161
2000-03	1.25744	1.26745	1.12399
2000-04	1.05981	1.06813	0.95101
2000-05	1.02035	1.02732	0.91572
2000-06	1.01092	1.01890	0.90932
2000-07	1.08587	1.09856	0.97657
2000-08	1.16514	1.18276	1.04572
2000-09	1.20173	1.21659	1.08100
2000-10	1.30747	1.32657	1.17334
2000-11	1.30941	1.32547	1.17232
2000-12	1.23029	1.24677	1.10096
2001-01	1.10059	1.11589	0.98262
2001-02	1.07075	1.08409	0.95404
2001-03	1.02694	1.04136	0.91541
2001-04	1.12102	1.13360	0.99802
2001-05	1.17007	1.17620	1.03911
2001-06	1.18021	1.18903	1.04614
2001-07	1.23498	1.24727	1.09482
2001-08	1.41481	1.42867	1.25184
2001-09	1.50806	1.52607	1.33424
2001-10	1.44158	1.45729	1.27580
2001-11	1.43411	1.45278	1.26086
2001-12	1.44844	1.47178	1.26954
2002-01	1.52479	1.55319	1.32420
2002-02	1.47876	1.50107	1.27179
2002-03	1.30454	1.32435	1.12246
2002-04	1.31886	1.33801	1.13139
2002-05	1.19790	1.21180	1.03095
2002-06	1.19750	1.21206	1.02932
2002-07	1.16930	1.18236	1.00412
2002-08	1.30928	1.33152	1.12320
2002-09	1.40534	1.42609	1.20667
2002-10	1.42771	1.45043	1.22631
2002-11	1.32801	1.34557	1.13857
2002-12	1.29608	1.31664	1.11343
Mean Diff.	0	0.01089	–0.10549

The last row in the above Table lists the average difference between the three indexes and the preferred RYGEKS indexes (excluding the first observation when all of the indexes are equal to unity). Thus on average, the fixed base month to month Fisher indexes were 1.1 percentage points above their RYGEKS counterparts while the chained month to month Fishers were a whopping 10.9 percentage points below their RYGEKS counterparts.

Looking at geometric rates of growth of the three indexes over the sample period, the monthly average geometric rates were 1.00366 , 1.00388 and 1.00151 respectively. These monthly rates translate into the following annualized geometric rates of growth: 1.04481 for the RYGEKS index, 1.04759 for the fixed base Fisher index and 1.01832 for the chained Fisher index. Thus the annualized downward "bias" in the chained Fisher relative to the RYGEKS index is 2.65 percentage points per year, which is huge. The fixed base Fisher index grew on average 0.28 percentage points more rapidly than the RYGEKS index, which is not a large difference given the volatility in the underlying data (but it is not negligible either). We prefer the RYGEKS indexes to the fixed base Fisher indexes because the fixed base Fisher indexes depend too heavily on the data for the first month in the sample, which may not be representative.[16] The indexes listed in Table 12 are plotted in Chart 12 below.

Chart 12: Monthly Rolling Year GEKS, Fixed Base and Chained Fisher Price Indexes for Seven Kinds of Vegetables, 1997:1-2002:12



It can be seen that visually, there is very little difference between the Rolling Year GEKS index and the fixed base Fisher index. However, the peaks in the fixed base Fisher indexes seem to be consistently higher than the corresponding peaks in the RYGEKS

[16] Another problem with the fixed base indexes is that at some stage, the base period observation must be updated for various reasons. But at what point should this updating occur? The use of Rolling Year GEKS indexes avoids this problem. However, the RYGEKS indexes are dependent to a certain extent on the length of window when computing the GEKS indexes in the first stage. It is not certain that a window length of 13 months is the "optimal" window length. This is a topic that requires further research.

indexes, which leads to the differences between these two indexes. The downward drift in the chained Fisher indexes is again apparent.

Looking at the seasonality in the above indexes, it can be seen that the seasonal peaks do not remain constant across the years but there is a definite peak during the last 3 months of each year and there is a definite valley around June of each year but these peaks and valleys are not completely regular.

In the following section, we will compare the above RYGEKS indexes with the various month to month "practical" indexes that are used by statistical agencies. These practical indexes rely on out of date annual quantity baskets or expenditure shares to weight the monthly prices. It will be of interest to see how these practical indexes perform using our Israeli data.

