Elementary Indices for Purchasing Power Parities

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The paper reviews the different methods that can be used to calculate elementary PPPs for basic headings. Comparisons are made with the methods used to calculate elementary indices for elementary aggregates in temporal price indices. The selection of the common list of products to be used for price collection in different countries can have a considerable impact on the results. Representative products are defined and their role in the process of drawing up the product list explained. Information about representativity can also be used to correct potential biases in the estimated elementary PPPs. The choice of index formula can have a significant effect on the results even when a complete set of prices can be collected for the products on the common list.

Introduction

The methodologies used to calculate temporal indices within a country and international price indices between countries, or Purchasing Power Parities, have many points in common even though they are often obscured by differences in terminology. This paper focuses on elementary indices. An elementary price index is a price index that is calculated directly from a sample of price observations without the use of quantities or expenditures as weights. Quantities and expenditures are ignored not as a matter of choice but simply because in every country there is a certain level of economic aggregation below which it is not possible to obtain detailed information on quantities or expenditures. This is entirely a practical matter that depends on the availability of expenditure data within a country or group of countries. Expenditure data are usually collected quite separately from price data and much less frequently.

The calculation of aggregate price indices, whether temporal or international, is therefore generally carried out in two steps. The first step is to identify the smallest economic aggregates for which it is possible to

[1] Parts of this paper draw upon material contained in Chapter 11 of the *ICP Handbook* (2006) on the "Estimation of PPPs for Basic Headings Within Regions " written by the author. Responsibility for the present paper, however, rests solely with the author. The views expressed here do not necessarily reflect the views of the ICP or the World Bank.

obtain expenditure data. These are called 'basic headings' in the literature on international comparisons and 'elementary aggregates' in the literature on temporal indices. The price index for each basic heading or elementary aggregate is estimated on the basis of a sample of prices collected in each of the periods or countries compared. Price indices for higher level aggregates are then obtained by combining the elementary indices with the corresponding expenditure data.

Elementary indices and their properties are explained in some detail in the recent international Manuals on CPIs and PPIs. One of the purposes of this paper is to re-examine the elementary indices used in international comparison in the light of the recent literature on elementary temporal indices.

Binary and multilateral approaches

There are two approaches to the calculation of a set of transitive multilateral PPPs for a group of countries: the binary approach and the multilateral approach.

- The binary approach proceeds by calculating binary PPPs between each pair of countries. Most types of binary PPPs are not transitive, but transitivity can be imposed by adjusting the PPPs, usually by means of the EKS formula.
- The multilateral approach proceeds by calculating a set of transitive PPPs for all countries in the group simultaneously. At the level of an elementary price index the CPD is the method generally used.

The binary approach will be considered first in this paper. There is a close formal similarity between a price index between two periods of time for the same country and a price index between two countries in the same period of time. Most of the theory and practical experience gained in the calculation of temporal indices is directly relevant to binary international indices. It should be noted that even when a binary approach is used, the ultimate objective is to arrive at a set of transitive multilateral indices.

The binary approach to multilateral comparisons was adopted by the Statistical Office of the European Communities at the start of their PPP program in the late 1960s. The on-going joint Eurostat-OECD PPP program

covering 46 countries continues to use the binary approach. The methodology has been considerably changed and improved over the years.

Elementary binary PPPs

As just noted, most types of binary indices are not transitive. When the binary approach is adopted therefore, the EKS formula is usually applied to transform the original intransitive PPPs into a set of transitive PPPs. EKS PPPs are defined as the set of transitive PPPs that minimize the sum of the squares of the logarithmic differences between the original intransitive PPPs and the transitive PPPs. It can be shown that the EKS PPP for country k on country j is equal to the geometric average of the direct PPP for k on j and all the indirect PPPs for k on j *via* third countries, the direct PPP carrying twice the weight of each indirect PPP.

Thus, EKS is not itself a type of index but a method of imposing transitivity on an already existing set of indices, whether elementary or aggregate. Simply describing a PPP as EKS provides no information whatsoever about the type of elementary or aggregate index used or its properties. The relative merits of the binary and multilateral approaches at the basic heading level depend mainly on the nature and properties of the underlying elementary binary indices before the EKS formula is applied.

It is useful to establish what kind of indices would provide appropriate targets in an ideal situation in which data were available on expenditures as well as prices within the basic heading. After that, it is possible to consider how best to approximate to such indices using samples of price observations only.

In the case of temporal indices, the properties and behaviour of elementary indices are discussed in some depth in the recent international CPI and PPI Manuals[2]. Most elementary price indices are either averages of the price relatives for the individual products or ratios of their average prices. To facilitate discussion of the various indices the *2004 CPI Manual* decided to introduce names for some of them as follows.

- The *Dutot* index: the ratio of the arithmetic average prices

[2] See Chapter 20 by Erwin Diewert of the *2004 CPI Manual*. In this paper reference will be made to the *CPI Manual* only as the PPI manual is fully consistent methodologically with the CPI Manual.

- The *Carli* index : the arithmetic average of the price relatives
- The harmonic index: the harmonic average of the price relatives
- The *CSWD* index: the geometric average of the Carli and harmonic indices
- The *Jevons* index: the geometric average of the price relatives (≡ the ratio of the geometric average prices)

All the averages should be understood to be unweighted unless stated to the contrary.

The relative merits of the various types of elementary index are systematically evaluated in the *2004 CPI Manual* using both the axiomatic and economic approaches to index number theory. The conclusion reached is that on both the axiomatic or the economic approaches the preferred type of elementary index is the *Jevons* index[3]. Some countries already use the Jevons although many others have traditionally preferred the Carli.

The Jevons index has been widely used for purposes of international binary price comparisons. At the level of an individual product, the ratio of the average national prices actually provides a definition of purchasing power parity. It is the rate of currency exchange which, if used to convert a given sum of money from one currency into the other, will ensure that the same quantity of the product in question can be purchased in both countries. Taking a geometric average of all the individual product PPPs in a basic heading in order to arrive at the elementary PPP seems to have been regarded as the obvious way to proceed. In the early years of the European Economic Communities' PPP program the elementary PPPs were simple unweighted geometric averages of the individual *product PPPs* within the basic heading. However, they were soon abandoned in favour of weighted Jevons indices, as explained below.

One advantage of Jevons indices is that binary Jevons PPPs are transitive *provided* that each individual product PPP carries the same weight in each binary comparison. More remarkably, it has been proved that no other type of price index is transitive[4]. This is a particularly significant

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[3] See paragraph 20.115 of the *2004 CPI Manual*.

