The interpretation of the PPPs: a method for measuring the factors that affect the comparisons and the integration with the CPI work at regional level

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1. Introduction

Purchasing Power Parities (PPPs) deal with the comparisons of price levels across different countries, as well as across different areas or regions within a country. They are used for a wide variety of purposes, including comparisons of Gross Domestic Product and/or its components and comparisons of revenues, salaries and living conditions.

When PPPs are published by the World Bank within the International Comparisons Program (ICP), or by Eurostat/OECD, the users usually put forward some criticisms on their reliability essentially with reference to binary comparisons between two countries or areas. This is due to the PPP interpretation which is considered difficult as the products included in the computation are not representative enough of the actual consumption (expenditure, or other reference aggregates of the PPPs) of the countries or areas concerned.

In this context, we should underlined that the purpose of the computed PPPs affects both the underlying concepts, definitions and procedures of their construction. As many authors have pointed out, it is clear that in the estimation of parities for Basic Headings the main roles are played by product selection and price data quality, and obviously the relative procedures adopted may influence the measurement and the reliability of the final results (Castels, 1997).

However, in general the main criticism is due to the conflicting choice between two important principles in products and price selection for spatial comparisons, that is comparability and representativeness, as were widely recognized by Locker (1984), Heston (1996) and Rao (2001) among others.

Focusing on the comparisons of household consumption, which is the object of this paper, if the products for which the prices are collected are strictly comparable, they may have a different degree of representativeness in different areas. Therefore, the weights to be assigned to each product in two different areas are usually different. For this reason the comparisons of consumer price levels between two areas can be affected by a *pure price effect* (measured only by the differences in the elementary price indices, which may be traditional PPPs) and by the differences on the *system of weights*, that is on the share of the household expenditure concerning the different products and services

in the two areas (which is a kind of *representativity effect*). Moreover, if some products are typical (or characteristic) in one area and not in the other and vice-versa, they cannot be considered as comparable. If these products were included in the comparisons of consumer price levels between two areas the results would be affected by the different value of their prices as well (this can be called *characteristicity effect*).

The aim of this paper is to investigate the above mentioned effects (components) and to suggest a simple method to evaluate the importance of the different factors that affect the value of binary spatial indices and therefore get a clear interpretation of the comparisons of consumer price levels between two areas, focusing on the comparisons across different areas or regions within a country. Our proposal is theoretical in order to point out the methods and the detailed data which are necessary to interpret adequately the binary consumer spatial indices at this stage. Therefore in this paper we do not refer to the definitions and practical procedures followed by the ICP and Eurostat-Oecd to compute the international PPPs.

Nevertheless, the measures of the above mentioned effects could be helpful for deciding to what extent the products included in the list for the computation of spatial indices are comparable and/or representative and characteristic of the household consumption of the two areas. In this way we can balance comparability and representativeness positively whenever the National Statistical Institutes (NSIs) can collect all the necessary information on prices and weights at product level in one benchmark period.

Finally, the proposed measures can also be useful when the NSIs decide to integrate CPI and PPP work (Rao, 2001a) - obviously for the computation of PPPs referred to comparison of household expenditures - in order to assess the feasibility of using and integrating the collection of price data of CPIs to compute adequate PPPs for areas across a country.

The paper is organised as follows. In section 2, we present specific definitions of comparability, and representativeness in the comparisons of consumer price levels between two areas suitable for our aim. Section 3 illustrates the methods for decomposing the comparison between the consumer price levels concerning the two areas in different components (effects), both at level of Basic Headings (BH) and at the aggregate level. This is accomplished by considering different hypotheses concerning comparability, representativeness and characteristicity in order to understand the meaning of the effects measured. In section 4, in order to highlight the usefulness of measuring these effects, some preliminary and approximate estimations of them are presented. The computations were carried out with reference to an experiment conducted in Italy aimed at using and integrating the data collected for CPIs to compute PPPs for some divisions of household expenditures and for 20 cities for the year 2006 (De Carli, 2008). Finally, the concluding remarks focus on the usefulness of the methods suggested and on possible further development of the methods and of the analysis.

2. Difficulties in interpreting PPPs: comparability and representativeness

Many methods have been developed to compute and aggregate PPPs, both for binary and multilateral comparisons (cfr. World Bank, 2007; OECD/Eurostat, 2006; ILO, 2004; Balk, 1996 and 2001; Hill 1997). In this paper, we only consider the *binary*

comparisons in order to focus the analysis on the difficulties in interpretation. Each binary PPP[1] is estimated separately using data specifically concerning the two areas in question, thus in general the resulting binary PPPs are not transitive, while transitivity is considered a necessary property for a set of multilateral PPPs if not they would be mutually inconsistent[2]. The transitivity aspects are set aside at this stage however the conclusions and the results we obtain can be used to get approximate measures of the decomposition of binary comparisons of consumer price levels which give us a better interpretation of the results obtained[3]. Whatever method is chosen, PPPs are computed and aggregated in two stages: at level of Basic Headings and at the aggregate levels above BHs.