7. Month to Month Indexes Based on Annual Quantity and Expenditure Share Data

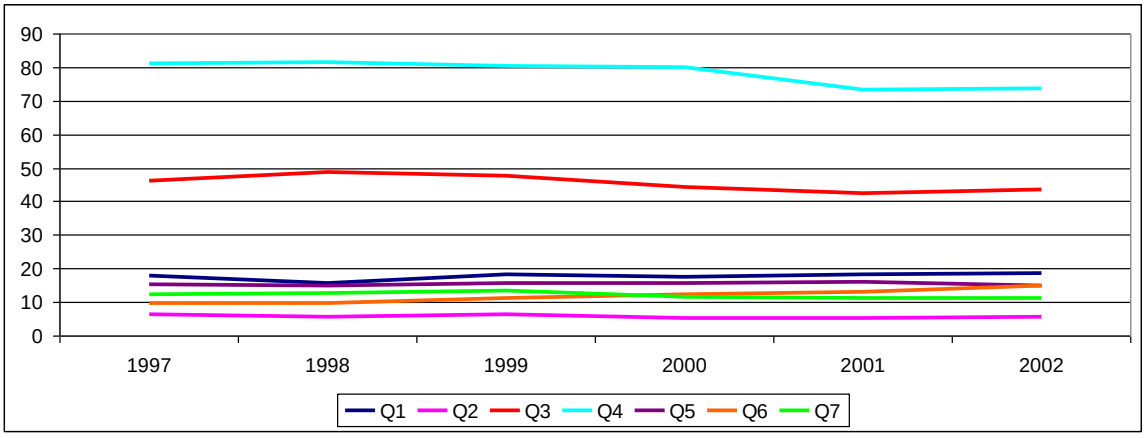
In Table 13 below, we list the annual quantity vectors that are generated by the Israeli vegetable data. These quantity vectors are used in various Lowe indexes which will be constructed later in this section.

Table 13: Annual Quantity Baskets for Seven Kinds of Vegetable, 1997-2002

Year	q_1^y	q_2^y	q_3^y	q_4^y	q_5^y	q_6^y	q_7^y
1997	17.68194	6.19676	46.24627	80.99184	15.36361	9.56315	12.40576
1998	15.49471	5.70233	48.78737	81.36412	14.79188	9.67397	12.61540
1999	18.21388	6.14720	47.52807	80.18156	15.47696	11.28593	13.25660
2000	17.48393	5.29060	44.11214	79.90462	15.70111	12.09495	11.36736
2001	18.16580	5.29550	42.39826	73.34825	15.81546	12.92449	11.23581
2002	18.58990	5.74526	43.47046	73.49378	15.03684	14.77450	11.20061

The annual quantity series in Table 13 are plotted in Chart 13 below.

Chart 13: Annual Quantity Baskets for Seven Kinds of Vegetable, 1997-2002



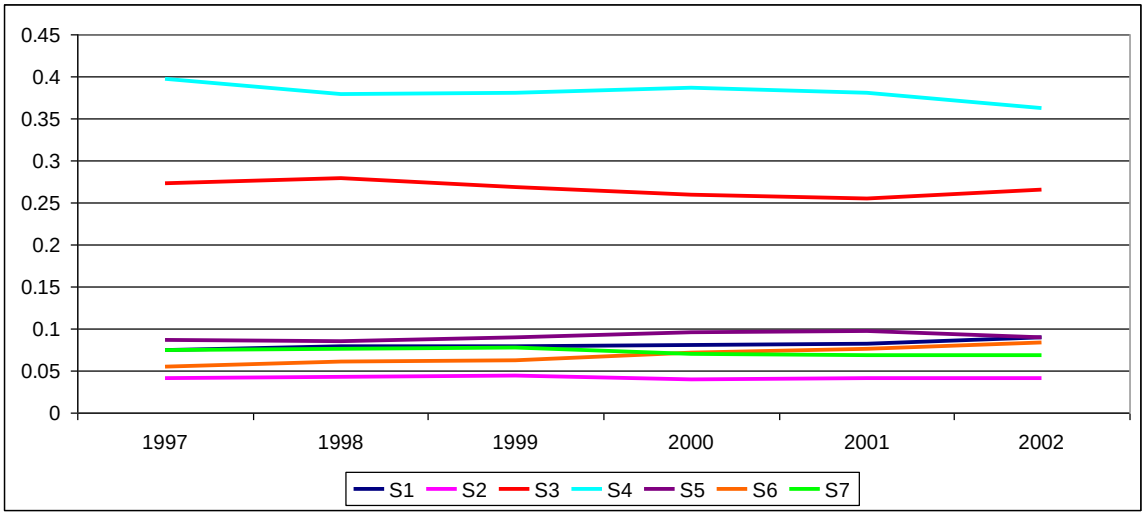
Comparing Charts 10 and 13, it can be seen that there is a lot less volatility in the annual quantity baskets as compared to the monthly baskets. However, some trends in the annual quantities are apparent and this could lead to bias in fixed base Lowe indexes if these trends reflect substitution towards vegetables whose relative price has declined.

