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Sub-national projections

Bayesian multiregional population forecasting: England

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Summary

In this paper, we extend the well-known multiregional population projection model developed by Andrei Rogers and colleagues to be fully probabilistic. Multiregional models provide a general and flexible platform for modelling and analysing population change over time. They allow combining all the main components of population change by age with various transitions that population groups may experience throughout their life course. What distinguishes these models from ordinary projections is that they include transition matrices of interregional migration by age. This information is an important component of subnational population change yet models for forecasting the patterns for use in population projections are largely non-existent. To provide measures of uncertainty, we develop a Bayesian hierarchical model to forecast age-specific interregional migration, and then include this information with probabilistic forecasts of regional births, deaths, immigration and emigration. The results demonstrate the differences that arise from different specifications and the promise of the general approach. The data used in computations relate to five regions of England and were obtained from the Office for National Statistics.

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I. Introduction

1. This research substantially extends earlier efforts for multiregional estimation and forecasting population, namely Gullickson and Moen (2001), Sweeney and Konty (2002), Raymer et al. (2006), Wilson and Bell (2007), and Bryant and Graham (2013) by the inclusion of probabilistic information within the multiregional population projection model. Despite over forty years of development and evidence for reduced biased projections, multiregional projection models are still under-utilised by national statistical offices and the production of probabilistic forecasts is largely non-existent. Instead, national statistical offices tend to rely on very simple deterministic assumptions regarding net migration or gross flows of in-migration and out-migration that are often held constant for the foreseeable future. These models do not take into account the linkages between origins and destinations and often have to be adjusted to ensure zero net migration and the same totals for in-migration and out-migration. This is concerning, particularly since both internal migration and international migration are increasingly becoming the dominant sources of population growth.

2. The forecasting model developed for this paper embeds forecasts of age-specific births (by age of mother), deaths, internal migration and international migration for subnational populations in England. The forecasting approach utilises Bayesian techniques and extends the cohort component forecasting model developed by Wiśniowski et al. (2015) to include subnational regions.

II. Background

3. Multistate population models provide a general and flexible platform for modelling and analysing subnational population changes over time. They allow the combination of all the main components of population change by age and sex with various transitions that population group may experience throughout their life course. The transitions may include those between states of residences, employment, marriage or health. However, despite the many theoretical and analytical advantages, these models have been relatively unexplored because of the large amount of input data required (which are not always available over time, particularly between censuses) and complex calculations required to perform the estimations and analyses. In fact, most national statistical agencies choose to rely on relatively simple accounting models to produce estimates of population by age, sex and region, which do not include uncertainty or population covariates, such as state of employment, health or marriage, that are of particular interest to policy makers and local planning agencies. In this paper, we hope to change this reliance on simple models by (1)

creating frameworks for dealing with the data and (2) for estimation of the components of change and for the dynamic modelling of subnational populations.

4. More specifically, we develop a probabilistic population modelling framework that brings together recent advances in multi-state (or multiregional) life tables and projections (Rogers 1995; Schoen 2006) and probabilistic forecasting (Wiśniowski et al. 2015). With the exception of Rees and Turton (1998) and Gullickson (2001), very little has been carried out in this area (see also discussions in Wilson and Rees 2005 and Wilson and Bell 2007). With the combination of empirical data, statistical modelling techniques and expert judgements, this research advances the building and application of dynamic population models.

5. There have been many examples of multiregional or multistate population models applied to study population change from which we can draw experiences from (e.g., Espenshade 1983; Kalogirou and Murphy 2006; Land, Guralnik and Blazer 1994; Rogers, Little and Raymer 1999; Rogers and Raymer 1999, 2001; Rogers, Rogers and Belanger 1989; Rogers and Willekens 1986; Willekens 1980; Willekens et al. 1982), including several analyses focused on the United Kingdom (Murphy, Glaser and Grundy 1997; Rees 1986; Rees and Willekens 1989).

6. Multi-state or multiregional population models may be considered extensions of the life table and the cohort-component projection model. These models allow populations to move between various states in their life course, providing the analyst with a means to better model and understand the mechanisms behind population change. Early developments of this modelling framework can be found in Rogers (1975), Land and Rogers (1982) and Schoen (1988). Note, "multi-state" is a more general form of population modelling, whereas "multiregional" specifically refers to the inclusion of origin-destination specific migration rates or proportions associated with the correct populations "at risk" in the models (Rogers 1975, 1995).

7. Most standard cohort-component population projections models ignore migration transitions and instead rely on net migration or (slightly better) out-migration and in-migration rates to account for the change due to migration. The problem with net migration and in-migration rates is that they include the incorrect population at risk of migrating in the denominator, which can seriously bias the results (Rogers 1990), particularly over the long term. The models developed in this paper extend current multi-state models by (1) including all of the main demographic components of change (i.e., fertility, mortality, interregional migration and international migration) and their (expected) variations over time, and (2) by including uncertainty using statistical modelling approaches for rates and categorical data.

