# POST-LASPEYRES: THE CASE FOR A NEW FORMULA FOR COMPILING CONSUMER PRICE INDEXES


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# LOWER AND HIGHER-LEVEL INDEXES

**CPIs are compiled in two stages:**

- the lower “elementary” level using unweighted averages. The *CPI Manual* strongly supports geometric means (Jevons), a widely-accepted innovation.

- the higher level using expenditure-share weighted averages of the lower level indexes. Arithmetic means widely used with price-updating.

# ARITHMETIC HIGHER LEVEL

**Laspeyres:**

$$I_L^t = \sum_{i=1}^{n} \frac{p_i^t q_i^0}{p_i^0 q_i^0} = \sum_{i=1}^{n} \frac{p_i^0 q_i^0 \left( \frac{p_i^t}{p_i^0} \right)}{\sum_{i=1}^{n} p_i^0 q_i^0} = \sum_{i=1}^{n} s_i^0 \left( \frac{p_i^t}{p_i^0} \right), \text{ where } \quad s_i^0 = \frac{p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}$$

**Young:**

$$I_Y^t = \sum_{i=1}^{n} s_i^b \left( \frac{p_i^t}{p_i^0} \right), \text{ where } \quad s_i^b = \frac{p_i^b q_i^b}{\sum_{i=1}^{n} p_i^b q_i^b}$$

**Lowe:**

$$I_{Lo}^t = \frac{\sum_{i=1}^{I} \left[ p_i^b q_i^b \frac{p_i^0}{p_i^b} \right] \frac{p_i^t}{p_i^0}}{\sum_{i=1}^{I} p_i^b q_i^b \frac{p_i^0}{p_i^b}} = \frac{\sum_{i=1}^{I} p_i^t q_i^b}{\sum_{i=1}^{I} p_i^0 q_i^b} = \frac{\sum_{i=1}^{I} p_i^0 q_i^b \frac{p_i^t}{p_i^0}}{\sum_{i=1}^{I} p_i^0 q_i^b}$$

# GEOMETRIC HIGHER LEVEL

**Geometric Lowe:**

$$I_{GLo}^{t} = \prod_{i=1}^{n} \left( \frac{p_i^t}{p_i^0} \right)^{s_i^{b0}}, \quad \text{where} \quad s_i^{b0} = \frac{p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}$$

**Geometric Young (Cobb-Douglas):**

$$I_{CD}^{t} = \prod_{i=1}^{n} \left( \frac{p_i^t}{p_i^0} \right)^{s_i^{b}}, \quad \text{where} \quad s_i^{b} = \frac{p_i^b q_i^b}{\sum_{i=1}^{n} p_i^b q_i^b}$$

# TARGET INDEX AND APPROXIMATIONS:

**Törnqvist:**

$$I_T^t = \prod_{i=1}^{n} \left( \frac{p_i^t}{p_i^0} \right)^{\left(s_i^0 + s_i^t\right)/2}, \text{where } s_i^0 = \frac{p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0} \text{ and } s_i^t = \frac{p_i^t q_i^t}{\sum_{i=1}^{n} p_i^t q_i^t}$$

**Lloyd-Moulton:**

$$I_{LM}^t = \left[ \sum_{i=1}^{n} s_i^b \left( p_i^t / p_i^b \right)^{1-\eta} \right]^{1/(1-\eta)}$$

**Lent-Dorfman:**

$$I_{AG}^t = \eta \prod_{i=1}^{n} \left( p_i^t / p_i^0 \right)^{s_i^0} + (1-\eta) \left( \sum_{i=1}^{n} s_i^0 \, p_i^t / p_i^0 \right)$$

# ARITHMETIC vs. GEOMETRIC INDEXES

►Young and Lowe indexes are biased approximations of the Laspeyres and Laspeyres is biased in relation to Törnqvist (Balk and Diewert, 2003, Balk, 2009).

► Denmark's CPI: in some periods Young exceeds Lowe, in others the reverse. (Boldsen-Hansen, 2006, 2007).