The annual expenditure shares for the 7 kinds of vegetables are listed in Table 14 and they are plotted in Chart 14 below.

Table 14: Annual Expenditure Shares for Seven Kinds of Vegetable, 1997-2002

Year	s_1^y	s_2^y	s_3^y	s_4^y	s_5^y	s_6^y	s_7^y
1997	0.07382	0.04149	0.27299	0.39696	0.08696	0.05394	0.07382
1998	0.07894	0.04257	0.27870	0.37883	0.08445	0.06067	0.07584
1999	0.07939	0.04392	0.26774	0.38024	0.08868	0.06250	0.07753
2000	0.08103	0.03868	0.25859	0.38613	0.09520	0.07069	0.06969
2001	0.08238	0.04046	0.25441	0.38097	0.09743	0.07623	0.06813
2002	0.08887	0.04150	0.26586	0.36241	0.08962	0.08391	0.06782
Mean	0.08074	0.04144	0.26638	0.38092	0.09039	0.06799	0.07214

Chart 14: Annual Expenditure Shares for Seven Kinds of Vegetable, 1997-2002



Again, some gradual trends in the annual expenditure shares can be seen in the above Chart.

Recall that various "practical" indexes were defined in section 2 in the context of year over year monthly indexes. We will adapt three of these indexes to the present context where we have monthly prices and annual (out of date) weights. In section 2, we defined the vector of prices for month m of year y as $p^{y,m} \equiv [p_1^{y,m},...,p_7^{y,m}]$ for y = 1997, 1998, ..., 2002 and m = 1, ..., 12. Now define the vector of annual quantity weights for year y (see Table 13 above) as $q^y \equiv [q_1^y,...,q_7^y]$ and the vector of annual expenditure shares for year y (see Table 14 above) as $s^y \equiv [s_1^y,...,s_7^y]$ for y = 1997, 1998, ..., 2002.

The *month to month fixed base* (using the weights of 1997) *Lowe index*, P_{Lo}, can be defined as follows:

$$(7)\ P_{Lo}(p^{y,m},p^{1998,1},q^{1997}) \equiv \sum_{n=1}^{7} p_n^{y,m}q_n^{1997}/\sum_{n=1}^{7} p_n^{1998,1}q_n^{1997} \equiv p^{y,m}\cdot q^{1997}/p^{1998,1}\cdot q^{1997}\ ; \qquad m = 1,...,12;\ y = 1998,....,2002.$$

Thus the annual quantity basket for 1997 is used for all years and months, starting at month 1 in 1998. Expenditure on this basket in month m of year y is $\sum_{n=1}^{7} p_n^{y,m}q_n^{1997}$ and this expenditure is compared to expenditure on the same 1997 annual basket in month 1 of 1998, $\sum_{n=1}^{7} p_n^{1998,1}q_n^{1997}$ and so the fixed base Lowe index for month m of year y is given by (7).

The construction of the *month to month chained Lowe indexes* is more complicated. For the first 13 months, the levels of the chained Lowe index, say $P_{LoCh}^{1998,m}$, is set equal to the fixed base Lowe indexes defined by (7); i.e., we have

$$(8)\ P_{LoCh}^{1998,m} \equiv P_{Lo}(p^{1998,1},p^{1998,m},q^{1997})\ ; \qquad m = 1,2,...,13.^{17}$$

In order to define the remainder of the chained Lowe indexes, we need to define the following sequence of chain links, P_{LoCL}, which are indexes which compare the prices of month 1 in year y, $p^{y,1}$, to the prices of the subsequent 12 months, $p^{y,m}$, using the annual quantity weights of year y–1, q^{y-1}:

$$(9)\ P_{LoCL}(p^{y,1},p^{y,m},q^{y-1}) \equiv p^{y,m}\cdot q^{y-1}/p^{y,1}\cdot q^{y-1}\ ; \qquad m = 1,...,13^{18};\ y = 1999,....,2002.$$