The BH is normally defined as the lowest level of aggregation of products at which expenditure data and weights are available (Kravis et. Al 1975, Locker, H.K. 1984; OECD/Eurostat, 2006; World Bank, 2007; ILO, 2004). Each BH usually consists of a fairly homogeneous group of items (products and services) that are sold in different outlets in the area for which it is possible to compute the PPPs. The subsequent level of aggregation leads to PPPs for broad household expenditure categories, and finally to the whole household consumption basket.

As it is impossible to enumerate all the products inside each BH, a selection must be made in the pricing exercise. This selection should be carried out satisfying two main principles for spatial comparisons, that is *comparability* and *representativeness*, which are actually in conflict with each other. The grade of this incompatibility depends on the strictness of the definition of each product within a detailed category or Basic Heading.

Comparability depends on the way of defining each product, as a "product" could cover a large variety of types depending on the different characteristics such as raw materials used, weight, packing, etc. that affect the price. A product must have the same characteristics in order to be *strictly comparable* over the different areas. In this case alone the ratio of its prices in two areas (single parity, PPP) measures the *genuine (pure) price differences*, that is when the same product is sold at different prices in the two areas. This definition of comparability may be broadened as suggested by many authors, but in this case the comparisons do not show only pure price differences. We will focus on this problem in the following sections.

Representativeness is imprecisely defined in some cases and therefore may be subject to different interpretations. It is possible to give a precise definition of representativeness of the products included within the BH considering each of them to be representative in terms of sample design, referring to a sub-set of products in which the products included in the BH can be subdivided. In this case the product is representative in terms of share of expenditure when retain a weight equal to the share of expenditure of the sub-set it represents. Therefore the *degree of representativeness* of the products strictly comparable included within the BH depends on the *system of weights* connected to the products which must represent the consumption basket in the considered area for the BH. This is similar to the requirement of "characteristicity" mentioned by Kravis et al.

[1] As already mentioned, in this paper the term PPP does not refer to the indices computed by ICP and Eursotat/Oecd programs.

[2] Normally the *EKS* formula and its modifications are used to transform the matrix of binary PPPs into a set of transitive parities (Balk, 2001)

[3] On the other hand, all the methods suggested for achieving transitivity are aimed at differing as little as possible from the original binary PPPs, that we consider in this study.

(1975) [4], which should be satisfied by each binary comparison in order to be optimal for that pair of areas.
In order to assure that the PPPs at level of BH measure the pure price differences, when we compare the consumer price levels in two areas, the representativeness should be achieved for both areas and the system of weights should be the same in the two areas.
There are two reasons for which the comparisons between two areas using PPPs at BH level became unrepresentative : (i) the products selected are the same but the system of weights is different in the two areas (the situation mentioned above); (ii) one o more products are not purchased in one of the two areas and vice-versa, since they are *typical or characteristic* of only one area.
Considering comparability and representativeness together it is evident that from a practical point of view the strict comparability of products, obtained through a detailed specification, leads to PPPs for which it is possible to measure pure price differences, but this strict comparability will lower the degree of general representativeness and characteristicity of a given product in different areas, even across a country, so the real consumption basket of the areas are not taken into account. On the other hand, by broadening the definitions of the selected products the comparability will be lowered and the calculated PPPs may correspond to different products, thus reflecting both pure price differences and the different representativeness and characteristicity of the selected products in the different areas.
Much research has been carried out to solve the conflict between comparability (including the problem of the quality of the products) and representativeness (see among others, Kravis et al, 1975, and more recently Rao, 2001b) but in this paper we do not consider those methods and we focus on measuring the importance of the different factors that affect the value of binary PPPs, in order to achieve a clear interpretation of the comparisons made.

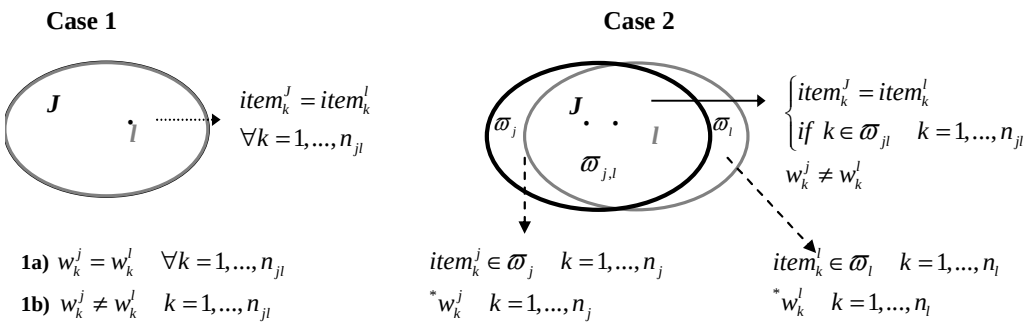
3. A method for measuring the factors that affect the comparisons of price levels

3.1 A framework for the Comparison at BH level

Considering the aim of this paper, we limit the analysis to binary comparison approach, *binary PPPs*, at the level of BHs and at the subsequent aggregate levels.
The hypothesis is that all data on prices and weights (defined as expenditure share) at product level are available for every area and for every BH[5].
In order to suggest a method for decomposing and interpreting the PPPs constructed to measure the consumer price level differences between area *j* and *l* and taking into account the definitions of comparability and representativeness reported in section 2, we can consider two different hypothetical situations involving each binary comparison at BH level as represented in Figure 1.