[4] Paragraph 15.89 of the *2004 CPI Manual* states that "under very weak regularity conditions the only [temporal] price index satisfying the circularity [transitivity] test is a weighted geometric average of all the individual price ratios, the weights being constant through time." This result is attributed to Funke, Hacker and Voeller (1979)

result in the context of *multilateral* international comparisons where the need for transitivity is paramount. One important special case is the unweighted Jevons PPP in which each product PPP carries a weight of *1/n* in each binary PPP, where *n* is the number of products covered.

The economic approach to elementary PPPs[5]

A particular type of index is said be exact if it coincides with some economic theoretic index. In the case of consumption price indices, the economic index depends on the form of the consumers' preferences. If the preferences can be represented by a Cobb-Douglas function a weighted Jevons index with the weights equal to the expenditure shares is exact[6]. The shares should be the same in both the situations (time periods or countries) compared as the cross elasticities of substitution are unity.

In an international context, it is not realistic to assume that the same set of Cobb-Douglas type preferences holds across a large group of countries, but it is nevertheless interesting to note that *if* they did hold the binary economic indices would be transitive. The expenditure shares would be the same across all countries so that binary Jevons indices using the shares as weights would be transitive.

From a theoretical perspective, therefore, it seems that a binary Jevons PPP with average expenditure shares as weights would provide a suitable target index for a basic heading PPP. The target index is not actually an elementary index itself as an elementary index has to be calculated from price data only without the use of either quantity or expenditure data. However, it is possible to devise weights based on other kinds of information that may provide rough approximations for expenditure weights and to use these proxy weights in a weighted Jevons PPP. Methods of deriving rough weights have been developed by Eurostat. First, it is necessary to explain the concept or representativity.

Product lists and representativity

When the objective is to arrive at a set of elementary PPPs for a group of countries, it is necessary to draw up a common list of products for pricing

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[5] This section draws upon paragraphs 20.71 to 20.86 of Chapter 20 of the *CPI Manual* (Diewert, 2004).

[6] See paragraphs 20.81 to 20.83 of the *2004 CPI Manual*.

in the various countries. In order to ensure that price matches can be made between all the countries in the group all countries must try to price the same set of products. When making an individual binary comparison the list of products can be tailor made for that particular comparison, but when many binary comparisons have to be made countries cannot be expected to collect a different set of prices for each comparison. With a single list for a group of countries the number of individual product PPPs available for each binary comparison will tend to be less than would be available in an independent binary comparison. A binary comparison based on a common list of products will not be as robust or reliable as the same binary carried out separately.

A common product list to be used by a group of countries should contain some products that are representative for every country in the group. However, there may no consensus about the exact meaning or 'representative' in this context. In this paper, a product is said to be representative in a particular country if it is purchased in relatively large quantities in that country compared with other countries. Given that relative quantities and relative prices tend to be negatively correlated, representative products are likely to have relatively low prices. Indeed, some products will be purchased in relatively large quantities in some countries precisely because their relative prices are low.

The concept of representativity is examined in some detail in the Annex to this paper. It is shown how representative products could be identified from quantity data if they were available. It is also shown how a product list that is dominated by the representative products of one country or sub-group of countries can introduce bias into the estimated elementary PPPs. As data on quantities and expenditures will not be available in practice below the level of the basic heading, local price and market experts have to use their knowledge and judgment to try to identify products that are representative.

Assuming that representative products can be distinguished from unrepresentative products, the information can be used in two ways. First, it can be used to draw up balanced lists of products for pricing. A well balanced list would be one containing some of the representative products of each country. As the same product may be representative in a number of countries such a list need not be very large. Second, information about

representativity can be utilized in the process of calculating the elementary basic heading PPP. It is this second function that is of more interest here.

Representativity may be more important for international comparisons than for inter-temporal indices. In temporal indices the same product can be priced repeatedly in successive periods of time. However representativity is defined, a product that is representative in one period does not suddenly become unrepresentative in the next period. A representative product remains representative from one period to the next in the same country. Like is compared with like. In international comparisons, however, an individual product can be representative one country but unrepresentative in another. If its price is relatively low in some countries it must be relatively high in some others. It is necessary to take account of this when estimating the elementary PPP for the basic heading.

The distinction between representative and unrepresentative products is not very clear cut in practice. To the extent that it has to rely on the expertise and judgment of experts it is inevitably partly subjective. It may also be new for many countries. There is a risk therefore that different countries may interpret it differently. If so, the distinction may not serve its intended purpose and may have to be ignored when estimating the elementary PPPs.

The calculation of elementary PPPs

The starting point for the calculation of elementary PPPs is a set of national average prices reported by countries for the products on the agreed common product list. The prices may be presented in the form of a tableau or matrix in which it is customary for the rows to denote products and the columns to denote countries.

	Country j			
Product i	1	2	…	C
1	p_1^1	p_1^2	…	p_1^C
2	p_2^1	p_2^2	…	p_2^C

.	.	.		.
.	.	.		.
n	p_n^1	p_n^2	…	p_n^C

The list of products in a tableau of this kind could be either the entire list of products within a basic heading or the sample list of products used of purposes of price collection. Elementary PPPs are calculated using the sample list. It must not be treated as if it were a random sample of products from within a basic heading. On the contrary, the products on the common list are typically selected purposively. For example, the construction of the product list in the Eurostat-OECD program is a lengthy and elaborate process extending over many months. It involves all the countries in the program. It is designed to ensure that the list is as balanced as possible.

Three types of elementary binary PPPs

Three types of elementary binary PPPs are described in the following sections. Each has been used by Eurostat. Although the EKS formula may be required to impose transitivity, this section is concerned with the type of binary indices used and their properties *before* the EKS formula is applied. They cannot therefore be described as EKS indices. The first index is the simple unweighted Jevons PPP.

Simple Jevons

The first stage of the original procedure used by Eurostat was to calculate each elementary binary PPP as a simple Jevons index. The simple or unweighted Jevons PPP for country k based on country j is defined in (1). It is the geometric mean of the price ratios p_i^k / p_i^j for the n_{jk} products for which *both* counties have reported average prices: $n_{jk} \leq nC$.

$$(1) \qquad PPP_{Jev}^{j.k} = \left(\prod_{i=1}^{n_{jk}} \frac{p_i^k}{p_i^j} \right)^{\frac{1}{n_{jk}}}$$

In practice, countries seldom report a complete set of national average prices for all the products on the list. The PPP for an individual product may carry a positive weight in some binary Jevons indices and a zero weight in others

because of missing prices. As the weights vary in this way, the simple Jevons PPPs are generally not transitive.