Using the above definitions, we can now explain how the remainder of the chained Lowe indexes can be calculated. For months m = 2,3,...,12 of 1999, the index level $P_{LoCh}^{1999,m}$ is calculated as the index level in month 1, $P_{LoCh}^{1999,1}$, times the 1999 chain link $P_{LoCL}(p^{1999,1},p^{1999,m},q^{1998})$ and for the first month of 2000, the index level $P_{LoCh}^{2000,1}$ is calculated as the index level in month 1, $P_{LoCh}^{1999,1}$, times the chain link between the first month of 2000 and the first month of 1999, $P_{LoCL}(p^{1999,1},p^{2000,1},q^{1998})$, using the annual quantity vector for 1998, q^{1998}. Similarly, for months 2,3,...,12,13 of the augmented year for 2000, the index level $P_{LoCh}^{2000,m}$ is calculated as the index level in month 1, $P_{LoCh}^{2000,1}$, times the 2000 chain link $P_{LoCL}(p^{2000,1},p^{2000,m},q^{1999})$. Thus in February of each year y, the base year quantity vector is updated from year y–1 to the year y vector and a new sequence of 12 chain link Lowe indexes is calculated (which compare the monthly price vectors $p^{y,m}$ with the January price vector for that year $p^{y,1}$) and these indexes are linked to the previously defined January level for year y. The fixed base and chained Lowe indexes are listed in Table 15 below.

[17] When m = 13, the definitions must be augmented: the price level $P_{LoCh}^{1998,13}$ is defined to be $P_{LoCh}^{1999,1}$ and the price vector $p^{1998,13}$ is defined to be $p^{1999,1}$ as was explained in Chapter 5 where the concept of an augmented year was introduced.

[18] Again, when m = 13, $p^{y,13}$ is defined to be $p^{y+1,1}$, the price vector of dimension 7 for the first month of year y+1.

Turning now to Young indexes using annual weights, the sequence of *monthly fixed base Young indexes*, P_{Yo}, starting in January of 1998 and using the annual expenditure share weights of 1997 is defined by (10) below:

$$(10)\ P_{Yo}(p^{1998,1},p^{y,m},s^{1997}) \equiv \sum_{n=1}^{7} s_n^{1997} (p_n^{y,m}/p_n^{1998,1}) ; \quad m = 1,2,...,12;\ y = 1998,...,2002.$$

Note that the annual expenditure shares for 1997 are used throughout the period 1998:1 to 2002:12.

The *chained month to month Young indexes* using annual weights are more complex to describe. For the first 13 months, going from January 1998 to January 1999, the chained Young indexes, $P_{YoCh}^{1998,1}$, $P_{YoCh}^{1998,2}$,..., $P_{YoCh}^{1998,12}$, $P_{YoCh}^{1999,1}$ are equal to the corresponding fixed base Young indexes defined by (10). The chain link Young indexes for augmented year y are defined as follows:[19]

$$(11)\ P_{YoCL}(p^{y,1},p^{y,m},s^{y-1}) \equiv \sum_{n=1}^{7} s_n^{y-1} (p_n^{y,m}/p_n^{y,1}) ; \quad m = 1, 2,...,12, 13;\ y = 1999, ..., 2002.$$

Note that the prices of month m in year y are compared with the corresponding prices of month 1 in year y and the annual expenditure shares for year y−1 are used as weights in definitions (11). Given the level for the chained Young index for month 1 in year y, $P_{YoCh}^{y,1}$, the index level for month m in year y, $P_{YoCh}^{y,m}$, is given by the month 1 level for year y, $P_{YoCh}^{y,1}$, times the chain link index going from month 1 to month m of year y, $P_{YoCL}(p^{y,1},p^{y,m},s^{y-1})$, for m = 2,3,...,12. The index level for month 1 in year y+1, $P_{YoCh}^{y+1,1}$, is given by the month 1 level for year y, $P_{YoCh}^{y,1}$, times the chain link index going from month 1 of year y to month 1 of year y+1, $P_{YoCL}(p^{y,1},p^{y+1,1},s^{y-1})$. These definitions enable us to construct the sequence of Young chained month to month indexes and they are listed in Table 15 below.