8. The modelling of the demographic components should make use of the best available data when possible and estimate the missing or inadequate data by relying on model schedules (Coale and McNeil 1972; Congdon 2008; Heligman and Pollard 1980; Rogers and Castro 1981; Rogers and Little 1994) and by using statistical methods for combining data (Raymer, Abel and Smith 2007; Smith, Raymer and Giulietti 2010) or expert judgements (Wiśniowski et al. 2013).

III. Methods

Forecasting population components

9. Models for forecasting population components utilized in this research are based on the combination of multiplicative component or log-linear models (e.g. Raymer et al. 2006) and bilinear models (e.g. Lee and Carter 1992). They are also a direct extension of methods implemented in Wiśniowski et al. (2015) to forecast population of the UK by age and sex.

10. The general approach to building such a model includes decomposing a high dimensional array of, say, region by age by sex by time data into lower dimensional arrays, such as region by age, region by sex or region by time. This is particularly important when analysing detailed origin-destination migration flow data, which has *n* by *n* dimensions. Lower dimensional arrays are more manageable and can be forecast by integrating a time component as in the Lee and Carter (1992) approach for forecasting age-specific mortality, subject to identifiability constraints placed on model parameters. This general approach allows forecasting both rates and counts, depending on the input required for the multiregional projection model.

11. For our demographic component forecast models, we assume that counts of region-, age-, sex- and time-specific events Y_{rast} (for interregional migration subscript r is replaced with od) are Poisson distributed:

$$Y_{rast} \sim \text{Poisson}(\mu_{rast} E_{rast}), \tag{1}$$

where μ denotes mean (expressed as rate) of the process and E is exposure, i.e., the population at risk of experiencing demographic events. Further, the logarithm of μ is assumed to be normally distributed:

$$\log \mu_{rast} \sim \text{Normal}(M, \tau), \tag{2}$$

where mean M denotes the log-linear model and τ is precision (inverse variance). The log-linear model is specific to the given population component and contains parameters that are relevant for capturing age-sex-region-specific patterns of that component. It also contains one or more time effects κ_t. These effects are then forecast using time series model. In our illustration with data for England, we rely on univariate (stationary) autoregressive model, i.e. AR(1), with the following specification:

$$\kappa_t \sim \text{Normal}\ (\varphi_1 + \varphi_2\, \kappa_{t-1}, \tau_\kappa). \tag{3}$$

For a multivariate example of this approach, refer to Wiśniowski et al. (2015).

12. To estimate model parameters, we utilise Bayesian inference. In this fully probabilistic approach, all unknown parameters are treated as random variables that have probability distributions. This allows forecast uncertainty to be included from both the data and the model parameters. Additionally, the Bayesian approach enables us to include subjective information that can be elicited from the experts. Computations for our models presented in this paper were carried out in R 3.2.2 using *rjags* package for Bayesian inference (Plummer 2003).

a) Interregional migration

13. To forecast patterns of interregional (origin-destination) migration, Eqs. (1) and (2) become $Y_{odast} \sim \text{Poisson}(\mu_{odast} E_{oast})$ and $\log \mu_{odast} \sim \text{Normal}(M, \tau)$, respectively. In this case, parameters μ can be interpreted as out-migration rates for all origin-destination specific interregional flows. For M we consider the following specification:

$$M = OA + DA + OS + DS + OD_1 + OD_2\, \kappa_t, \tag{4}$$

where OA, DA, OS, DS denote model parameters (two-way interactions) that are origin-age, destination-age, origin-sex and destination-sex specific, OD_1 is a parameter capturing average origin and destination 'profile' across time, age and both sexes, and OD_2 is a parameter reflecting change of origin-destination profile across time in response to time effect κ_t. Further, we assume a hierarchical structure for two-way interaction terms:

$$OA \sim \text{Normal}(A, \tau_1), \qquad DA \sim \text{Normal}(A, \tau_2),$$
$$OS \sim \text{Normal}(S, \tau_3), \qquad DS \sim \text{Normal}(S, \tau_4), \tag{5}$$

which allow borrowing of strength across interactions and improve performance of the estimation algorithm. To forecast κ_t we use Eq. (3).

b) Mortality

14. For mortality, we assume the following specification of model M:

$$M = RA + RS + AS_1 + AS_2\, \kappa_t, \tag{6}$$

where RA and RS denote region-age and region-sex specific interactions, and AS_1 and AS_2 capture the average age profiles of mortality for males and females, and changes in their profiles over time, respectively. This model implies that mortality is changing at the same pace in all regions. Two-way interactions RA and RS are assumed to have similar hierarchical structure as in Eq. (5), i.e., $RA \sim \text{Normal}(R, \tau_5)$ and $RS \sim \text{Normal}(R, \tau_6)$.To forecast κ_t we use Eq. (3).