[4] To quote Kravis *et al*. "This requires that the comparison between each pair should be based on the best sample of representative items that can be obtained for that pair so that the prices will be most directly comparable, and that the weights used in the comparison be based solely on the spending patterns of the countries."
[5] This could be possible in a benchmark collection of data and it is surely more easy for the areas across a county involved in the price and consumption surveys for the construction of CPIs.

Figure 1 Comparability versus Representativeness



In the first hypothetical situation, shown in Case 1, we assume that the same number of products n_{jl} with the same characteristics, therefore *strictly comparable*, are available both in the areas *j* and *l*.

In this case, we can consider two different situations: (i) the first one (case 1a) describes a purely theoretical condition, where the systems of weights are equal in the two areas; (ii) the second one (case 1b), refers to a more probable situation, where even if the products are the same, the two systems of weights are different, with a different degree of representativeness in the two areas.

On the contrary, Case 2 reveals the second hypothesis, which represents the usual situation when making binary comparisons. In this case, the overlapping area ϖ_{jl} includes a group of identical products (strictly comparable) but with different systems of weights in the two areas (*j* and *l*) as in case 1b and other separate groups of products considered in the comparisons which can be found in area *j* but not in area *l*, and vice-versa. The sets marked ϖ_j and ϖ_l consist of some goods and services that are typical regarding the consumption behaviour in areas *j* and *l* respectively. These products are not strictly comparable and so they should not be considered in a calculation of PPPs based on identical products.

If we want to consider the *typical* products in areas *j* and *l* for the computation of PPPs, as in the case 2, it is important to underline that these products should be considered separately in the outer areas ϖ_j and ϖ_l in order to assess their importance in determining the computed PPPs. As specified in Figure 1, the number of typical products in each area is usually different ($n_j \neq n_l$) although in some cases it can be the same in both areas ($n_j = n_l$). It is clear that the higher the number of typical products, the larger the outer areas will be.

Case 2 in Figure 1 can also describe the situation when making binary comparisons by using a broader definition of comparability referring to a general specification of product[6]. The strictly comparable products are still included in the overlapping area w_{jl},

[6] On the other hand, following Krijnse-Locker H. (1983), the solution of broadening the definition of product, would give just an average price of the BH in each country, which is equal to the price of the most representative variety within a BH. Similarly, Kravis et al. (1975) suggested that when exactly

while the less comparable products are included in the outer areas ϖ_j and ϖ_l but as separate sets because it is necessary to evaluate their importance in determining the possibly different computed PPPs using a broader definition of comparability.
We can follow the same approach for comparing a spatial consumer price index computed using all the price data collected for the CPIs and the PPPs calculated using only comparable products.
The representation of the different situations reported in Figure 1 is also applicable to the comparisons of the levels of consumer prices at aggregate levels when we consider the BHs to be aggregated, instead of products.
The decomposition and interpretation of the consumer price spatial indices (PPPs) is presented in the following sub-sections with reference to the mentioned hypothesis, at BHs and aggregate level. A general approach in describing spatial indices formulae and PPPs is followed, without specific reference to the procedure applied by the international organisations that compile PPPs at present.

3.2 Binary Comparison at BH level: Comparability and representativeness

By assuming that the requirement of comparability is perfectly well-matched with the principle of representativeness (case 1 of Figure 1) and following the suggestion made by Kravis et al. (1975), an "equal weighting system" is used and a simple geometric mean of price relatives is selected as a method for averaging when more than one price ratio is available for a detailed category[7]. This formula was named "average relative price or PPP" by the above mentioned Authors and is identical to the ratio of the unweighted geometric mean prices, of the *j* area relative to the *l.*
Therefore it is possible to calculate the following spatial index PPP, that we call *Average Prices' Parity* (*APP),* to differentiate it from the usually computed PPP:

$$APP_{l,j} = \left(\prod_{k=1}^{n_{jl}} \frac{p_k^j}{p_k^l} \right)^{\frac{1}{n_{ij}}} = \frac{\left(\prod_{k=1}^{n_{jl}} p_k^j \right)^{\frac{1}{n_{jl}}}}{\left(\prod_{k=1}^{n_{jl}} p_k^l \right)^{\frac{1}{n_{jl}}}} \qquad [1]$$

identical commodities can not be found, it is possible to make a comparison between products in terms of relative costs or goods that serve the same purpose (equivalence in use). In this context, the new approach for product classification and identification, called the Structured Product Description (SPD) developed by the ICP, can be considered as well.

[7] It should be emphasized that a geometric mean is preferred to an arithmetic one because the former meets the country-reversal test, whereas the latter does not. Among the consistency conditions, the country reversal test or symmetric treatment of countries requires that in a given binary comparison it should be no matter which country is used as the base country. In other words, if we interchange the role of the countries in the spatial index number formula, then the resulting value of the index equals the reciprocal of the original value of the index (Diewert, 1987).