Finally, we need to define the sequence of fixed base and chained month to month Geometric Young indexes using annual weights. The *fixed base Geometric Young indexes* using the annual expenditure share weights of 1997, P_{GY}, are defined by (12) below:[20]

$$(12)\ \ln P_{GY}(p^{1998,1},p^{y,m},s^{1997}) \equiv \sum_{n=1}^{7} s_n^{1997} \ln(p_n^{y,m}/p_n^{1998,1}) ; \quad m = 1,2, ...,12;\ y = 1998,...,2002.$$

Thus P_{GY} for month m of year y is a weighted geometric average of the long term prices relative to the prices of month 1 in 1998, $p_n^{y,m}/p_n^{1998,1}$, where the weights are the annual expenditure shares on the 7 commodities in 1997. Hence P_{GY} is a weighted Jevons index.

Again, it is more complicated to construct the chained counterparts to P_{GY} where the annual expenditure share weights are updated each year. The chain link Geometric Young indexes for augmented year y are defined as follows:[21]

[19] As usual, $p^{y,13}$ is defined as $p^{y+1,1}$.
[20] More precisely, (12) defines the natural logarithm of the Geometric Young index P_{GY}.
[21] As usual, $p^{y,13}$ is defined as $p^{y+1,1}$.

(13) $\ln P_{GYCL}(p^{y,1},p^{y,m},s^{y-1}) \equiv \sum_{n=1}^{7} s_n^{y-1} \ln(p_n^{y,m}/p_n^{y,1})$; m = 1,2,...12,13; y = 1999, ..., 2002.

Given the level for the chained Geometric Young index for month 1 in year y, $P_{GYCh}^{y,1}$, the index level for month m in year y, $P_{GYCh}^{y,m}$, is given by the month 1 level for year y, $P_{GYCh}^{y,1}$, times the chain link index going from month 1 to month m of year y, $P_{GYCL}(p^{y,1},p^{y,m},s^{y-1})$, for m = 2,3,...,12. The index level for month 1 in year y+1, $P_{YoCh}^{y+1,1}$, is given by the month 1 level for year y, $P_{GYCh}^{y,1}$, times the chain link index going from month 1 of year y to month 1 of year y+1, $P_{GYCL}(p^{y,1},p^{y+1,1},s^{y-1})$. These definitions enable us to construct the sequence of *Geometric Young chained month to month indexes* and they are listed in Table 15 below.

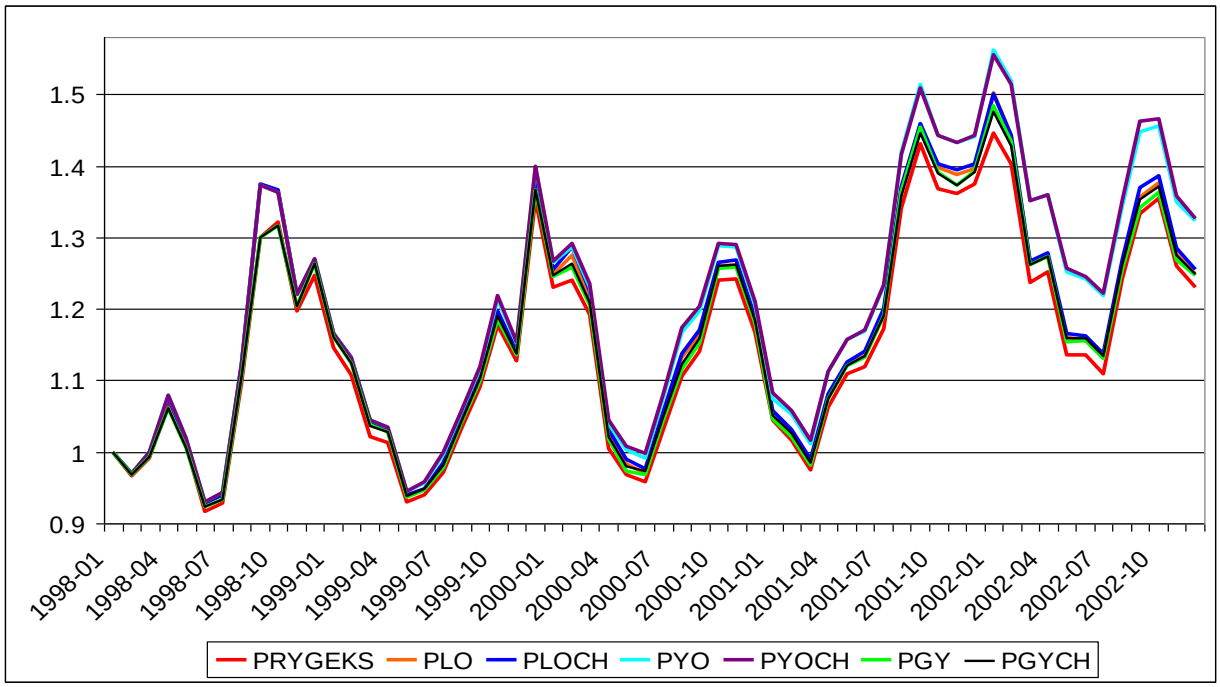
Table 15: Month to Month Rolling Year GEKS, Lowe, Chained Lowe, Young, Chained Young, Geometric Young and Chained Geometric Young Price Indexes, 1998:1-2002:12.