If transitivity is imposed by means of the EKS formula the resulting indices are described here as *EKS/Jevons*. It is also possible to use a weighted version of the EKS formula.[7] For example, if some of the Jevons PPPs are considered to be relatively unreliable, they could be given less weight than the others.

Eurostat soon abandoned EKS/Jevons on the grounds that simple Jevons PPPs are liable to be biased if the product list for the basic heading is not balanced. For example, if it happens that most the *p*'s for country *k* refer to products that representative in *k* whereas most of them are unrepresentative in *j* it may be presumed that $PPP_{Jev}^{j,k}$ will have a downward bias as compared with $PPP_{Jev}^{j,k}$ for the entire list of products in the basic heading. As it may be very difficult to ensure that product lists are evenly balanced in practice there is inevitably a risk that at least some of the Jevons PPPs may be biased, possibly substantially. The next version of the Eurostat method was specifically designed to correct for potential biases of this kind.

*Jevons**

The simple Jevons indices were replaced by indices described here as Jevons*.[8] The 'asterisk' method, is so called because it makes use of the distinction between representative and unrepresentative products, the representative products being flagged in the product lists by an *. A detailed exposition of Jevons* PPPs and their properties is given by Sergey Sergeev (2003).

The Jevons* binary index exploits the fact that the prices of representative products are likely to be relatively low and the prices of unrepresentative products relatively high. The index is the geometric mean of two separate simple Jevons indices, one covering products that are

[7] See Prasada Rao (2001).

[8] See Eurostat (1983). This publication contains a major section on methodology including a description of both simple Jevons indices and the Jevons * indices which replaced them: see Section 4.1, pp. 43 to 48. It also explains why Eurostat decided to change from Jevons to Jevons *. A description and evaluation of Jevons* indices was also given by Peter Hill (1982).

representative in the first country and the other covering products that are representative in the second country. Some products may be representative in both countries and included in both indices.

Let M_{jk} denote the number of products that are representative in either country j or in k, and for which average prices are reported by *both* j and k. M_{jk} will generally be smaller than n, the total number of products on the list for the basic heading.

Let M^R_{jk} denote the number of products that are representative in country j and priced in both j and k.

Let M^R_{kj} denote the number of products that are representative in country k and priced in both j and k.

Next, let $m = 1, \ldots, M_{jk}$ index the set of products that are representative in country j or in country k, and for which average price are reported by both j and k.

Now, define the following weights.

$w^m_{jk} = 1/M^R_{jk}$ if product m is representative in country j.

$w^m_{jk} = 0$ if product m is not representative in country j.

$w^m_{kj} = 1/M^R_{kj}$ if product m is representative in country k.

$w^m_{kj} = 0$ if product m is not representative in country k.

The simple *Jevons* j index, $P_j^{j,k}$, for j's representative products is the simple geometric mean of the price ratios for the products representative of j: that is,

$$(2) \quad P_j^{j,k} = \prod_{m=1}^{M_{jk}} \left[\left(\frac{p_m^k}{p_m^j} \right)^{w^m_{jk}} \right]$$

Similarly, *Jevons* k based on k's representative products is defined as follows.

$$(3) \quad P_k^{j,k} = \prod_{m=1}^{M_{jk}} \left[\left(\frac{p_m^k}{p_m^j} \right)^{w_{kj}^m} \right]$$

Jevons j, or $P_j^{j,k}$, may be expected to be greater than *Jevons k*, or $P_k^{j,k}$. The denominator of each individual product PPP, p_m^k / p_m^j, entering into Jevons *j* consists of a product that is representative in the base country *j*, whereas the product in the numerator could be either representative or unrepresentative of *k*. Conversely, the denominator of each individual product PPP in Jevons *k* consists of a product that may be representative or unrepresentative, whereas the numerator includes only representative products. Given that unrepresentative products tend to have relatively high prices, Jevone *j* may be expected to be greater than Jevons *k*. This expectation seems to be borne out in practice[9].

Assume that the target index for the basic heading PPP is the weighted Jevons index based on the assumption of a common set of Cobb-Douglas preferences. Jevons *j* is likely to be above the target index and Jevons *k* below the target.

In these circumstances a better estimate of the target index is likely to be obtained by calculating the geometric average of Jevons *j* and Jevons *k*, the $P_j^{j,k}$ and $P_k^{j,k}$ indices, as follows.

$$(4) \quad P_{J*}^{j,k} = \sqrt{P_j^{j,k} P_k^{j,k}} = \prod_{m=1}^{M_{jk}} \left[\left(\frac{p_m^k}{p_m^j} \right)^{\frac{w_{jk}^m + w_{kj}^m}{2}} \right]$$

This index, denoted by $P_{J*}^{j,k}$, is defined as the *Jevons** index. It is a weighted Jevons index that is intended to provide a rough estimate of the target index, also a weighted Jevons index[10].

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[9] Jevons *j* and Jevons *k* are described in paragraphs 7.7 to 7.9 of the Eurostat-OECD *Manual on Purchasing Power Parities* (2006) as "Laspeyres type" and "Paasche type" indices respectively. However, the indices are actually types of elementary geometric indices whose properties are different from Laspeyres and Paasche indices. Different terminology is preferred here even if it is less familiar.

[10] The Eurostat-OECD *Manual on Purchasing Power Parities* describes the Jevons* index as a "Fisher type" index because it is the geometric mean of their "Laspeyres type" and "Paasche type" indices. In fact, however, the index resembles a Törnqvist index more than a Fisher, as pointed out on p. 7 of the paper by Sergey Sergeev (2003).

It can be seen from (4) that it is not actually necessary to go through the process of calculating the two separate Jevons indices shown in (2) and (3) as $P_T^{j,k}$ could be calculated directly as a weighted geometric mean of the individual product PPPs or price ratios, using the weights shown.

The sets of products that are representative will tend to vary from country to country. It follows that the weights in the various binary Jevons* indices will also vary from one index to another so that the indices cannot be transitive. The final step is therefore impose transitivity on the Jevons* indices by means of the EKS formula. The resulting PPPs are described here as EKS/Jevons* PPPs. Eurostat describes them simply as EKS* indices.