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where p_k^j denotes the price of item specification *k* in country *j* , p_k^l the price of item specification *k* in country *l* and n_{jl} is the number of items within the BH priced in both areas, which in this case is the same in the two area compared.
In order to take account of the possible dispersion of price ratios within BH, a weighting system is introduced in formula [1] through relative weights w_k^j and w_k^l of item specification *k* in country *j* and *l*, respectively.
Considering the further hypothesis (case 1a of Fig. 1) that the share of expenditure concerning each item *k* is identical in the two areas considered, that is $w_k^j = w_k^l = w_k$, and $\sum_k w_k = 1$, formula [1] can be expressed as:

$$APP_{l,j} = \prod_{k=1}^{n_{jl}} \left(\frac{p_k^j}{p_k^l} \right)^{w_k} = \frac{\prod_{k=1}^{n_{jl}} \left(p_k^j \right)^{w_k}}{\prod_{k=1}^{n_{jl}} \left(p_k^l \right)^{w_k}} \qquad [2]$$

It can be seen that the above formula is a weighted version of the Jevons index used in formula [1].
By relaxing the hypothesis of identical weighting systems in the two areas (case 1b) it is possible to consider the more probable situation of the different share of expenditure for the comparable products in the areas *j* and *l*. In this case, two different *APPs* can be calculated depending on which area is selected as the base or reference area and on the weighting system chosen. Considering for each area the corresponding weighting system, formula [2], which considers *l* as the base area, becomes:

$$APP_{l,j} = \frac{\prod_{k=1}^{n_{jl}} \left(p_k^j \right)^{w_k^j}}{\prod_{k=1}^{n_{jl}} \left(p_k^l \right)^{w_k^l}} \qquad [3]$$

where $\sum_k w_k^j = \sum_k w_k^l = 1$ and n_{ij} is once again the number of items within the BH priced in both countries.
From a different point of view, by considering *j* as the base area, we can obtain a similar APP, comparing area *l* to area *j*:

$$APP_{j,l} = \frac{\prod_{k=1}^{n_{jl}} \left(p_k^l \right)^{w_k^l}}{\prod_{k=1}^{n_{jl}} \left(p_k^j \right)^{w_k^j}} \qquad [4]$$

By examining formulae [3] and [4] we can get two different decompositions of each, following two different ways.

Starting with formula [3] and therefore with the index which compares area *j* relative to area *l*, by taking the natural logarithms and by adding and subtracting the hybrid product between the weights of the base area *l* and the log price of area *j* we can state that:

$$\begin{aligned}\ln\left(APP_{l,j}\right) &= \sum_{k=1}^{n_{jl}} w_k^j \ln\left(p_k^j\right) - \sum_{k=1}^{n_{jl}} w_k^l \ln\left(p_k^l\right) + \sum_{k=1}^{n_{jl}} w_k^l \ln\left(p_k^j\right) - \sum_{k=1}^{n_{jl}} w_k^l \ln\left(p_k^j\right) \\ &= \sum_{k=1}^{n_{jl}} w_k^l \left[\ln\left(p_k^j\right) - \ln\left(p_k^l\right)\right] + \sum_{k=1}^{n_{jl}} \ln p_k^j \left(w_k^j - w_k^l\right)\end{aligned} \qquad [5]$$

Looking at the exponential function, decomposition [5] can be equivalently expressed as:

$$APP_{l,j} = \frac{\prod_{k=1}^{n_{jl}} \left(p_k^j\right)^{w_k^j}}{\prod_{k=1}^{n_{jl}} \left(p_k^l\right)^{w_k^l}} = \prod_{k=1}^{n_{jl}} \left(\frac{p_k^j}{p_k^l}\right)^{w_k^l} \cdot \prod_{k=1}^{n_{jl}} \left(p_k^j\right)^{w_k^j - w_k^l} \qquad [5bis]$$

The first product on the right hand side of [5bis] represents the *pure price effect (PPE)*, corresponding to a bilateral PPP, calculated considering strictly comparable products, using a weighted Jevons index with weights of area *j*.
The second product, which refers to the *weight effect (WE)*, is related to the impact of the difference in consumption structures, expressed by $d_k^{j,l} = \left(w_k^j - w_k^l\right)$. When the product is not much representative of the consumer behaviour in area *l* the weight corresponding to the item *k* tends to be nearly equal to zero.
We must emphasize that by using this multiplicative decomposition, as well as those illustrated later in the paper, the WE can be greater than 1, indicating a positive influence of the weighting system, or it can be less than 1 denoting that the representativeness has a negative impact on the comparisons between the two areas based on APPs.
Obviously, if we take the natural logarithms in [3] and add and subtract the hybrid product between the weights of area *j* and the log price of the base area *l* we can obtain a second decomposition, which after applying the exponential function, is expressed as:

$$APP_{l,j} = \frac{\prod_{k=1}^{n_{jl}} \left(p_k^j\right)^{w_k^j}}{\prod_{k=1}^{n_{jl}} \left(p_k^l\right)^{w_k^l}} = \prod_{k=1}^{n_{jl}} \left(\frac{p_k^j}{p_k^l}\right)^{w_k^j} \cdot \prod_{k=1}^{n_{jl}} \left(p_k^l\right)^{w_k^j - w_k^l} \qquad [6]$$