► Greenlees and Williams (2009) US: Dec 1990 to Dec 2008: annual growth rates of (chain CD) 2.4, (chain Törnqvist) 2.5, and (chain Lowe) 2.8 percent, respectively. [Lent and Dorfman (2009]

► What drives differences between CD and Geo-Lowe?

# DECOMPOSITION OF DIFFERENCE BETWEEN GEO-LOWE AND CD

**Geo-Lowe to CD:**

$$\frac{I^t_{GLo}}{I^t_{CD}} = \exp\left( \rho^{w^b}_{x,y} cv^{w^b}_x \sigma^{w^b}_y \right)$$

where $y^t_i \equiv \ln\left( \frac{p^t_i}{p^0_i} \right)$ $x^t_i \equiv \frac{p^0_i}{p^b_i}$

# IMPLICATIONS

► **Geometric Lowe to equal CD:**

- for all *i*, period *b* to 0 price changes OR the logarithms of all *i* period 0 to *t* price changes are the same, OR
- there is no (weighted) correlation between period *b* to 0 and period 0 to *t* price changes. These are extreme conditions, but multiplicative.

► **Multiplicative: Difficult to predict extent/ direction. It is a constant, at least between rebasing.**

► **A *potential* drift in the formulas difference –** $CV_x^{w^b}$ **dispersion increases with increases in inflation.**