c) Fertility

For fertility, we simplify model M to:

$$M = RA + A_1 + A_2\, \kappa_t, \quad (7)$$

where RA and RS denote region-age and region-sex specific interactions, and A_1 and A_2 capture the average age profile of fertility and changes over time in this profile, respectively. Again, we use Eq. (3) to forecast time effects.

d) International immigration and emigration

15. The model for forecasting international flows of migrants to and from England, we veer from our specification to better reflect the nature of the data, which are derived from the International Passenger Survey (this issue is further discussed in Section III). In the case of immigration, we model counts of inflows, whereas for emigration, we model the rates. We assume that the natural logarithm of either counts or rates follows normal distribution:

$$\log\,(Y_{rast} / E_{rast}) \sim \text{Normal}\,(RSA + \kappa_t, \tau), \quad (8)$$

where RSA is a three-way interaction term, subscript r denotes region of origin for emigration and region of destination for immigration. One person was added throughout the raw data on counts before constructing rates to ensure the logarithm is well defined and $E_{rast}=1$ for all r, a, s and t in the case of immigration. This model virtually captures distribution of the total flows in a given year across all regions, age groups and both sexes.

Priors

16. For all model parameters we assume approximately non-informative or weakly informative prior distributions. In particular, all means of the two-way interactions (e.g. Eq. (5)) follow normal distribution with mean zero and precision 10^{-4} (standard deviation equal to 100). All priors for precision terms are truncated normal distributions:

$$\tau \sim \text{Normal}\,(0, 10^{-4})\ \text{I}[\tau > 0]. \quad (9)$$

17. For the parameters of the time series models, we assume

$$\varphi_1 \sim \text{Normal}\,(0, 10^{-4}), \qquad \varphi_2 \sim \text{Normal}\,(0, 10^{-4})\ \mathbf{1}[0 \le \varphi_2 \le 1], \quad (10)$$

except for the fertility model, where we use $\varphi_2 \sim \text{Normal}\,(0, 9)\ \mathbf{1}[-1 \le \varphi_2 \le 1]$. These specifications imply stationarity of the time series effect. In the case of fertility, the prior is more centred on zero and allows negative values to ensure that the resulting Total Fertility Rate does not increase unrealistically over the forecast period.

Multiregional cohort component projection model

18. In our application, we rely on the classical specification of the multiregional projection model as described by Rogers (1975) but with an open population (Preston et al. 2001). As mentioned previously, we forecast emigration rates where we have a clear population ‘at

risk' of migration, and immigration counts where we do not. The baseline year for the forecasts is 2007 and we forecast six five-year periods until the year 2037.

19. Projections for the population aged $x+5$ in year $(t+5)$, denoted by $K_{x+5}(t+5)$, are made using the following equations:

$$K_0(t+5) = \frac{5}{2} S_{-5} \sum_{x=\alpha}^{\beta} \left(F_x + F_{x+5} S_x\right)\left(K_x(t) + \frac{1}{2} G_x(t)\right) + \frac{1}{2} G_0(t), \qquad (11)$$

$$K_{x+5}(t+5) = S_x \left(K_x(t) + \frac{1}{2} G_x(t)\right) + \frac{1}{2} G_{x+1}(t), \qquad (12)$$

where F denotes fertility rates for all R regions, α and β denote the first and last reproductive age groups, respectively (i.e. 15-19 and 45-49) and G denotes a vector of international immigrants during entire 5-year period. Births K_0 are then split between males and females using the male birth to female birth ratio of 1.05 to 1.00. The matrix S represents survivorship proportions that are computed using approximations derived by Rogers & Ledent (1976) and Ledent (1978: 48-49):

$$S_{-5} = \left(I + \frac{5}{2} M_0\right)^{-1} \qquad (13)$$

$$S_x = \left(I + \frac{5}{2} M_{x+5}\right)^{-1} \left(I - \frac{5}{2} M_x\right) \text{ for } x = 0, 5, 10, \ldots, 5(z\text{-}2), \qquad (14)$$

$$S_{5(z-1)} = \left(5 M_{5z}\right)^{-1} \left(I + \frac{5}{2} M_{5(z-1)}\right), \qquad (15)$$

where I denotes identity matrix, M_x denote a matrix of mortality ($m_x^{r\delta}$), international emigration (m_x^{rE}) and internal out-migration (m_x^{od}) rates as estimated by models described in the first part of this section. The M_x matrix is specified as

$$M_x = \begin{bmatrix} \left(m_x^{1\delta} + m_x^{1E} + \sum_{j \neq i} m_x^{1j}\right) & -m_x^{21} & \cdots & -m_x^{R1} \\ -m_x^{12} & \left(m_x^{2\delta} + m_x^{2E} + \sum_{j \neq i} m_x^{2j}\right) & \cdots & -m_x^{R2} \\ \vdots & \vdots & & \vdots \\ -m_x^{1R} & -m_x^{2R} & \cdots & \left(m_x^{R\delta} + m_x^{RE} + \sum_{j \neq i} m_x^{Rj}\right) \end{bmatrix} . \qquad (16)$$