where the pure price effect, expressed as a weighted Jevons index using the weights of the base area *l*, is multiplied by the weight effect obtaining the APP.
From a different perspective, taking into account formula [4] and thus referring to the comparison between *j* and *l*, with *j* as base area, by taking the natural logarithms and by adding and subtracting the hybrid product between the weights of the base area *j* and the log price of area *l* we can obtain a similar decomposition, which after applying the exponential function is expressed as:

$$APP_{j,l} = \frac{\prod_{k=1}^{n_{jl}} \left(p_k^l \right)^{w_k^l}}{\prod_{k=1}^{n_{jl}} \left(p_k^j \right)^{w_k^j}} = \prod_{k=1}^{n_{jl}} \left(\frac{p_k^l}{p_k^j} \right)^{w_k^j} \cdot \prod_{k=1}^{n_{jl}} \left(p_k^l \right)^{w_k^l - w_k^j} \qquad [7]$$

where the pure price effect is now expressed by the weighted geometric mean of price ratio between area *l* and *j* using the expenditure shares of the base country and the weight effect is related to the influence of the difference $d_k^{l,j} = \left(w_k^l - w_k^j \right)$.

On the other hand, by taking the natural logarithms of [4] if we add and subtract the hybrid product between the weights of the area *l* and the log price of the base area *j* the decomposition, after using the exponential, becomes:

$$APP_{j,l} = \frac{\prod_{k=1}^{n_{jl}} \left(p_k^l \right)^{w_k^l}}{\prod_{k=1}^{n_{jl}} \left(p_k^j \right)^{w_k^j}} = \prod_{k=1}^{n_{jl}} \left(\frac{p_k^l}{p_k^j} \right)^{w_k^l} \cdot \prod_{k=1}^{n_{jl}} \left(p_k^j \right)^{w_k^l - w_k^j} \qquad [8]$$

where the Jevons index which identifies the pure price effects is calculated using the weights of area *l*.

As we can see, by using equations [5bis] and [6] the APPs are the same, although the values of the pure price effect and of the weight effect differ in the two decompositions. On the other hand, by applying decomposition [7] and [8] it is possible to obtain overall identical results differing however in the pure price and weight effects. It is important to emphasize that the results obtained from [5bis] and [6] are the reciprocal of those obtained from [8] and [7] respectively.

Besides, it is evident that when the decomposition [5bis] and [8] or [6] and [7] are used, the area reversal property is easily satisfied since the results obtained imply that $APP_{l,j} = \frac{1}{APP_{j,l}}$, $PPE_{l,j} = \frac{1}{PPE_{j,l}}$ and $WE_{l,j} = \frac{1}{WE_{j,l}}$

On the other hand, the symmetric treatment of areas could be achieved for the pure price effect and for the weight effect by using a geometric mean of the indices obtained from [5bis] and [6] initially, and then by applying a geometric average to the results from [7] and [8] and therefore obtaining Törnqvist indices[8]. Considering for example the PPE, we can state:

$$^{T}PPE_{l,j} = \sqrt{\prod_{k=1}^{n_{jl}} \left(\frac{p_k^j}{p_k^l} \right)^{w_k^l} \cdot \prod_{k=1}^{n_{jl}} \left(\frac{p_k^j}{p_k^l} \right)^{w_k^j}}$$

[8] As it can be easily seen, the indices obtained are proper Törnqvist indices since they use a simple average of the expenditure shares in the two areas as weights.

$$^{T}PPE_{j,l} = \sqrt{\prod_{k=1}^{n_{jl}} \left(\frac{p_k^l}{p_k^j} \right)^{w_k^j} \cdot \prod_{k=1}^{n_{jl}} \left(\frac{p_k^l}{p_k^j} \right)^{w_k^l}}$$

where $^{T}PPE_{l,j} = \frac{1}{^{T}PPE_{j,l}}$

Finally, looking at the different "weight effects" shown in the four decompositions (5bis, 6, 7 and 8), it could be interesting to decompose them in order to understand the reasons for their values due to the price distributions and their correlation with the weight difference distributions and thus provide interesting information on its characteristics. This analysis can be carried out following the same approach presented in Biggeri, Brunetti and Laureti (2008) so that each weight effect can be expressed as a product of factors which have a unique interpretation although they differ in value since they are defined relating to different distributions of prices and weight differences.
Focusing for example on decomposition [5bis] the weight effect can be expressed as:

$$\prod_{k=1}^{n_{jl}} \left(p_k^j \right)^{w_k^l - w_k^j} = \exp\left(n_{ij} \cdot s_{\ln P^j} \cdot s_d \cdot R_{\ln P^j, d} \right) \qquad [9]$$

From the above formula it is possible to identify the factors that influence the weight effect, that is $s_{\ln P^j}$, the standard deviations of prices of area *j*, s_d, the standard deviation of the difference between the weights in the two areas compared $d_k^{j,l} = \left(w_k^l - w_k^j \right)$ and $R_{\ln P^j, d}$, the linear correlation coefficient between the prices and the differences in the corresponding weights.