Year/Month	P_{RYGEKS}	P_{Lo}	P_{LoCh}	P_{Yo}	P_{YoCh}	P_{GY}	P_{GYCh}
1998-01	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1998-02	0.96606	0.96894	0.96894	0.96916	0.96916	0.96818	0.96818
1998-03	0.99077	0.99785	0.99785	0.99983	0.99983	0.99279	0.99279
1998-04	1.06556	1.07577	1.07577	1.07935	1.07935	1.06105	1.06105
1998-05	1.00981	1.01660	1.01660	1.01898	1.01898	1.00638	1.00638
1998-06	0.91740	0.92776	0.92776	0.92993	0.92993	0.92264	0.92264
1998-07	0.92833	0.93967	0.93967	0.94237	0.94237	0.93332	0.93332
1998-08	1.09561	1.11996	1.11996	1.12214	1.12214	1.10417	1.10417
1998-09	1.29941	1.37436	1.37436	1.37201	1.37201	1.29939	1.29939
1998-10	1.32077	1.36538	1.36538	1.36294	1.36294	1.31593	1.31593
1998-11	1.19699	1.21981	1.21981	1.21936	1.21936	1.20312	1.20312
1998-12	1.24693	1.26695	1.26695	1.26994	1.26994	1.26303	1.26303
1999-01	1.14586	1.16568	1.16568	1.16571	1.16571	1.16041	1.16041
1999-02	1.10704	1.13027	1.13272	1.13110	1.13079	1.12419	1.12380
1999-03	1.02145	1.03942	1.04221	1.04439	1.04398	1.03694	1.03638
1999-04	1.01202	1.02828	1.03159	1.03516	1.03514	1.02782	1.02768
1999-05	0.93028	0.93800	0.93974	0.94419	0.94545	0.93708	0.93820
1999-06	0.93968	0.94767	0.94881	0.95594	0.95795	0.94588	0.94773
1999-07	0.97188	0.98409	0.98430	0.99580	0.99902	0.9775	0.98045
1999-08	1.03027	1.04635	1.04496	1.05423	1.05663	1.03894	1.04099
1999-09	1.09142	1.10325	1.10396	1.11574	1.11946	1.09941	1.10285
1999-10	1.17677	1.19155	1.19665	1.21378	1.21828	1.18536	1.18950
1999-11	1.12724	1.13751	1.14020	1.15263	1.15416	1.13590	1.13708
1999-12	1.35334	1.37231	1.38138	1.39812	1.39893	1.36633	1.36643
2000-01	1.23022	1.24798	1.25563	1.26490	1.26627	1.24538	1.24637
2000-02	1.24060	1.27509	1.28618	1.28632	1.29122	1.25857	1.26276
2000-03	1.19267	1.21630	1.22546	1.23059	1.23476	1.20573	1.20925
2000-04	1.00522	1.02401	1.03179	1.04014	1.04511	1.01709	1.02145
2000-05	0.96780	0.98228	0.98997	1.00213	1.00844	0.97367	0.97932
2000-06	0.95885	0.96962	0.97678	0.99125	0.99720	0.96771	0.97283
2000-07	1.02994	1.04749	1.05401	1.07332	1.07952	1.04278	1.04773
2000-08	1.10513	1.13145	1.13684	1.16719	1.17436	1.11599	1.12148