20. The specification of the projection model as above has several limitations. First, survivorship and birth rates are specific to the region where deaths, emigration or births occur, not to the group of population that experiences them. Second, the fact that we work with 16 five-year age groups including the youngest being 0-4 and the oldest 75+, implies that mortality patterns of children aged less than one year and of the older population are aggregated. This may potentially lead to errors in the projections, especially in the long term. Third, simple linear approximations are assumed for proportions of person-years lived

in a period (Rogers 1975: 66), which again is likely to influence the youngest and the oldest age groups. To precisely estimate survivorship of the oldest population, more advanced techniques, such as described in Preston et al. (2001: 44) and Bijak et al. (2015) may be applied. Fourth, due to limitations of the data at hand we neglect migration flows between England and Wales, Scotland and Northern Ireland.

a) Data and results

Data

21. The data on vital events and international migration were obtained from the Office for National Statistics. We use data on births for 2000-2007, deaths for 2003-2007, immigration and emigration flows for 1991-2007 and mid-year population totals for 1991-2007 (adjusted to 2011 Census). Data on interregional migration were obtained from the census and administrative data in form of estimates by Raymer et al. (2011).

22. To illustrate the methodological approach, the official nine regions (former Government Offices for Regions) of England are aggregated into five regions: (1) North East with Yorkshire and Humber, (2) North West, (3) Midlands (West Midlands and East Midlands), (4) East and London, and (5) South (Southwest and Southeast).

23. Since the data on international migration come from the International Passenger Survey, the detailed characteristics are subject to high variation. Moreover, the estimated immigration and emigration flows by region, age and sex very often do not sum up to the total reported flow (ONS, 2016). Hence, we assume the average region, age and sex specific profile for all years 1991-2007.

Results

24. Figures 1-5 present the data and the forecasts of population components, whereas Figures 6-9 and Table 1 show the population forecasts from the multiregional projection model. Since all of the models assumed AR(1) process for the developments of the time effect, the forecasts and their uncertainty converge and remain constant during the entire forecasting period.

25. In Figure 1, the forecasts of interregional out-migration rates reflect the patterns that have been observed since the late 1990's and are specific to a given origin and destination. The largest forecasted migration rates are from the East and London to the South, whereas the smallest from the East and London to the North West.

26. In Figure 2, we observe the forecasts of Total Fertility Rates (TFR) for five regions. While the East and London are forecast to have the largest TFR by 2037, the differences between regions are relatively small. The relatively ‘tight around zero’ prior distribution assumed for the autoregressive parameter ensures that the TFR does not increase dramatically and converges to a stable value of around 2.3.

27. Life Expectancy at birth (LE), however, differs considerably between regions. The highest is forecast for the South as well as the East and London (around 83.5-84 for females and 80-80.5 for males), whereas the lowest is forecast for the North West (81.5 for females and 78 for males). Here, a more sophisticated model with separate region-specific interactions for time effects would be required if a forecaster would like, for example, ensure all life expectancies converge to a common value for males and females, or to increase according to a pre-specified function.

28. The largest international immigration counts (Figure 4) are forecast for the flows into the East and London region. The posterior distribution of the autoregressive parameter tends strongly towards a random walk model (see Bijak 2010; Wiśniowski et al. 2015), which results in a slightly increasing trend of immigration overall. A random walk model, however, would lead to ever increasing flows and their uncertainty, which would dominate the population forecasts in the long term. We believe that a stationary autoregressive model provides sufficient description of uncertainty in this case.

29. Virtually the same model assumed for emigration rates (Figure 5) leads to stable forecasts with relatively small uncertainty. Again, the largest rates and their variability are observed for the East and London region. An improved model for international migration would incorporate correlations between inflows and outflows.