It is worth noting that the Jevons* index does not include products that are unrepresentative in both countries even if prices are reported for them by both countries. However, the prices of these products will exert some indirect influence on the final Jevons* EKS PPP between the two countries as they must be used in the calculation of Jevons* PPPs with third countries.

It is useful to illustrate the Jevons* index by means of a numerical example.

An example of a Jevons PPP*[11]

Suppose two countries report prices for 12 of the products on the common list. Country *j* has 10 representative products and the country *k* has 5 representative products. Let the product PPPs for *j*'s representative product be numbered 1 to 10, and those for *k*'s products numbered 8 to 12. Products 8, 9 and 10 are representative in both countries.

Let PPP_m denote the *logarithm* of the product PPP p_m^k / p_m^j for product *m*. By definition,

(5) $$Ln\, P_j^{j,k} = 1/10\,(PPP_1 + PPP_2 + \ldots + PPP_9 + PPP_{10})$$

(6) $$Ln\, P_k^{j,k} = 1/5\,(PPP_8 + PPP_9 + PPP_{10} + PPP_{11} + PPP_{12})$$

[11] The example is the same as that given in Chapter 11 of the *ICP Handbook*

(7) $Ln\ P_{J*}^{j,k} = ½ \{ Ln\ P_j^{j,k} + Ln\ P_k^{j,k} \}$

$= 1/100 \{ 5(PPP_1 + PPP_2 + \ldots + PPP_6 + PPP_7) + 15(PPP_8 + PPP_9 + PPP_{10}) + 10 (PPP_{11} + PPP_{12}) \}$

- Although *j* has twice as many representative products as *k*, the imbalance is compensated by the fact that each individual *PPP* for a representative product of *k* carries twice as much weight in $P_{J*}^{j,k}$ as each *PPP* for a representative product of *j*.

- Because products 8, 9 and 10 are representative in both countries, their *PPP*s enter into the calculation twice. They carry a weight of 15 % reflecting the fact that each receives a weight of 5 % in respect of country *j* plus a weight of 10 % in respect of country *k*.

The binary Jevons* PPP between this pair of countries is therefore a weighted Jevons in which

- the PPPs for products 1 to 7 carry a weight of 5 %;
- the PPPs for products 8 to 10 carry a weight of 15 %,
- the PPPs for products 11 and 12 carry a weight of 10 %.

The pattern of the weights depends on the relative numbers of representative products in the pair of countries in question, that is on the ratio $M^R jk / M^R kj$, and also on the relative size of the set of products that are representative in both countries. The pattern of weights will therefore tend vary from one pair of countries to another. The weights are easily worked out case by case.

It is worth noting that even if there are no empty cells in the price tableau, the weights will still vary from one Jevons*PPP to another. Having a complete set of prices does not ensure transitivity.

Sergey Sergeev (2003, pp. 9 to 13) has pointed out that Jevons* does not necessarily lead to 'balanced' or 'unbiased' estimates in all circumstances. It has therefore been proposed to replace Jevons* by a third type of Jevons PPP labeled *Jevons-S* by Eurostat. It is easiest to explain the index by utilizing the example given above.

Jevons-S

In general, the products within a basic heading used to calculate a Jevons* PPP between two countries A and B can be divided into three groups.

1. Products that are representative in A but not in B: (products 1 to 7 in the example)
2. Products that are representative in both countries: (products 8 to 10)
3. Products that are representative in B but not in A: (products 11 and 12).

It can be seen from equation (9) that the Jevons* PPP can be regarded as a weighted average of the product PPPs in the three groups of products. As already shown, each product PPP in group one carries a weight of 5%; each in group 2 carries a weight of 15% and each in group 3 a weight of 10%

However, the Jevons* can also be viewed as a weighted geometric average of the simple geometric averages (or *simple* Jevons PPPs) for each of the three *groups*. In the example, the group weights are 35 %, 45 % and 20% respectively.

The geometric average (or simple Jevons) for the product PPPs in group 2 should provide an unbiased estimate of the target basic heading PPP because representative products are being compared with representative products. The geometric average for group 1 is likely to have an upward bias, while that for group 3 is likely to have a downward bias. However, the Jevons* PPP gives more weight to the geometric average for group 1 than to that for group 3. The Jevons*PPP seems to have some upward bias in this example.

In order to have an unbiased estimate, equal weight should be given to the geometric average PPPs for the first and third groups. In general, however, the weights for groups 1 and 3 do not have to be equal, so that the Jevons* PPP is liable to produce biased results.

The alternative index proposed by Sergey Sergeev is designed to eliminate this kind of potential bias. There are two changes from Jevons*. First, groups 1 and 3 should be allocated equal weight. Second, the individual product PPPs in group 2 should be given double the weight of those in groups 1 and 3 on the grounds that a PPP that is estimated from the

prices of products that are representative in both countries is likely to be reliable.

In the example, counting each product *PPP* in group 2 twice would double the number of *PPP*s in group 2 from 3 to 6. The numbers in the three groups become 7, 6 and 2, so that the weights of the groups become 46.7%, 40.0% and 13.3 %. Equalizing the weights for groups 1 and 3, while keeping the total weight of the two groups unchanged means that the weights for the 3 groups become 30%, 40 % and 30% respectively in Jevons-S as compared with 35%, 45% and 20% in Jevons*. By reducing the weight for group 1 and increasing that for group 3 the presumed upward bias in Jevons* should be eliminated.

Jevons-S can also be viewed as a weighted average of the individual product PPPs, the weights being 4.3%, 13.3% and 15% in Jevons-S as compared with 5, 15% and 10% in Jevons *.

From a theoretical viewpoint, Jevons-S seems to be marginally superior to Jevons*. While the two methods are likely to produce very similar results in most cases, there may be exceptional cases in which they yield significantly different results. Both methods introduce differential weights for the individual PPPs that are by no means intuitively obvious, and which are liable to vary considerably from one pair of countries to the next depending on the relative sizes of the three groups of products. The variation in the patterns of weights means that Jevons-S indices, like the Jevons* PPPs, are not transitive so that the final step is to apply the EKS formula to obtain the Jevons-S/EKS PPPs.

