The use of Superlative Index Links in the Swedish CPI[‡]

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10 May, 2012

Abstract: In this paper we present the index construction of the national Swedish CPI. We also deal with some aspects regarding revision of the CPI. From 2005 the CPI is based on a superlative index formula by the use of Walsh index for macro-level aggregation. For elementary aggregation the Jevons index formula is used. The use of a Walsh index serves to transparently avoid a substitution bias. As a result, there is now a clear theoretical basis for the ability of the CPI to closely approximate a cost-of-living index.

Key Words: CPI; COLI, Superlative Index; Walsh Index; Price Index;

[‡] This paper draws to some extent on an earlier paper presented by M. Ribe of Statistics Sweden in a seminar on inflation measures, held in Paris by the OECD in June 2005.

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1. Introduction

In the early 1940´s a Government Commission established that the Swedish Consumer Price Index (CPI) should follow the cost for households to maintain a constant standard of living, i.e. measure the change over time in the cost of keeping a constant consumer utility. As the Swedish CPI to a great extent is used for compensation purposes, there was a desire to find an index that reflected the consumer gains and losses from substitution between products. The index should avoid the typical upward bias from the use of a Laspeyres-type index, which was recognized as an undesirable feature in the CPI computation.

The Commission came in principle to the conclusion that an annually chained index with "long-term links" would be most comparable to a Cost of Living Index (COLI)[**]. That practice meant that the index baskets for past years' index links were retrospectively updated to reflect the consumption pattern of the years in question, rather than that of earlier years. (SOU, 1943).

In 1992 an enquiry on the National CPI was made by the Swedish National Audit Office (Riksrevisionen), and it was suggested that an evaluation of the CPI should be considered. This led the Swedish Government to appoint a Commission in 1997 to generally review the CPI practices.

Following one suggestion of the latter Commission, a superlative index formula (Walsh index) for macro-level aggregation was implemented in 2005. The new index construction takes advantage of the modern theory of superlative indices and serves to transparently avoid a substitution bias. As a result, there is now a clear theoretical basis for the ability of the CPI to closely approximate a cost-of-living index.[††]

In this paper we present the index construction of the Swedish CPI. We also deal with some aspects regarding revision of the CPI.

2. The Chained CPI

From 2005, the national Swedish CPI is based on a superlative index formula by the use of Walsh index links for macro-level aggregation and Jevons index formula for elementary aggregation.

The Swedish CPI is constructed as a chain index with annual links. Consider for example the computation of the CPI for January 2012 with index reference year 1980, denoted below as $I_{1980}^{2012,jan}$.

$$(1) \qquad I_{1980}^{2012,jan} = I_{1980}^{1980,dec} \times I_{1980,dec}^{1981,dec} \times I_{1981,dec}^{1982,dec} \times \dots \times I_{2002,dec}^{2003,dec} \times I_{2003,dec}^{2004} \times I_{2004}^{2005} \times \dots \times I_{2009}^{2010} \times I_{2010}^{2012,jan}$$

In the CPI construction that was used before 2005, the annual link measured how much the price level had changed from December of the preceding year to December of the year considered, e.g. $I_{1980,dec}^{1981,dec}$. This link was known as the "long-term link". Also, before 2005, a final link measured the change in price level up to the current month from December of the preceding year.

[**] See Allen, 1975, p. 65ff, on cost-of-living indices.
[††] See Diewert, 1976, on superlative indices.

In the index construction that is in use since 2005, the annual link measures the change in average price level in the year concerned from the average price level of the preceding year. Hence, the CPI basket of goods and services of an annual link reflects a blend of consumption patterns of the year concerned and the preceding year, e.g. I_{2009}^{2010}. This blend is key in understanding the Swedish approach towards a superlative index and is more closely examined in the next section. Also, a final link measures the change in price level of the current month, as compared to the average price level of the year before the preceding year, e.g. $I_{2010}^{2012,jan}$ above for the price level of January of 2012.