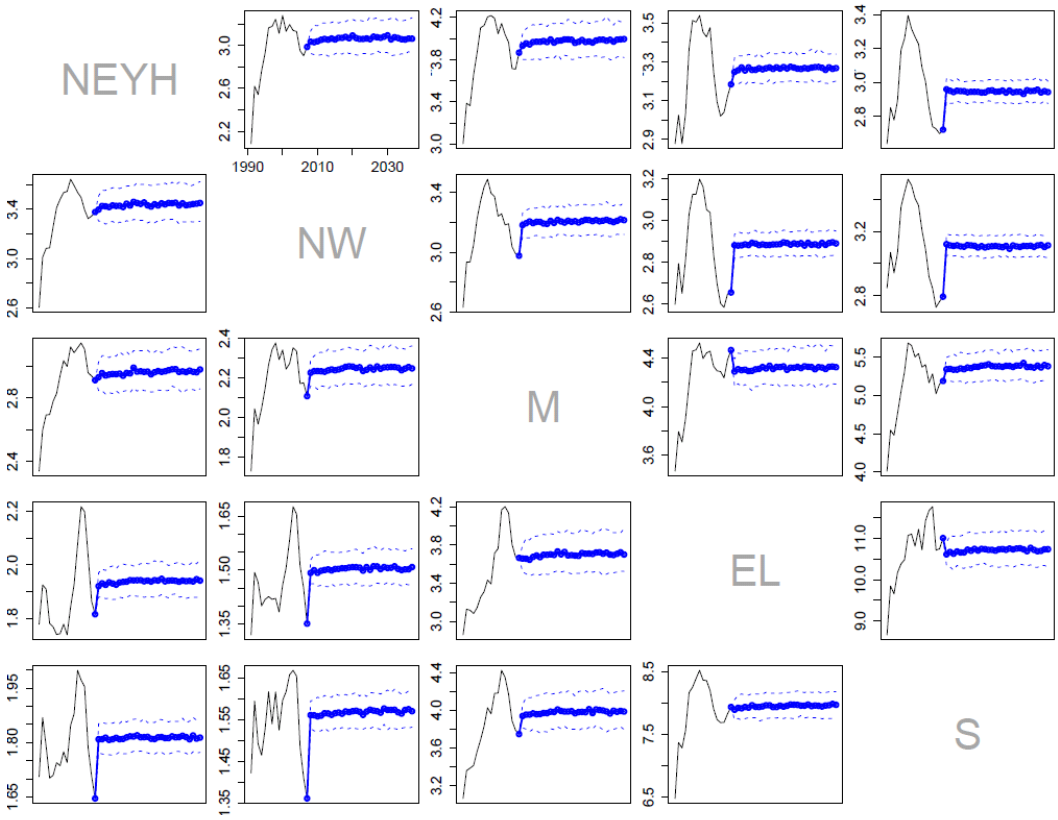

Figure 1: Origin (row) – destination (column) internal migration between five regions of England. The X axis in each plot denotes out-migration rate per 1000; regions: NEYH – North East with Yorkshire and

Humber, NW – North West, M – Midlands, EL – East and London, S – South. Thick blue line represents median, dashed lines denote interquartile range.

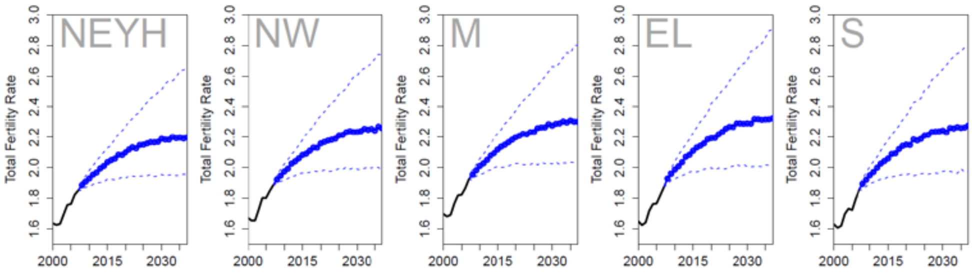

Figure 2: Total Fertility Rate for five regions of England. Thick blue dotted line represents median, dashed lines denote interquartile range. Note: see Figure 1.

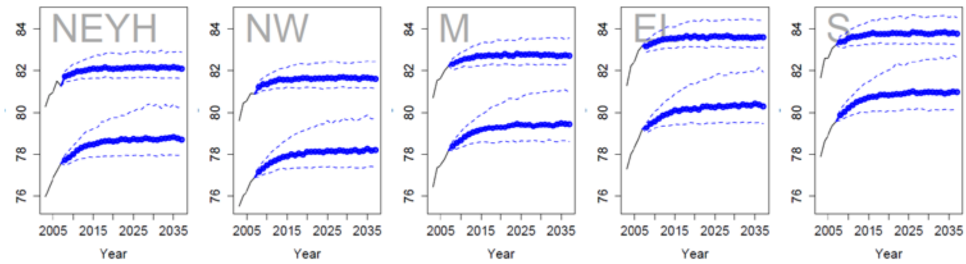

Figure 3: Life Expectancy at birth for five regions of England; females (upper line) and males (lower line). Thick blue dotted line represents median, dashed lines denote interquartile range. Note: see Figure 1.