There can be difficulties with both the Jevons* and Jevons-S PPPs if the absolute numbers of products in any of the three groups become very small or zero. In the above example, suppose the number of products in group 3 falls from 2 to 1; in the Jevons-S PPP the single product PPP in group 3 would be given as much weight as the average of all 7 product PPPs in group 1. If there were to be no products in group 3 the group 1 product PPPs would have to be ignored in Jevons-S, the basic heading PPP being estimated using the product PPPs for group 2 only. Discarding PPPs in this way would be a matter of concern. They would not have to discarded using Jevons*, but it can be argued that Jevons* produces biased results in these circumstances (as would a simple unweighted Jevons PPP).

Spanning trees

Another multilateral method that uses the binary approach is the Spanning Tree method developed by Robert Hill (1999) which can be applied to elementary PPPs as well as aggregate ones. The method uses graph theory. The countries in the group can be represented on a graph. A connected graph is one in which it is possible to travel from any one country to every other country in the group by means of 'edges' linking pairs of countries. A cycle exists if it is possible to travel from one country to another by two or more distinct paths. A spanning tree is then defined as a connected graph which has no cycles. In a PPP context the edges linking the countries consist of binary PPPs. If there are C countries in the group a spanning tree has C-1 edges or links.

A spanning tree can take many different forms. One example is a star with one country at the centre with every other country being linked to it by a binary PPP. Another example is a chain of binary PPPs. Alternatively, parts of spanning tree can be stars and other parts chains. In a spanning tree, each country must engage in at least one binary comparison but some countries may engage in several.

The problem is to find which out of the many possible spanning trees is optimal in some sense. For example, suppose that a measure of reliability can be attached to each binary PPP. One possibility would then be to find the spanning tree consisting of the C-1 binary PPPs that are *collectively* the most reliable. In other words, the optimal spanning tree could be viewed as the one with the strongest possible set of links. A straightforward computing algorithm, Kruskal's algorithm, exists to find such an optimal spanning tree. Other criteria besides reliability could be used to determine an optimal spanning tree.

Suppose the links in the graph are weighted Jevons PPPs. If they all used the same weights the PPPs would be transitive in which case every possible spanning tree would generate the same set of PPPs. There would be no need to search for an optimal spanning tree. This case includes the situation in which there are no missing prices and the Jevons are unweighted. For example, if the binaries are all transitive, any chain with C-1 links that connects all C countries gives the same results as any other chain. The order in which the countries are chained is immaterial: the indirect PPP between any given pair of countries is the same as the direct

PPP whatever the order. If they are represented in the form of a star, it makes no difference which country is chosen to be at the centre of the star.

On the other hand, the elementary Jevons will not be transitive in practice, in which case the determination of an optimal spanning tree provides an alternative way of achieving transitivity to the EKS. As a spanning tree requires only C-1 binaries most of the binaries would not be needed. Ignoring them after they have been estimated may not be desirable[12]. However, once the form of the optimal spanning tree is known, and also possible alternative trees based on other criteria, it may become possible to use the knowledge to conduct subsequent follow–up PPP programs on a reduced scale by concentrating on only a relatively small number of binaries.

The multilateral approach to elementary PPPs

In the multilateral approach, the entire set of elementary PPPs for a group of countries is estimated simultaneously. The prime example of the group approach at the level of the basic heading is the CPD method proposed by Robert Summers (1973). It has been used in all the various phases of he ICP. It uses the stochastic approach to price indices[13]. The model underlying the CPD is that

$$(8)\quad p_{ij} = \kappa\, \alpha_j\, \beta_i\, \nu_{ij} \qquad i = 1, 2, \ldots n: \quad j = 1, 2, \ldots c$$

$$(9)\quad \alpha_1 = \beta_1 = 1$$

where κ is a constant, α_j is a parameter for country j, βi is a parameter for product i and ν_{ij} is a random error term. As the model is concerned with price ratios, there are only n+c-1 parameters to estimate. Equation (9) is needed to determine the absolute levels of the prices. When both i =1 and j = 1, the expected value of $p_{ij} = \kappa$. In effect, product 1 in country 1 becomes

[12] When the EKS formula is used some binaries may also be ignored in practice if they are considered to be unreliable. It is possible to calculate a set of transitive EKS PPPs from an incomplete set of binary PPPs.

[13] See Chapter 16 by Erwin Diewert of the *CPI Manual* (2004) on *The Axiomatic and Stochastic Approaches to Index Number Theory*. For a discussion of the stochastic approach below the basic heading level in a time series context, see the section on *A Simple Stochastic Approach to Elementary Indices*, paras. 20.100 to 20.111 of Chapter 20, also by Erwin Diewert, in the *CPI Manual*. See also Prasada Rao (2004) and Erwin Diewert (2004)

the 'reference' product, all prices being measured relatively to its price. Country 1 therefore acts as the reference country for the PPPs. [14]

Taking natural logarithms of both sides of (10) and (11) we have:

(10) $\ln p_{ij} = \ln \kappa + \ln \alpha_j + \ln \beta_i + \varepsilon_{ij}$

(11) $\ln \alpha_1 = \ln \beta_1 = 0$

Equation (10) can be rewritten as follows using two sets of dummy variables X_{ij} and Y_{ij} that take the values of either unity or zero:

(12) $\ln p_{ij} = \ln \kappa + \ln \alpha_2 x_{i2} + \ln \alpha_3 x_{i3} + \ldots \ln \alpha_c x_{ic} + \ln \beta_2 y_{2j} + \ln \beta_3 y_{3j} + \ln \beta_c y_{2c} + \varepsilon_{ij}$

The parameters of equation (12) can then be estimated by least squares or multiple regression[15]. The number of parameters to be estimated, including the constant κ, is equal to n + c – 1. The number of simultaneous equations to be solved therefore also equals n + c – 1.

The least squares estimate of $\ln \kappa$ can be interpreted as the log of the expected price of the reference product: *i.e.*, of $\ln p_{11}$. Denoting the least squares estimates of the parameters $\ln \alpha_j$ and $\ln \beta_i$ by $\ln a_j$ and $\ln b_i$, $\ln a_j$ measures the expected logarithm of the price ratio p_{ij} / p_{i1}, both prices being measured in their own currencies, this ratio being assumed to be a constant, α_j, for all products. Similarly, $\ln b_i$ measures the expected logarithm of the ratio p_{ij} / p_{1j} this ratio being assumed to be constant for all countries. a_j is the estimated PPP of country j with reference to country 1. The reference country can be changed from country *1* to any other country, such as country j simply by dividing all the estimated PPPs by a_j. The estimated PPPs are

[14] Most presentations of the CPD model have no constant term, in which case normalization can be achieved simply by letting $\alpha_1 = 1$ without requiring $\beta_1 = 1$. However, when a third type of variable, representativity, is introduced into the model, an additional constraint has to be imposed anyway, in which case the approach adopted here is more convenient and symmetrical. For a simple exposition of regression with dummy variables, see David Huang (1970). He remarks, p. 166 that: "The rule of thumb is that , whenever there are two or more dummy systems, drop one variable from each system (preserving the constant, say) for OLS estimation." When representativity is introduced into the CPD, there are three or more dummy systems, depending on whether interaction terms are included.