► **Difference not due to substitution effects.**

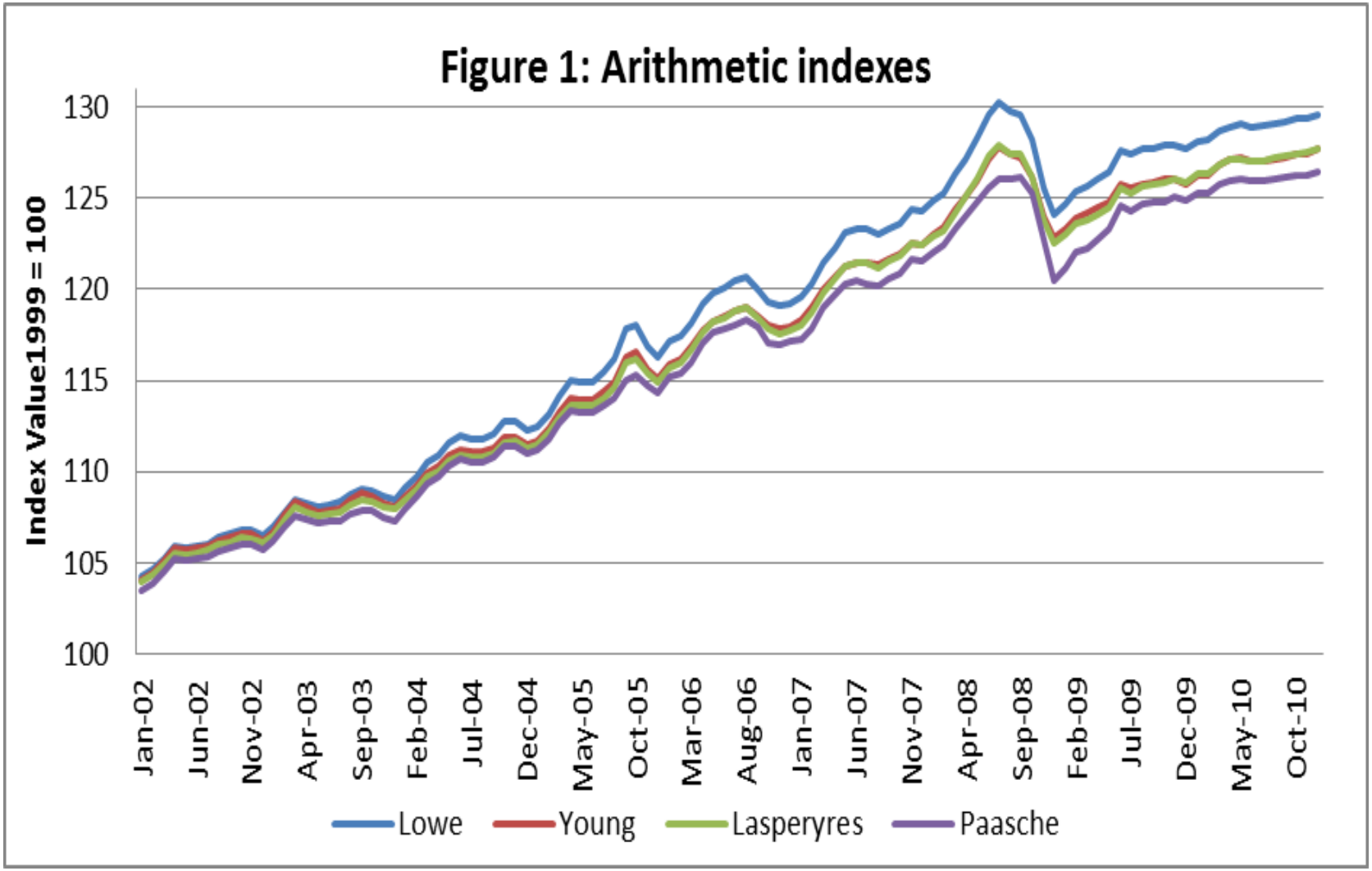
# DATA USED FOR TEST INDEXES

**Data: elementary aggregate indexes for the U.S. CPI and their weights, January 1998 to December 2011 (U.S. Bureau of Labor Statistics).