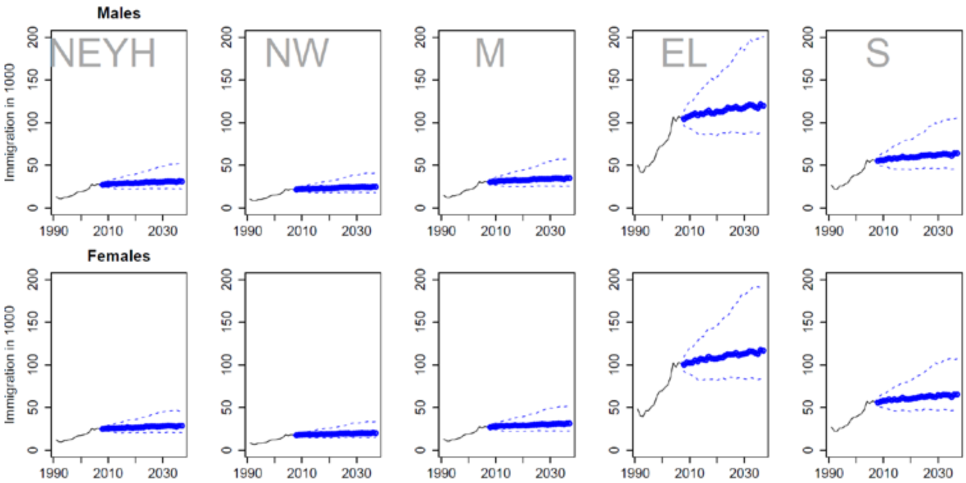

Figure 4: International immigration to five regions of England and by sex. Thick blue dotted line is median, dashed lines denote interquartile range. Note: see Figure 1.

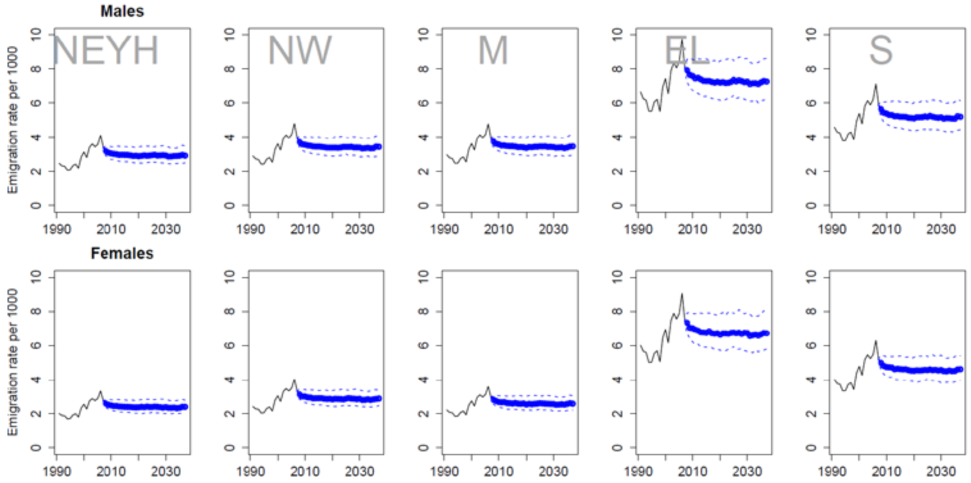

Figure 5: International emigration rates from five regions of England and by sex. Thick blue dotted line is median, dashed lines denote interquartile range. Note: see Figure 1.

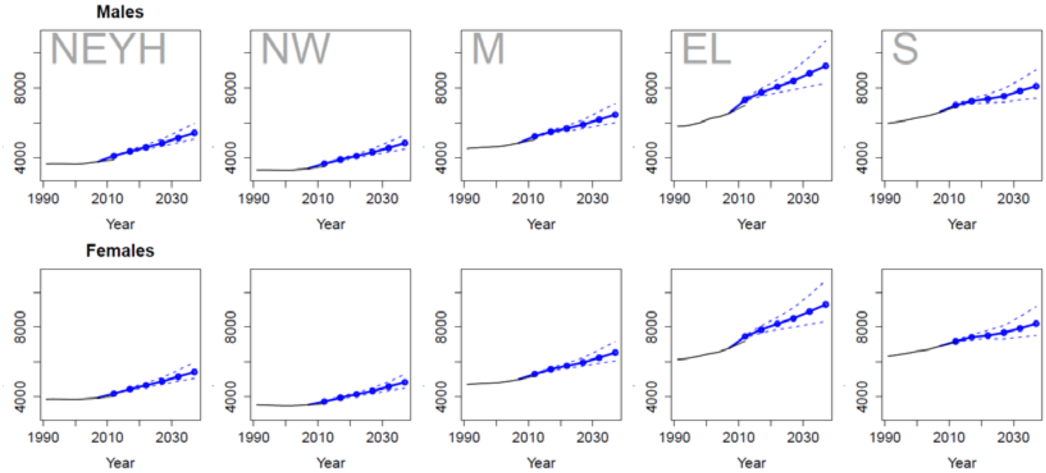

Figure 6: Forecasts of population totals for five regions of England by sex. Note: see Figure 1.