[15] The use of regressions with dummy variables originated in the analysis of experimental data where the variables are often non-numerical or qualitative attributes such as location or plant variety. See, for example, Oskar Kempthorne (1952) chapters 5 and 7.

transitive. One advantage gained by using the CPD method is that sampling errors can be estimated for all the coefficients, including the PPPs.

One important question is how the estimates obtained from a simple (unweighted) CPD relate to the various Jevons indices used in the binary approach. Consider first the special case in which every country reports an average national price for every product on the list. In other words, the price tableau is complete. From a CPD perspective, the data are orthogonal. The estimated PPP for each pair of countries is independent of that for every other pair of countries. Bearing in mind that the dependent variables in the CPD are the logarithms of the prices, the estimated log PPP for any pair of countries is equal to the difference between their arithmetic average log prices. The PPP therefore equals the ratio of the geometric average prices for the two countries compared. This is, of course, the simple Jevons index.

Given that the price tableau is complete and each Jevons covers exactly the same set of products, the Jevons indices are transitive. The CPD estimates coincide with the Jevons indices. In the conclusion to his original paper on the CPD Robert Summers (1973) remarked as follows. "In the limiting case in which there are no missing observations in the price tableau, it can be shown that the regression procedure amounts to the computation of a set of geometric means. Since this is just what one normally would compute to estimate relative price levels if he did not introduce stochastic considerations into his framework of analysis, it is reassuring to see that the regression method is consistent with ordinary practice."

The coincidence between the simple CPD estimates and simple Jevons indices when the price tableau is complete is a well known result in the design and analysis of experiments[16]. Two points should be noted. The first is that the coincidence is not between CPD and EKS as the EKS formula is not involved. Indeed, whenever the EKS formula actually has to be used because the Jevons indices are not transitive, the CPD and EKS results do not coincide. The second point is that a simple CPD and weighted Jevons indices of the kind actually used by Eurostat do not coincide even when there are no missing prices. The choice of method can make a big difference even when the price tableau is complete.

[16] See, for example, Prasada Rao (2004) for a formal proof in a PPP context.

It is useful to clarify the role of missing prices. If some prices are missing the simple Jevons PPPs will not be transitive because the coverage of the various Jevons PPPs will vary from binary to binary. In this case, the EKS formula is needed to impose transitivity. From a CPD perspective the data are not orthogonal and the PPPs cannot be estimated independently of each other. They have to be estimated simultaneously by solving the normal regression equations.

Once the country and product parameters have been estimated they can used to estimate the missing prices and fill the holes in the price tableau, if desired. Of course, it is not necessary to fill the holes in order to estimate the parameters. Quite the reverse, the holes cannot be filled without already knowing the estimated product parameters and PPPs.

It is useful, however, to consider the interpretation that can be placed on the PPPs when all the holes in the price tableau are filled. If a new CPD were to be run on the resulting complete tableau of actual and expected prices the estimated country and product parameters would remain identical with the original estimates. Adding observations to a regression that are predicted by the regression cannot change the regression.

Suppose that simple Jevons PPPs are calculated for the complete tableau of actual and expected prices. They must be transitive because the coverage of each Jevons is the same. Thus, adding the expected to the actual prices can be viewed as a method of imposing transitivity on a set of binary Jevons PPPs. It is an alternative to the EKS formula. Conceptually, it is more transparent than EKS.

Finally, suppose a CPD were to be calculated for all the products in the basic heading as a whole. The ratios of the expected prices in one country to those in another are constant for every product (and equal to the CPD PPP). Given the expected prices in one country, the expected prices in other countries are obtained simply by scaling them all up or down by a constant, the PPP. The expected prices can therefore be viewed as tracing out the general price level in each country. The actual prices of representative products can be expected to be below the general price level as so defined and the prices of unrepresentative products to be above. If the products selected for pricing are all representative in one country and unrepresentative in another, it is clear that a PPP calculated by comparing

their prices will not reflect the relative price levels in the two countries whatever method of calculation is used.

Weighted Jevons and weighted CPDs

In practice, there are no data on quantities or expenditures within a basic heading which can be used as weights. If there were, it would make no sense from an economic point of view to calculate elementary price indices that ignore the expenditure information. PPPs for basic headings could be calculated in the same way as PPPs for higher level aggregates. For example, when the binary approach is used, the appropriate index to calculate for each basic heading would be superlative index[17] such Fisher or Törnqvist. The Törnqvist index can also be viewed as a weighted Jevons index in which the weights are simple arithmetic averages of the expenditure shares in the two countries compared. As expenditure shares vary from country to country in practice, the weights would vary from one binary index to another so that the weighted Jevons indices would not be transitive, even with complete information on prices and expenditures. To achieve transitivity they would need to be adjusted, presumably by using the EKS formula

Using the multilateral approach if, hypothetically, there were to be expenditure data available within a basic heading, a weighted CPD in which the weights were the expenditure shares within each country would provide better estimates of the PPPs than a simple CPD[18]. The estimated PPPs obtained from a weighted CPD are transitive. It follows that they would differ from the corresponding binary weighted Törnqvist/Jevons PPPs, even with a complete set of prices.

The special case in which the simple Jevons and simple CPD coincide when there are no missing prices has attracted attention in the literature. The appropriate generalization of this case is the situation in which different products carry different weights but the weights are the same from country to country. In the context of a set of multilateral comparisons, simple averages of the expenditure shares in each of the countries in the group provide an appropriate set of weights. As the weights would be the same for each binary comparison, the weighted Jevons indices would be transitive.

[17] The concept of a superlative index was introduced in Diewert (1976).

[18] Erwin Diewert (2004) has argued that "transaction share weighted CPD purchasing power parities … provide a reasonable target set of parities that could be used in international comparisons."

From the CPD perspective, when the product weights are the same for each country, the data are orthogonal so that each country PPP can be estimated independently of those for the others. For this reason, the weighted CPD PPPs coincide with the weighted Jevons. The binary and the multilateral approaches converge.