The index constructions used before and after 2005 were chained together using the average price level of 2004 by the link denoted $I_{2003,dec}^{2004}$ above. This was done in order to avoid any interruption in the index series between the previous and the new index construction.

In the following description of the Swedish CPI the index construction used after 2005 is considered in more detail. A more detailed description of the construction used before 2005 can be found in Statistics Sweden (2001).

3. Index links

After 2005, the annual links in the Swedish CPI refers to the average price level of the year considered from the preceding year and are calculated using a *Walsh index formula*. For example, in (1), the index link I_{2009}^{2010} measures the change in price level from the full year 2009 to the full year 2010, i.e. from the average price level of 2009 to the average price level of 2010. This index link can be written as

$$I_{2009}^{2010} = \frac{\sum_i P_i^{2010} \times \sqrt{Q_i^{2009} \times Q_i^{2010}}}{\sum_i P_i^{2009} \times \sqrt{Q_i^{2009} \times Q_i^{2010}}} \quad (2)$$

for annual average price P and consumed quantities Q for consumer product i.

The final link in (1), $I_{2010}^{2012,jan}$, is calculated using a *Laspeyres index formula* and measures the change in price level from the average price level of 2010 to the price level of January 2012. This index link can be written as

$$I_{2010}^{2012,jan} = \frac{\sum_i P_i^{2012,jan} \times Q_i^{2010}}{\sum_i P_i^{2010} \times Q_i^{2010}} \quad (3)$$

for annual average price P and consumed quantities Q for consumer product i.

Lastly, the transition from the old index construction to the new one calls for some special consideration. As mentioned earlier, the transition was made by the use of an index link denoted $I_{2003,dec}^{2004}$ in (1). This link is computed as

$$I_{2003,dec}^{2004} = \left(\frac{1}{12}\sum_{m=1}^{12} I_{2003,dec}^{2004,m}\right) \times \frac{I_{2002}^{2003} \times I_{2003}^{2004}}{\frac{1}{12}\sum_{m=1}^{12} I_{2002}^{2004,m}} \quad (4)$$

See Ribe (2005) for more details regarding the transition.

4. Sub-indices, Elementary Aggregate Indices and Practical Calculations

The actual collection of price data in Sweden follows prices during a period of 13 months, from December of the preceding year to December of the year concerned. For every product group, *g*, an index is calculated that describes the change in price level from December of the preceding year to the price level of the current month of the year concerned. These elementary aggregate indices are the building blocks that are used in order to calculate the full chain index in (1).

The elementary aggregate indices are calculated from observed prices using, with very few exceptions[‡‡], a *Jevons index formula*. In a simple, unweighted case this index is calculated as

$$\left(\frac{P_t^A}{P_0^A} \times \frac{P_t^B}{P_0^B} \times \frac{P_t^C}{P_0^C}\right)^{1/n} \quad (5)$$

For price observations *A,B* and *C* in periods *t* and *0* and where n=3.

Using this, it is possible to compute an index number that describes the price development from December of the preceding year to the current month of the year concerned. These elementary aggregate indices, or link elements, are computed for each product group, *g*, and have the form

$$I_{2011,dec;g}^{2012,m} \quad (6)$$

where *m* runs through all months of 2012.

Using these link elements as building blocks the two final links in (1), i.e. (2) and (3) for each product group g can be computed

$$I_{2009;g}^{2010} = \frac{I_{2008,dec;g}^{2009,dec} \times \frac{1}{12}\sum_{m=1}^{12} I_{2009,dec;g}^{2010,m}}{\frac{1}{12}\sum_{m=1}^{12} I_{2008,dec;g}^{2009,m}} \quad (7)$$

$$I_{2010;g}^{2012,jan} = \frac{I_{2009,dec;g}^{2010,dec}}{\frac{1}{12}\sum_{m=1}^{12} I_{2009,dec;g}^{2010,m}} \times I_{2010,dec;g}^{2011,dec} \times I_{2011,dec;g}^{2012,jan} \quad (8)$$

[‡‡] These exceptions are muncipal services for home owners, such as water and severage. Instead, for those services a *Dutot-type index* is used.