3.3 Binary Comparison at BH level: Comparability versus Representativeness and Characteristicity

The other hypothesis represented in Case 2 of Figure 1 describes the most usual situation faced when making binary comparisons. In fact, since the patterns of consumption can differ greatly between the two areas considered, products that are representative and easily found in area *j* may be not easily found in *l*, because of differences in supply conditions, income levels, tastes, climate, customs, etc.
Besides, a comparison between areas *j* and *l* must be made using products that are comparable even if they are not identical. Therefore, methods for selecting products and for calculating PPPs are designed to try to respect both requirements. Inevitably compromises must be made in order to reach an overlapping area, marked ϖ_{jl} in Figure 1, which includes the set of items strictly comparable with different systems of weights in the two areas (*j* and *l*) and other not strictly comparable products considered in the comparisons which can be available in area *j* but not in area *l*, and vice-versa. On the other hand, those products which are characteristic or typical of one of the two areas are included in area ϖ_j or ϖ_l.

In this case[9], by using formulae [5bis] and [7], considering firstly area *l* and then area *j* as the base area it is possible to obtain an interesting decomposition of the spatial price indices relating to the two sets of consumer goods and services .
Firstly, considering the right hand side of expression [5bis] and taking into account all the products in the two areas, a new ${}^{*}APP_{l,j}$ could be obtained by introducing a multiplicative factor as follows:

$$ {}^{*}APP_{l,j} = \prod_{k=1}^{n_{jl}} \left(\frac{p_k^j}{p_k^l} \right)^{w_k^l} \cdot \prod_{k=1}^{n_{jl}} \left(p_k^j \right)^{w_k^j - w_k^l} \cdot \frac{\prod_{k=1}^{n_j} \left(p_k^j \right)^{{}^{*}w_k^j}}{\prod_{k=1}^{n_l} \left(p_k^l \right)^{{}^{*}w_k^l}} \qquad [10] $$

where the third ratio on the right hand side of [10], that is a ratio between the average prices of the characteristic products in the two areas, could measure the effect due to the characteristics of the area in its consumption basket (*characteristicity effect*). On the other hand, in this case the weight effect continues to evaluate the influence due to the different representativeness in terms of the weighting systems relating to the n_{jl} strictly comparable goods and services priced in both areas, thus included in area $\varpi_{j,l}$.
Similarly, by using the approach followed in [6] it is possible to obtain the expression:

$$ {}^{*}APP_{l,j} = \prod_{k=1}^{n_{jl}} \left(\frac{p_k^j}{p_k^l} \right)^{w_k^j} \cdot \prod_{k=1}^{n_{jl}} \left(p_k^l \right)^{w_k^j - w_k^l} \cdot \frac{\prod_{k=1}^{n_j} \left(p_k^j \right)^{{}^{*}w_k^j}}{\prod_{k=1}^{n_l} \left(p_k^l \right)^{{}^{*}w_k^l}} \qquad [11] $$

The pure price effect in [10] and [11] are weighted Jevons using the weights of area *l* and *j* respectively.
From a different perspective, by considering *j* as the base area as in [8] and [7], it is possible to obtain two different decompositions. Firstly, we can state:

$$ {}^{*}APP_{j,l} = \prod_{k=1}^{n_{jl}} \left(\frac{p_k^l}{p_k^j} \right)^{w_k^l} \cdot \prod_{k=1}^{n_{jl}} \left(p_k^j \right)^{w_k^l - w_k^j} \cdot \frac{\prod_{k=1}^{n_l} \left(p_k^l \right)^{{}^{*}w_k^l}}{\prod_{k=1}^{n_j} \left(p_k^j \right)^{{}^{*}w_k^j}} \qquad [12] $$

Then following a similar procedure we can obtain:

[9] The same approach can be followed, as already mentioned in section 3.1, when we would like to make binary comparisons by using a broader definition of comparability referring to a general specification of product and also for comparing a spatial consumer price index computed using all the price data collected for the CPIs and the PPPs, calculated using only comparable products.

$$^{*}APP_{j,l} = \prod_{k=1}^{n_{jl}} \left(\frac{p_k^l}{p_k^j} \right)^{w_k^j} \cdot \prod_{k=1}^{n_{jl}} \left(p_k^l \right)^{w_k^l - w_k^j} \cdot \frac{\prod_{k=1}^{n_l} \left(p_k^l \right)^{^{*}w_k^l}}{\prod_{k=1}^{n_j} \left(p_k^j \right)^{^{*}w_k^j}} \qquad [13]$$

Once again, the pure price effects in [12] and [13] are weighted Jevons indices, which use as weights the expenditure shares of area *l* and *j* respectively.

Although the results from [10] and [11] are the same for the APP as well as for the characteristicity effect, the values of the pure price effect and of weight effect differ in the two decompositions. The same occurs when decompositions [12] and [13] are applied. On the other hand, the pure price and the weight effects derived from [10] and [11] are the reciprocal of those obtained from [12] and [13] respectively.