A binary Jevons with average group shares can be viewed as a generalization of the Törnqvist index from a pair of countries to a group of countries[19]. The average expenditure share for the individual pair of countries in question is replaced by a simple average of the expenditures shares for the group as a whole. Taking a simple average assigns equal importance to each country in the group irrespectively of its size. Although the binary approach is retained, the characteristics of the group as a whole are imposed on each binary instead of the characteristics of the pair of countries on their own. This is a conceptually attractive way of achieving transitivity in a set of multilateral comparisons but it must be remembered that, in practice, there is no real possibility of being able to use group expenditure shares as weights within a basic heading.

The Extended CPD Method, or CPRD Method

Suppose that for each country the products that are representative are flagged by markers such as asterisks. This is the procedure followed in the Eurostat-OECD program. If a product is representative in a particular country it may be expected to have a relatively low price in that country. Representativity may be expected to exert an influence on the price of a product in the same kind of way that the type of product or the country in which it is sold influences its price. It ought to be included, therefore, as an additional explanatory variable in the CPD.

A given product will be representative in some countries and unrepresentative in others. Thus, the price of a given product may be expected to be relatively low in one country and relatively high in another country depending on whether or not it is representative. In the model underlying the CPD method the product parameters are the same for all countries which implies that if representativity is not included the pattern of expected relative prices is same in all countries. The inclusion of representativity enables this unrealistic assumption to be relaxed somewhat.

[19] See Hill (1997).

The expected price of a product relatively to another product in the same country is not constant when representativity is included as an additional explanatory variable[20].

Let γ_k denote the degree of representativity. Only two degrees are distinguished here: namely, representative denoted by γ_1 and unrepresentative denoted by γ_2. However, in principle, more than two different degrees of representativity could be recognized: for example, very representative, moderately representative and unrepresentative.

The basic model is now written as:

(20) $p_{ij} = \kappa\ \alpha_i\ \beta_j\ \gamma_k\ \nu_{ijk} \qquad i = 1, 2, \ldots n: \quad j = 1, 2, \ldots m$

(21) $\alpha_1 = \beta_1 = \gamma_1 = 1 \qquad k = 1, 2.$

Taking natural logarithms of both sides of (20) and (21) we have:

(22) $ln\, p_{ij} = ln\, \kappa + ln \alpha_j + ln\, \beta i + ln\, \gamma_k + \varepsilon_{ij}$

(23) $ln\, \alpha_1 = ln\, \beta_1 = ln\, \gamma_1 = 0$

The regression equation now requires three sets of dummy variables X_{ijk}, Y_{ijk} and Z_{ijk} . It becomes:

(24) $ln\, p_{ij} = ln\, \kappa + ln\, \alpha_2 xi_{i2} + ln\, \alpha_3 x_{i3} + \ldots ln\, \alpha_c x_{ic} + ln\, \beta_2 y_{2j} + ln\, \beta_3 y_{3j} + ln\, \beta_c y_{2c} + ln\, \gamma_2 z_{ij2} + \varepsilon_{ij}$

The expected price depends on the combined effect of three factors: the country, the product and its representativity. Given that the coefficient of a representative product is fixed at unity, the coefficient of an unrepresentative product may be expected to be greater than unity.

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[20] The extension of the CPD model to include representativity was first proposed by James and Margeret Cuthbert (1988, p. 55) who argued as follows: “The standard CPD technique makes no allowance for characteristic / non-characteristic bias. It is not difficult however to see how the basic CPD model could be extended to allow for the possibility of a differential price between characteristic and non-characteristic products, if information on the characteristic / non-characteristic classification of items is available.” ‘Characteristicity’ means roughly the same as ‘representativity’ here.

Consider two products. The traditional CPD model assumes that the ratio of their prices is the same in every country. The extended CPD model allows the ratio of their prices to vary between two countries depending on whether the products are representative (or unrepresentative) in both countries, or whether one is representative in one country and the other is representative in the other country. The inclusion of the dummy for representativity introduces some flexibility into the CPD model by permitting a limited amount of variation in the pattern of expected relative prices between countries.

The addition of the new variable, representativity, does not simply add another parameter to be estimated. It adds another dimension to the analysis. As there are three types of explanatory variables in the regression -- country, product and representativity -- the extended regression is described as the CPRD method.

If the product list is unbalanced, there is a risk that without the representativity term the estimated CPD PPPs will be biased for the same reason that the simple Jevons indices are liable to be biased. As representative products may be expected to have relatively low prices, if most of the products on the common product list are representative for a particular country its expected PPP using the CPD will be lower than the CPD PPP for the universe of products in the basic heading. The CPD estimate will have a downward bias[21]. With a random sample of products, there should be no systematic bias but purposive selection could well lead to situation in which most of the products are unrepresentative in the majority of countries: for example, if the common product were to be put together by merging lists that are representative of two or three countries only.

The need to avoid a situation in which the product list is dominated by products that are representative of a minority of countries has always been recognized in international comparisons. However, it is not easy to achieve a balanced list in practice. The advantage of flagging products that are representative is that even when the product is not balanced it is still possible

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[21] James and Margaret Cuthbert (1988), use actual data from the OECD PPP program to test for the significance of representativity. They conclude, p. 79:

"(a) There is evidence of a significant positive differential effect on the prices of non-characteristic items.

(b) The magnitude is such that, for some basic headings, it could potentially have a very serious effect in distorting inter-country PPP comparisons."

to use the information to correct for the potential bias in the process of estimating the elementary PPPs. The Jevons* and Jevons-S indices exploit this information when the binary approach is used while the CPRD method does so when the group approach is adopted. They are likely to produce similar results in most cases. On the other hand, the simple Jevons and the Jevons* indices are liable to produce different results, as also are the CPD and CPRD.

The distinction between representative and unrepresentative products has been successfully exploited in the Eurostat-OECD program for many years. However, countries with no previous experience of using it may have difficulties implementing it. Some countries may not have enough information to distinguish representative from unrepresentative products in a satisfactory way. Different countries may also interpret the distinction in different ways. If the information collected about representativity turns out to be inconsistent, incomplete or unreliable it may have to be discarded. In this case, it still possible to fall back on the CPD.