5. Revised Index Links

The Swedish CPI is formally established in association with the monthly publication of the index. After that, the index number is not revisable and can thus be unambiguously referred to in any law, decree, business agreement or so.

However, in the index construction for the CPI certain revised index links are used[§§]. Each year, a new index link of the form $I_{2010,dec;g}^{2011,m,rev}$ is computed for each product group with respect to revised price data, quality adjustments, updated weighting material or other revised information and practices that affects the calculations. These revised index links are then used in order to calculate the last two index links in (1), i.e. I_{2009}^{2010} and $I_{2010}^{2012,m}$ for the CPI calculated during 2012.

For example, the link $I_{2010}^{2012,jan}$ in (8) uses revised index numbers for $I_{2010,dec;g}^{2011,dec}$, $I_{2009,dec;g}^{2010,dec}$ and $I_{2009,dec;g}^{2010,m}$ for each product group. The link $I_{2009;g}^{2010}$ in (7) uses revised index numbers for all calculations for all product groups. Note however, that the link $I_{2011;g}^{2012}$ is not used until January of 2014.

In the case of a considerable error in the CPI, corrections are made in a parallel index series.[***] Any number that expresses the rate of change in the CPI, including the inflation rate, is then calculated and reported as the change in that parallel index series. When no corrections are made the established CPI and the parallel CPI are identical.

6. Weighting the Index

The equations (2) and (3) show the conceptual nature of the index links by the use of *consumed quantities*, Q_i. However, in practice these quantities cannot be used directly since they often are not measured. Instead and by traditional practice for almost all index computations, the *consumption values* for each product group are used as weighting mechanism for macro level aggregation.

In Sweden, the data for on consumption values are obtained primarily from the computations of household consumption made in the National Accounts. In the cases where more specified breakdown in product groups is needed the data is supplemented by data from the Household Budget Survey and certain other sources.

For the CPI during 2012 the latest data on consumption that is needed in the computations is for 2010 and earlier. This can be seen in (1), where the final link links the year 2010 to the current month in 2012. In comparison to the previous index construction that was used up until 2005, this is a relaxing in the need for rapidly produced consumption data. The longer time available has made it possible to obtain more accurate data and a less hurried production process (Ribe, 2005).

Consider the index link (6). In order to obtain the full index link that is used in (1), the sub-indices for product groups *g* has to be weighted together in accordance with

[§§] For the index construction used before 2005, see Statistics Sweden (2001) for details regarding revision and non-revision of the CPI.

[***] This index series is known as the *shadow index.*

(9) $$I_{2009}^{2010} = \sum_g W_g \times I_{2009;g}^{2010}$$

The weights are computed by the formula

(10) $$W_g = \frac{\sqrt{U_g^{2009} \times U_g^{2010} / I_{2009;g}^{2010}}}{\sum_{g'} \sqrt{U_{g'}^{2009} \times U_{g'}^{2010} / I_{2009;g'}^{2010}}}$$

where *Ug* is the consumption value of product group *g* during the year indicated and g' contains all product groups.

The weights used in the index link in (8) are computed in accordance with the Walsh index. That is, the link that measures the change in average price level from the year 2009 to the average price level in the year 2010, uses the geometric average of the consumption values for both years as weighting mechanism (note that $\sum W_g = 1$).

In the same way, the final link that is used in (1) is computed as a weighted average of the sub-indices for product groups

(11) $$I_{2010}^{2012,jan} = \sum_g W'_g \times I_{2010;g}^{2012,jan}$$

Above, the weights are computed as to produce a Laspeyres type index in accordance with the formula

(12) $$W'_g = \frac{U_g^{2010} / I_{2010;g}^{2012,jan}}{\sum_{g'} U_{g'}^{2010} / I_{2010;g'}^{2012,jan}}$$

That is, the final link in (1) only uses the consumption values for the year 2010 as weighting mechanism (also here $\sum W'_g = 1$).

In this way, roughly 350 sub-indices are weighted together and aggregated from product group level up to the overall CPI.

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