2000-09	1.13983	1.16367	1.16963	1.19755	1.20424	1.15276	1.15792
2000-10	1.24012	1.25941	1.26567	1.28782	1.29159	1.25735	1.26016
2000-11	1.24196	1.26100	1.26776	1.28698	1.29056	1.25889	1.26157
2000-12	1.16692	1.18126	1.18840	1.20567	1.21029	1.17979	1.18357
2001-01	1.04391	1.04968	1.05698	1.07476	1.08171	1.04561	1.05156
2001-02	1.01560	1.02503	1.03064	1.05190	1.05697	1.02113	1.02534
2001-03	0.97405	0.9852	0.98936	1.01157	1.01530	0.98100	0.98395
2001-04	1.06328	1.07930	1.08150	1.11220	1.11287	1.07407	1.07394
2001-05	1.10980	1.12148	1.12527	1.15699	1.15787	1.12099	1.12069
2001-06	1.11942	1.13453	1.14013	1.16804	1.17081	1.13244	1.13398
2001-07	1.17137	1.19446	1.19998	1.23130	1.23286	1.18950	1.18967
2001-08	1.34194	1.36764	1.37036	1.41988	1.41525	1.36366	1.35697
2001-09	1.43039	1.45799	1.45914	1.51356	1.50795	1.45322	1.44561
2001-10	1.36732	1.39687	1.40252	1.44274	1.44258	1.39115	1.38918
2001-11	1.36025	1.38706	1.39429	1.43255	1.43304	1.37320	1.37212
2001-12	1.37383	1.39557	1.40176	1.44131	1.44179	1.39260	1.39147
2002-01	1.44625	1.50135	1.50101	1.56238	1.55468	1.48446	1.47564
2002-02	1.40259	1.44190	1.44205	1.51923	1.51342	1.43378	1.42748
2002-03	1.23734	1.26410	1.26678	1.35097	1.35098	1.26233	1.26218
2002-04	1.25092	1.27521	1.27765	1.35984	1.35917	1.27349	1.27253
2002-05	1.13619	1.16092	1.16579	1.25193	1.25587	1.15457	1.15867
2002-06	1.13582	1.15616	1.16123	1.24138	1.24579	1.15526	1.15906
2002-07	1.10907	1.13401	1.13775	1.21764	1.22130	1.13121	1.13439
2002-08	1.24184	1.26002	1.26752	1.34315	1.35082	1.25546	1.26174
2002-09	1.33296	1.35668	1.36867	1.44787	1.46155	1.34180	1.35296
2002-10	1.35417	1.37557	1.38626	1.45504	1.46532	1.36287	1.37064
2002-11	1.25960	1.27716	1.28513	1.34999	1.35730	1.26849	1.27405
2002-12	1.22932	1.24903	1.25464	1.32220	1.32619	1.24652	1.24902
Mean Diff.	0	0.01987	0.02381	0.05056	0.05275	0.01198	0.01346

All 6 of the practical month to month indexes that use annual out of date expenditure or quantity information are above our preferred Rolling Year GEKS month to month indexes. On average, the fixed base Lowe indexes P_{Lo} are about 2.0 percentage points above their RYGEKS counterparts and the chained Lowe indexes P_{LoCh} are about 2.4 percentage points above the RYGEKS indexes. On average, the fixed base Young indexes P_{Yo} are about 5.1 percentage points above their RYGEKS counterparts and the chained Young indexes P_{YoCh} are about 5.3 percentage points above the RYGEKS indexes. Finally, on average, the fixed base Geometric Young indexes P_{GY} are about 1.2 percentage points above their RYGEKS counterparts and the chained Geometric Young indexes P_{GYCh} are about 1.3 percentage points above the RYGEKS indexes. Thus all of the practical indexes that use annual weights appear to be subject to some degree of substitution bias with the bias for the Young indexes being the most substantial. The indexes listed in Table 15 are plotted in Chart 15 below.

Chart 15: Month to Month Rolling Year GEKS, Lowe, Chained Lowe, Young, Chained Young, Geometric Young and Chained Geometric Young Price Indexes, 1998:1-2002:12.



The Young and Chained Young tend to be well above the other indexes. The Lowe indexes and the Geometric Young indexes are fairly close to each other but the Rolling Year GEKS index (the red line) tends to be markedly below the other indexes at peaks and valleys.

Our conclusion is that all of the indexes that rely on out of date annual quantity or expenditure information are subject to some substitution bias which can be quite considerable in the case of the Young indexes. An implication of this analysis is that it is important for statistical agencies to collect household expenditure data on a continuous basis so that substitution bias in practical and timely Consumer Price Indexes can be evaluated on an ex post basis.

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