**

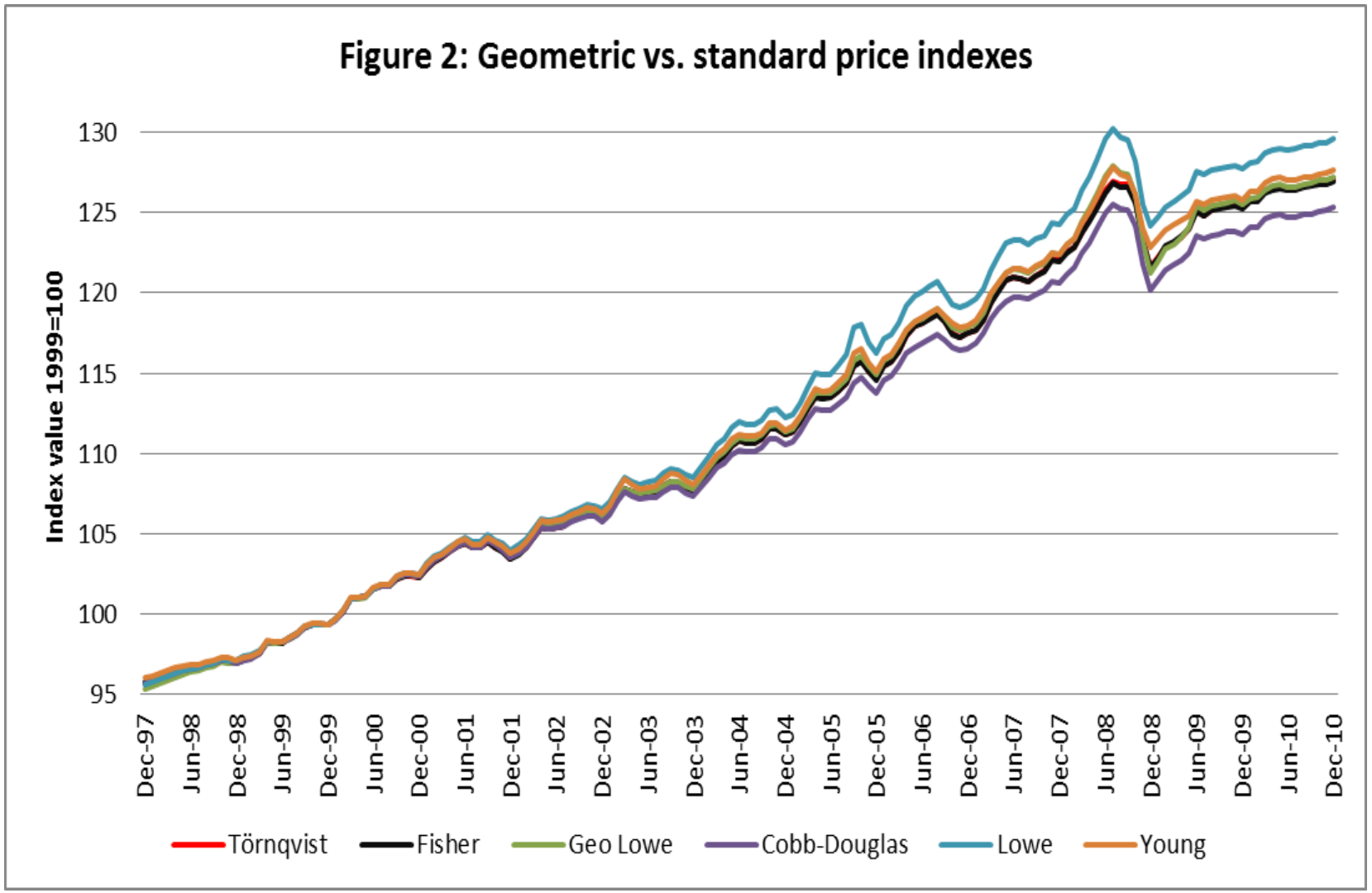
**► 211 product groups for the large part derived using geometric means. Price updated from mean of mid-point to December prior to reweighting (every two years).**