Conclusions

The selection of the products to be included on the common list used for purposes of price collection can be critical for international comparisons. The results seem to be more sensitive to the selection of list of products used for price collection in temporal price indices. However representativity is defined in temporal indices, a representative product can be expected to remain representative over time. When the same product is priced repeatedly in successive periods the price of a representative product in one period is not compared with the price of an unrepresentative product in another period. However, in international comparisons the same product can be representative in one country but unrepresentative in another. It is necessary to try to allow for this when calculating elementary PPPS.

The choice of elementary index is contingent on whether or not reliable information about representativity can be collected and utilized. Eurostat and OECD have been successful in collecting such information in their joint PPP program. The evidence and experience accumulated over several decades strongly suggest that methods that are able to take this information into account in the process of estimating the elementary PPPs are superior to those that do not. The former are able to correct for potential biases resulting from the use of unbalanced product lists. Jevons* and

Jevons-S indices are superior in theory and in practice to simple Jevons indices while the CPRD is theoretically superior to the CPD[22].

The differences between methods that make use of representativity (Jevons*, Jevons-S and the CPRD) and those that do not (simple Jevons and simple CPD) are not reduced as the number of prices collected for the products on the list increases. The fact that simple Jevons and simple CPD coincide when there are no missing prices provides useful insights into the relationships between binary and multilateral methods but does not imply that there is any tendency for the results obtained by other methods also to converge on the simple Jevons and simple CPD as the number of missing prices decreases. The choice of methodology does matter whether there is a complete set of prices or not.

Annex

Representativity, relative quantities and balanced product lists

The purpose of this Annex is to explain how information about representativity could be used to construct balanced product lists in the hypothetical situation in which quantity data are available for the universe of products within a basic heading. The objective is to try to establish what kind of product list can serve as a target in the real world of incomplete and fragmentary data.

Representative products are defined here as products that are purchased in relatively large quantities in a country. When there are only two countries it is easy to see which products are purchased in relatively large quantities in one country compared with the other. When there are many countries, however, each with a different pattern of relative quantities, situation is more complex.

[22] In a paper by Yuri Dikhanov on "Assessing the Efficiency of Elementary Indices with Monte Carlo Simulations" presented to the ICP Technical Advisory Group in September 2004 simulated data were used to evaluate the results obtained for various methods described in this paper. The conclusion reached was that "the CPRD was found to be superior to the other CPD and EKS style indices." It was also concluded that: "In general, CPRD index is found to be the most robust, especially with sparse price and representativity matrices."

An objective and efficient way in which to identify products that are purchased in relatively large quantities in a particular country is to compute a CPD in which logarithms of the quantities are regressed on the country and product dummies. The model is exactly the same as for a conventional CPD as given in equations (8) and (9) in the text except that log quantities replace log prices as the dependent variable. The estimated country and product parameters are then used to predict the expected log quantities for each product in each country. Finally, the residuals, *i.e*. the differences between the actual and expected log quantities, are calculated. After taking anti-logs, the residuals take the form of ratios of the actual to the expected quantities. Products with residual ratios in excess of unity are consumed in *relatively* large quantities in that country compared with other countries. They are representative products as defined here. A simple numerical example can be used for illustration.

Suppose there are 3 countries A, B and C, and the universe of products in the basic heading consists of 6 products. An illustrative set of quantities are shown in the first 3 columns of Table A.1. The quantity units can be quite different from product to product. The expected quantities given by the CPD are shown in the middle section of the table and the residuals, as measured by the ratios of the actual to the expected quantities, are shown in the third section. The entries in the column and row labeled GA are geometric averages.

Table A.1

Actual, expected and residual quantities using the CPD method

Product i	Country				Country				Country			
	A	B	C		A	B	C		A	B	C	
	Actual quantities				Expected quantities				Residual ratios			GA
1	80	200	16		53.04	215.44	22.26		1.51	0.93	0.72	1.00
2	25	100	18		29.71	121.45	12.47		0.84	0.82	1.44	1.00
3	20	72	8		18.87	77.11	7.92		1.06	0.93	1.01	1.00
4	15	90	9.5		19.55	79.92	8.21		0.77	1.13	1.16	1.00
5	50	225	12.5		43.44	177.56	18.23		1.15	1.27	0.68	1.00
6	70	336	42		83.19	340.05	34.92		0.84	0.99	1.20	1.00
GA	35.8	146.2	15.0		35.79	146.2	15.02		1.00	1.00	1.00	1.00

As just noted, a residual ratio above unity indicates that the actual quantity is greater than expected so that the quantity is relatively large compared with other countries. The country's share of the total consumption of the product in the group of countries is therefore above average. For example, the ratio of actual to expected for product 2 in country C is 1.44. Country C's share of product 2 is equal to 12.6% whereas the average share for country C is 7.6%.

When the CPD is used the geometric average of the residual ratios for *all* the products in a country must be unity (the arithmetic average of the log residuals is zero). When selecting a sample list of products for pricing the objective should be to choose a list that reflects the situation in the basic heading as a whole by trying to ensure that that the geometric average of the residual ratios of the selected products is as close to unity as possible for each country.

Suppose, for example, that products 1, 3 and 5 were to be selected to make up the list. The geometric averages of the residual ratios for these three products for countries A, B and C are 1.23, 1.03 and 0.79 respectively. This particular list favours the representative products of country A and discriminates against the representative products of country C. It can be seen from Table A.1 that the list consists of the three products with highest residual ratios in A and excludes the three products with the highest residual ratios in C. The list is very unbalanced.

Assuming a negative correlation between relative prices and relative quantities, all the products on the list are likely to be relatively cheap in A and relatively dear in C. If a CPD were to be run on the prices of the products on the list, the estimated PPPs would therefore be likely to be biased compared with the CPD PPPs for the basic heading as a whole.

A more balanced list can be obtained by simply selecting the most representative product for each of the three countries: *i.e.*, the product with the highest residual ratio. This is a rational and objective procedure. The list therefore consists of products 1, 2 and 5. The geometric averages of the residual ratios for these three products for countries A, B and C are 1.13, 0.99 and 0.89. This list is by no means perfectly balanced but it is an improvement over products 1, 3 and 5.

The optimal strategy for selecting a product list should be to try to ensure that it contains some products that are representative of every country. This is the kind of strategy followed in the Eurostat-OECD program. Ideally, countries could be asked to nominate the two or three products with the highest residual quantity ratios for inclusion on the common list, but of course the data needed to run quantity CPDs will not be available in practice. However, country experts can be asked to identify two or three products that they believe to be purchased in relatively large quantities in their countries.

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