



**UiO : University of Oslo**

**Probabilistic household forecasts for  
Denmark, Finland, and the Netherlands  
2011-2041:  
Combining the Brass relational method  
with a Random Walk model**

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## Task:

Compute stochastic household forecasts for Denmark, Finland, and the Netherlands

Selected results for three countries (+ Germany and Norway) at <http://mopact.group.shef.ac.uk/research-posts/householdforecasts/>

# Data

Population by five-year age group (0-4, 5-9, ..., 85-89, 90+), sex (M/F), and household position, 1 January of each year:

- child
- living alone
- cohabiting
- married
- lone parent
- other private hh pos
- institution

Such stock data are available for

- Denmark 1981-2007 (register), 2011 (census)
- Finland 1988-2009 (register), 2011 (census)
- Netherlands 1996-2010 (register), 2011 (census)

# Approach

1. Construct a time-series model for the share of persons who live in household position  $k$ , age group  $x$ , sex  $s$ , country  $c$
2. Predict each share  $h$  years ahead → **prediction interval**
3. Simulate shares from predictive distributions, combine with stochastic population forecast (age & sex)
4. Combine # persons by hh position into # households by type

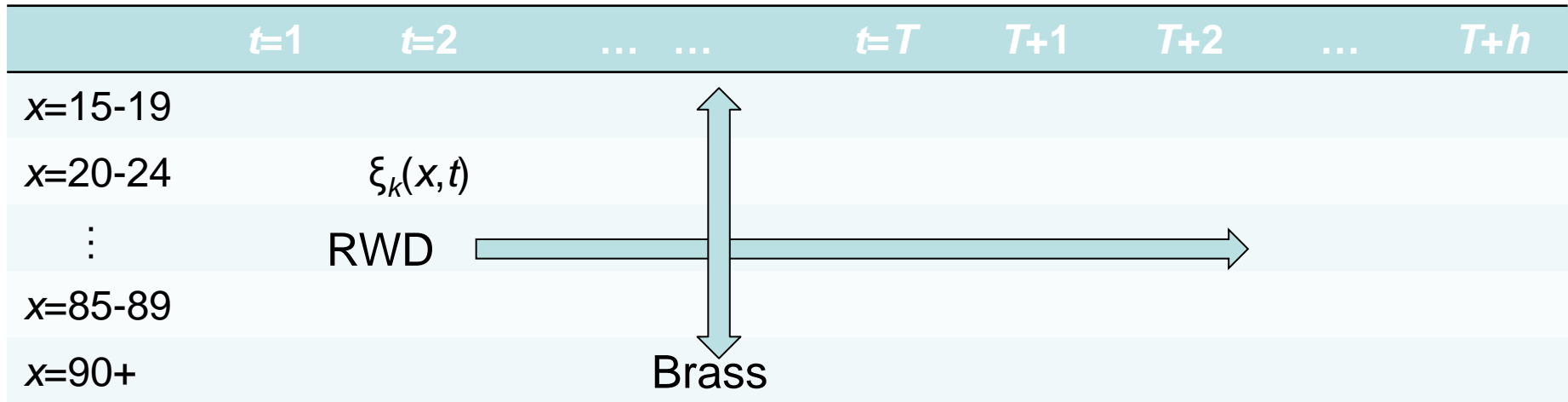
# Time-series model

For each household position (except child), sex, country:

Model the share (logit transformed) as

Random Walk with Drift (**RWD**), preserving age pattern by **Brass** relational model

Shares for children kept constant



Share  $\xi_k(x,t)$  for hh pos  $k$ , age group  $x$ , time  $t$  (given sex, country) :

$$\Delta \xi_k(x,t) = a_k + b_k \cdot \xi_k^S(x) + d_k(x,t),$$

$\xi_k^S(x)$  standard age pattern: average of  $\xi_k(x,t)$  over interval  $[1, T]$   
(given sex, country)

drift  $a_k + b_k \cdot \xi_k^S(x)$

$d_k(x,t)$  error term, „innovation“

Parameters  $a_k$  and  $b_k$  estimated by OLS, assuming innovation variance independent of age and time

Small differences across countries, not significant in many cases  
→ re-estimate model for the three countries combined