In this way the symmetric treatment of areas is easily satisfied since $^{*}APP_{l,j} = \frac{1}{^{*}APP_{j,l}}$, $^{*}PPE_{l,j} = \frac{1}{^{*}PPE_{j,l}}$ and $^{*}WE_{l,j} = \frac{1}{^{*}WE_{j,l}}$.

On the other hand, the same requirement could be achieved for the pure price effect and for the weight effect by using a geometric mean of the indices obtained from [10] and [11] initially, and then by applying a geometric average to the results from [12] and [13] and therefore obtaining Törnqvist indices. Considering for example the PPE, we can state:

$$^{T}PPE_{l,j} = \sqrt{\prod_{k=1}^{n_{jl}} \left(\frac{p_k^j}{p_k^l} \right)^{w_k^l} \cdot \prod_{k=1}^{n_{jl}} \left(\frac{p_k^j}{p_k^l} \right)^{w_k^j}}$$

$$^{T}PPE_{j,l} = \sqrt{\prod_{k=1}^{n_{jl}} \left(\frac{p_k^l}{p_k^j} \right)^{w_k^j} \cdot \prod_{k=1}^{n_{jl}} \left(\frac{p_k^l}{p_k^j} \right)^{w_k^l}}$$

where $^{T}PPE_{l,j} = \frac{1}{^{T}PPE_{j,l}}$

3.4 Aggregation above the BH level

In order to aggregate the basic heading APPs calculated as described in the previous two sub-sections, we can follow different procedures, which give different results, by using the expenditure shares for each BH as weights.

For the aggregation of the APPs it is possible to choose, for example, between the utilization of a weighted geometric average or a weighted arithmetic average.

Once a formula is selected the subsequent procedure is similar, therefore we limit the analysis to [10].

Considering the geometric mean as aggregation formula we obtain the aggregated APP, denoted by ${}^{*}APP_{l,j}^{A}$:

$$
\begin{aligned}
{}^{*}APP_{l,j}^{A} &= \prod_{h=1}^{M}\left({}^{*}APP_{l,j}{}^{BH_h}\right)^{W_{hl}} = \\
&= \prod_{h=1}^{M}\left({}^{*}PPE_{l,j}{}^{BH_h}\right)^{W_{hl}} \cdot \prod_{h=1}^{M}\left({}^{*}WE_{l,j}{}^{BH_h}\right)^{W_{hl}} \cdot \prod_{h=1}^{M}\left({}^{*}CE_{l,j}{}^{BH_h}\right)^{W_{hl}}
\end{aligned}
\qquad [14]
$$

where ${}^{*}APP_{l,j}{}^{BH_h}$ is the APP calculated at basic heading level h ($h=1,..,M$) and w_{hl} denotes the expenditure share of the basic heading h in the base area l and $\sum_{h} W_{hl} = 1$.

Obviously, by using w_{hj} , the expenditure share of the basic heading h in the area j as weights in [14], we can obtain a similar expression.

The use of a geometric mean is preferred as the decompositions shown in the above formulae are still valid at any level of aggregation. In other words, it is possible to analyse the degree of influence of each factor by categories and divisions of consumption expenditure and the degree of the influence of the various effects (pure price effect, weight and characteristicity) depends on the importance of the BH in the consumption behaviour in the two areas compared. In fact, each aggregated effect is a weighted geometric mean of the corresponding effect measured referring to each BH.

On the other hand, if we refer to an arithmetic mean, using as weights the expenditure share of the basic heading h in the base area l , in order to obtain the aggregated $APP_{l,j}^{A}$, the decomposition results found at the BH level in section 3 have not the same structure at higher level of aggregation. However, it is possible to have an approximate measure of the influence of each single factor by computing a weighted average of each factor at the BH level.

4. An attempt to measure the factors that affect APPs within an experiment conducted in Italy to compute territorial PPPs

The approach proposed to measure the factors that affect the value of the computed APPs and PPPs can be particularly useful for making comparisons of consumer price levels across different areas or cities within a country. Firstly, the consumption baskets may be more similar across areas in a country than between countries and besides it may be possible to find more strictly comparable products for all the areas within a country; secondly, more comparable data at detailed elementary level (products) are

usually available for the areas within a country, normally collected through the surveys carried out to compute the CPIs.

On the other hand, there is considerable demand: (i) for measuring spatial consumer price indices for different areas within a country to compare household expenditures, revenues and salaries (Cfr: Bretell and Gardiner, 2002; Melser and Hill, 2005) and (ii) for integrating the work of PPPs and CPIs in order to obtain benefits in terms of enlargement of coverage of products, coherence between temporal and spatial price indices, cost of the collection of data, and so on (Rao, 2001a; ILO, 2004; Ferrari and al., 2005).

Moreover, the decomposition approach can be applied not only to measure the factors that affect the PPP results, but also to consider the use of all data collected for the CPIs in order to calculate consumer spatial indices as APPs. These last indices as the CPIs are certainly more representative of the products purchased in each different areas, but they do not satisfy the principle of strict comparability of products among areas. However the computation of these indices and of PPPs allows us to assess the difference between them, and the factors that affect the differences. In fact, in this case the situation is similar to case 2 illustrated in Figure 1, where we can enlarge the outer areas considering one part of them referring to the products included in the list of CPIs, but not in the list of the PPPs.