| Mean-annual expenditures | Basis of weights for: |
|---|---|
| 1993-1995 | Jan98-Dec01 |
| 1999-2000 | Jan02-Dec03 |
| 2001-2002 | Jan04-Dec05 |
| 2003-2004 | Jan06-Dec07 |
| 2005-2006 | Jan08-Dec09 |
| 2007-2008 | Jan10-Dec11 |
| 2008-2009 | Jan11-Dec12 |

# STANDARD INDEXES



# GEOMETRIC vs. STANDARD INDEXES



# ALTERNATIVES: APPROXIMATIONS TO TARGET INDEXES

► Are there alternative measures that Statistical Offices (SOs) can compile in real time that approximate the target indexes?

► Lowe, Young, and Geo-Lowe have greater price change than the target indexes

► Geometric Young (Cobb Douglas) has lower price change than target indexes

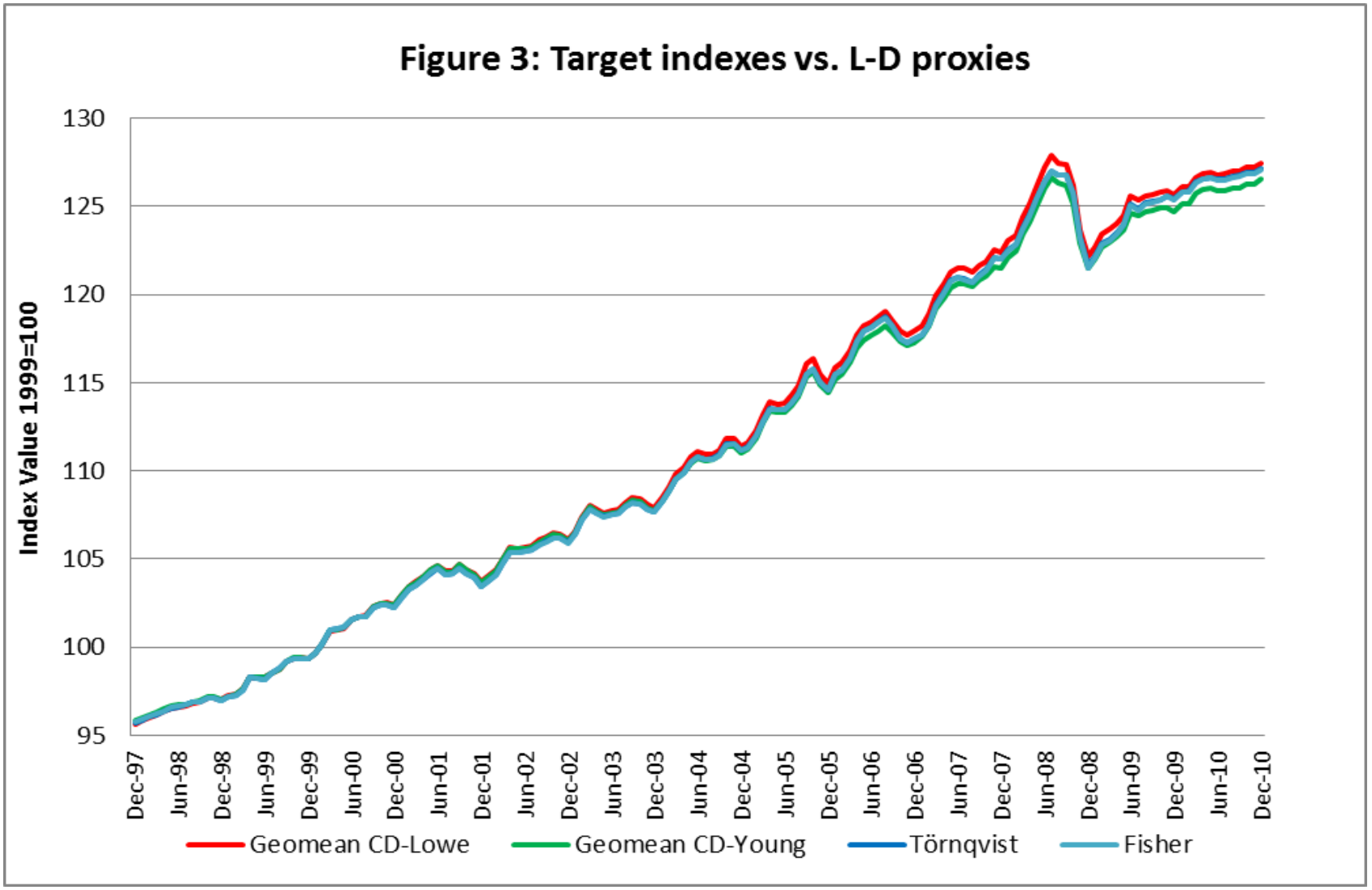
► All of these indexes can be compiled in real time by SOs; is it possible to use combinations of these similar to the L-D indexes to approximate the target indexes?

**TABLE 2: AVERAGE MONTHLY PERCENTAGE DIFFERENCES BETWEEN ALTERNATIVE VS. TARGET INDEXES:***

| | Target Indexes | |
|---|---|---|
| | Fisher | Törnqvist |
| ***Arithmetic formulas*** | | |
| Lowe | 1.712 | 1.689 |
| Young | 0.466 | 0.443 |
| ***Geometric formulas*** | | |
| Geometric Lowe (Glowe) | 0.265 | 0.242 |
| Cobb-Douglas (C-D) | -0.959 | -0.981 |
| ***Geometric means of formulas*** | | |
| CD-Young | -0.249 | -0.272 |
| CD-Lowe | 0.368 | 0.345 |
| Glowe:Young | 0.366 | 0.343 |
| Glowe:Lowe | 0.986 | 0.963 |
| ***Lent-Dorfman (η using lag)*** | | |
| CD:Young | -0.339 | -0.375 |
| CD:Lowe | -0.196 | -0.219 |
| Glowe:Young | 0.339 | 0.316 |
| Glowe:Lowe | 0.581 | 0.558 |

*Covers January 2004 to December 2010

# L-D TYPE PROXIES vs. TARGETS



# CONCLUDING REMARKS

1. **CD index is lower than target indexes; this is expected because CD is consistent with η = 1. In U.S. has η averages about 0.7 but varies between 0.5 and 0.9 (Greenlees, 2010)**

2. **In most countries, SOs do not have measures of η. It is close to 1, CD would be good proxy for target.**

3. **Using geometric averages of an upward biased index with one that is downward biased such as Lowe and CD or Young and CD can approximate the targets.**

# CONCLUDING REMARKS (CONTINUED)

4. **In the U.S. data, the Geo-Lowe performs well but there is no theoretical justification for its use at this time.**

5. **We believe this is an interesting area for additional research so that SOs can be rightly moving beyond a Laspeyres-centric world of price index calculations.**