30. When we decompose the forecasts from Figure 6 into age profiles in 2012 (Figure 7) and 2027 (Figure 8), we observe that these differences are driven by estimates for age groups 25-29 and 30-34, which are overestimated for the Northern regions and underestimated for the East and London. These age groups are most likely driven by international immigration, including migration between constituent countries of the UK. Hence, the lack of fit in these particular age groups suggests that the assumption about constant age pattern over time of IPS data is not valid. To overcome this limitation, the model for international migration should incorporate variation over time from the imperfect age, sex and region profiles reported by the IPS. Here, a solution can be a hierarchical model, such as the one developed in Wiśniowski et al. (forthcoming), that would allow borrowing of strength over time and regions. Further, capturing correlations and smoothing the data (originally rounded to the nearest thousand) as in Wiśniowski et al. (2015) would better reflect immigration uncertainty. The mismatch of age profiles between regions cancels out, however, for the forecast of the total population (Figure 7). The median forecast for total population in

England in 2012 is 53,333 thousand, which is practically the same as the observed 53,341 thousand. The interquartile range is (52,948; 53,761) thousand.

31. The other differences between observed and estimated profiles arise in the youngest and oldest age groups. In the former, the main driver is an over-optimistic forecast of fertility, based on the available data. In fact, TFR for the UK remained at around 1.9 for years 2008-2012 (ONS, 2014). Here, more data on fertility broken by desired characteristics, as well as stronger subjective prior information would be required to improve precision of the forecasts. As mentioned earlier, the last age group (75+) is broad and would require disaggregation into older age groups and employing more advanced techniques both in the forecasting model (capturing differences over time between regions) and within the projection model (graduation of the oldest groups and closing of the life table). Also, more than just five years of data would greatly increase precision of the estimated and forecast mortality rates.

Table 1: Forecasts of English population by region and sex for 2012 and 2027, rounded to the nearest thousand. Note: (1) Data for 2007 and 2012 obtained from the Office for National Statistics; p25% denotes 25th percentile of the posterior probability distribution.

	2007	2012	2012					2027				
Males	data	data	p2.5%	p25%	p50%	p75%	p97.5%	p2.5%	p25%	p50%	p75%	p97.5%
North East & Yorkshire and Humber	3779	3891	3934	3986	4013	4041	4111	4282	4620	4792	5033	5873
North West	3390	3484	3526	3569	3595	3622	3669	3826	4128	4293	4489	5201
Midlands	4852	5038	4973	5034	5069	5104	5174	5175	5583	5817	6089	7074
East and London	6561	7012	6690	6865	6947	7030	7220	6764	7635	8142	8773	11481
South	6618	6908	6604	6718	6773	6825	6950	6410	7055	7401	7822	9458
Females												
North East & Yorkshire and Humber	3947	4028	4038	4087	4114	4138	4203	4363	4671	4855	5071	5884
North West	3539	3601	3605	3648	3671	3696	3746	3895	4156	4313	4489	5095
Midlands	5005	5172	5062	5120	5153	5186	5263	5313	5671	5886	6156	7088
East and London	6786	7204	6847	6994	7076	7153	7346	6834	7739	8241	8845	11444
South	6904	7156	6767	6876	6929	6986	7107	6562	7191	7558	7968	9655

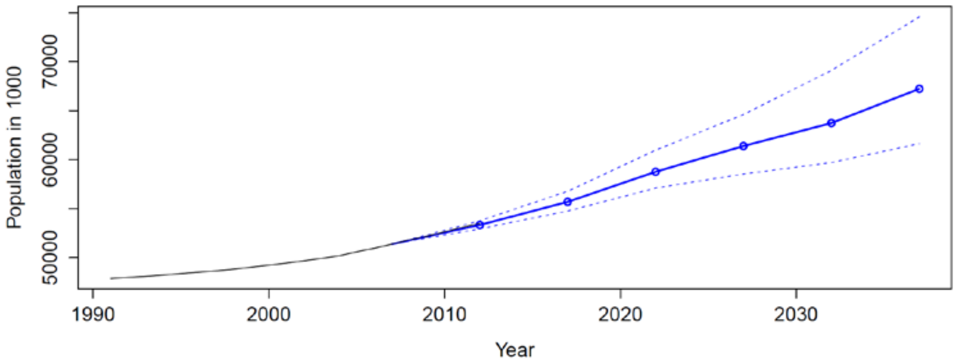

Figure 7: Forecasts of population total for England. Note: see Figure 1.

IV. Conclusion

32. There are several contributions that this research provides. First, we have produced a vehicle for probabilistic forecasting of subnational populations by age and sex. Second, the forecasts include measures of uncertainty over time for all desired characteristics of the population and population change. Third, we have developed and extended methods for estimating the subnational components of population change over time, making best use of available data and expert judgements. Finally, we have demonstrated how the results from these models provide us with a more in-depth understanding of future population change.

33. We found that relatively small differences in the age profile of international migration disaggregated by regions can have relatively large influence on the subnational population forecasts. This finding confirms the importance and need for describing uncertainty of the international migration, as well as the role of international migration in shaping population structure of England.

34. This modelling framework can be extended in various directions. First, more complex models can be employed for all population components, in particular for mortality and international migration. Second, the level of subnational disaggregation can be increased into smaller areas. Here, computational difficulties will have to be overcome in handling large arrays of data containing low counts of events. Third, expert-based information can be incorporated in the population component models, such as the example with fertility in our implementation.