Istat started a research project in 2005 and since then it has been carrying out an experiment to calculate PPPs in order to compare the consumer price levels at regional level (De Carli, 2008). Some preliminary analysis proposed in Section 3 were carried out using the data from the above experiment.

The research project was aimed at testing the possibility of using and integrating the statistical information currently supplied within the CPI surveys and only refers to some expenditure divisions: Food and beverages, Clothing and footwear and Furniture for the home, which represent about 34% of the total consumer expenditures.

In order to calculate the intra-country consumer PPPs for consumer prices for *20 cities*, the procedure for international comparison was used, by following the principle of strict comparability of the products. For this reason, a complicated analysis of data collected for CPIs was carried out to check if the characteristics of products included in that list were the same in the different cities. Regarding Food and beverage products Istat was able to use the CPI data (of about 1,300 products) after having achieved comparability; while for Clothing and Furniture products it was necessary to carry out an *ad hoc survey* in order to collect new data for strictly comparable products in the 20 cities.

The computation of PPPs were made for the BHs of each mentioned division of expenditure. Moreover, it was decided to compute the PPPs for some sub-groups of divisions such as processed and non processed products regarding foods, well known brand and brand-less products concerning clothing and furniture as well.

The results of the experiment to compute the PPPs, carried out for the *year 2006*, are very interesting (De Carli, 2008).

Having all the necessary price data we have attempted to calculate the APPs for the Foods and beverage products at the level of 20 cities, and to decompose them in the three components: PPEs, pure price effects (equal to the computed PPPs), WEs, weight effects, and CEs, characteristicy effects.

The data on the household expenditures were only available at the level of BHs (144) and not at the level of products. Therefore it was impossible to make the computation following exactly the formulae illustrated in section 3. Some very rough estimations

were calculated using different procedures and weights that do not satisfy the requested hypothesis and the decomposition "properties" mentioned in section 3.
Therefore, the estimations of the effects are very approximate and should be analysed with caution, but clearly show the importance of the proposed analysis. The results are different for the different sub-groups of products, but in any case it is clear that the weight effect and the characteristicity effect play an important role in the binary comparisons of consumer price levels between the different cities and in reference to Italian averages. Both results present a larger range of values than the PPPs. Furthermore, the results regarding food products show that for some cities (usually the northern cities) the *weight effect* is substantially *less than 1* (min = 0.700) and the *characteristicity effect* is substantially *more than 1* (max = 1.535); this means that in those cities the shares of expenditure regarding the different products have a negative impact on the comparisons between the average prices (APPs), while the expenses for characteristic products have a positive impact on that.
Vice versa for other cities (usually southern cities) the weight effect is substantially more than 1 (max = 1.309) and the characteristicity effect is substantially less than 1 (min = 0.586); this means that in those cities the shares of expenditure regarding the different products have a positive impact on the comparisons made between the average prices (APPs), while the expenses for characteristic products have a negative impact on that comparison. The results obtained are coherent with the common idea of the behaviour of prices and of household consumption in the Italian cities, especially if we take into account the differences in the consumer markets between the northern and southern cities in Italy.

5. Concluding remarks

When the PPPs are published at an international level, many users criticize the interpretation of binary comparisons between two countries or areas, because the products included in the computation are not representative enough of consumption expenditures (or other reference aggregates of the PPPs) of the countries or areas in question. In general, the main criticisms concern the conflicting choice between the two most important principles in products and price selection for spatial comparisons, that is comparability and representativeness.
Focusing on the comparisons of household consumption and the spatial comparisons for areas across a country, we propose a simple method to decompose the comparison of the weighted average price levels between two areas APP in three different components, that is: a *pure price effect*, calculated for strictly comparable products (that may correspond to the usual bilateral PPP); a *weight effect*, due to the difference in consumption structures; a *characteristicity effect*, due to the comparison of the prices of products that are characteristic or typical of each area. In this way, having the necessary data, the comparison between APPs and PPPs (the latter for strictly comparable products) allows us to obtain approximate measures for a better interpretation of the spatial comparisons of prices, and to verify which could be the effect of broadening the definition of comparability and the effect of considering the use of all data collected for the CPIs to calculate consumer spatial indices as APPs.
We attempted to apply the proposed method by using an experiment carried out by Istat to compute the PPPs for 20 cities and for three divisions of expenditures, but the lack of

adequate data do not allow us to respect the hypothesis and the necessary properties specified in the section 3. It was only possible to get very approximate measures of the divergences. However, the results obtained are very encouraging because they stress the importance of having a measure of the weigh effects and characteristicity effects in order to interpret the PPPs and make adequate decisions for their implementation.
Further research needs to confirm the importance of the proposed analysis for different practical situations in different countries and to develop our proposal, thus improving the methods of decomposition, in order to be directly applicable to multilateral comparisons and more precisely to the procedures followed for computing PPPs in present international programs.

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