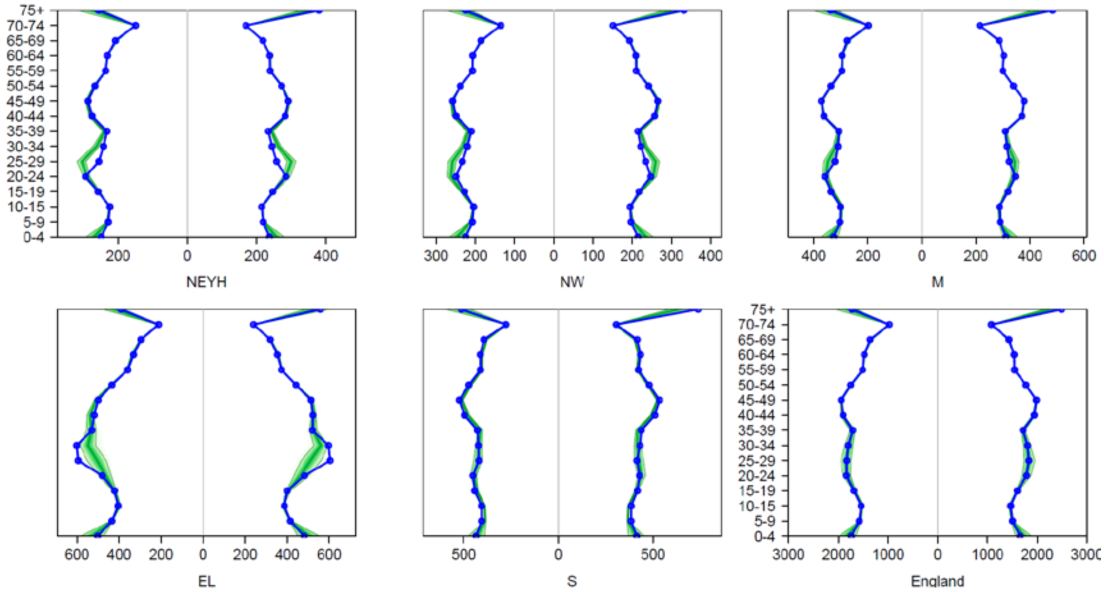

Figure 8: Forecasts of population pyramids for England by age and sex for 2012. Notes: (1) green fan plot denotes posterior probability density with three lines denoting 5th percentile, median and 95th percentile; (2) blue line with circles denotes reported population in 2012.

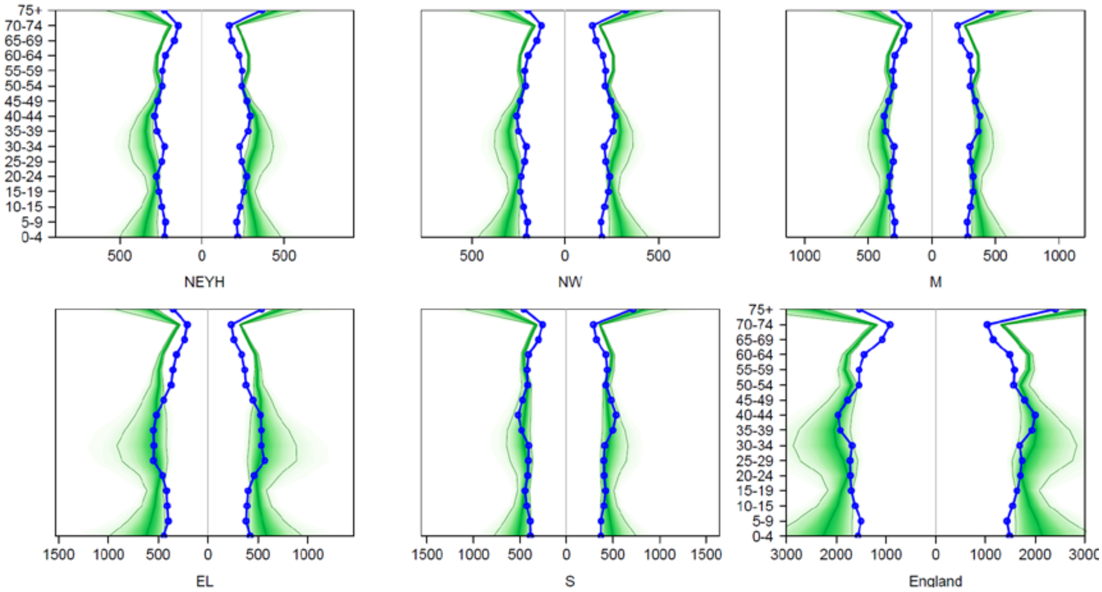

Figure 9: Forecasts of population pyramids for England by age and sex for 2027. Note: see Figure 8. Blue line with circles denotes baseline population in 2007.

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