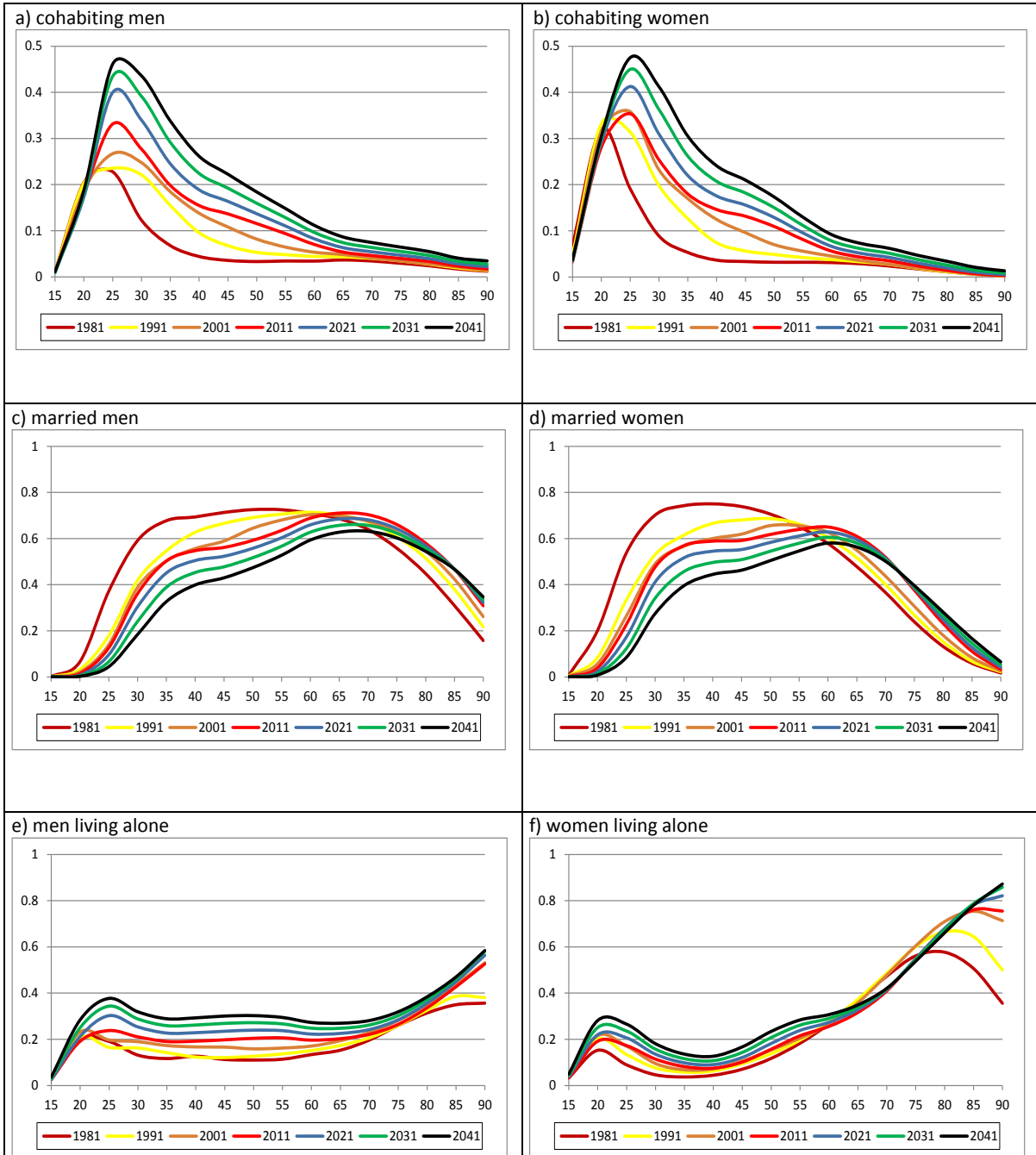
Small differences between men and women, except for **living alone** & living in **institution** → re-estimate model for sexes combined for remaining household positions

Reduced the dimension of the problem by almost a factor 1000

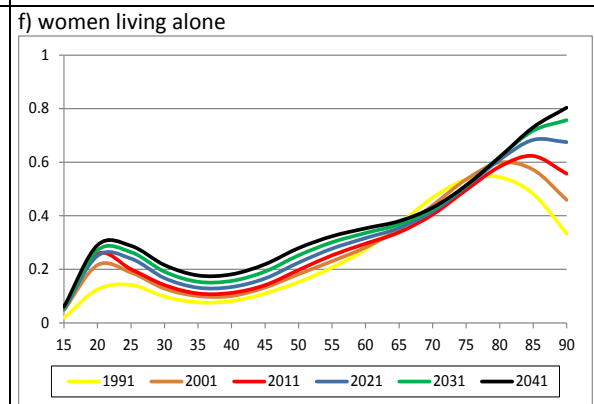
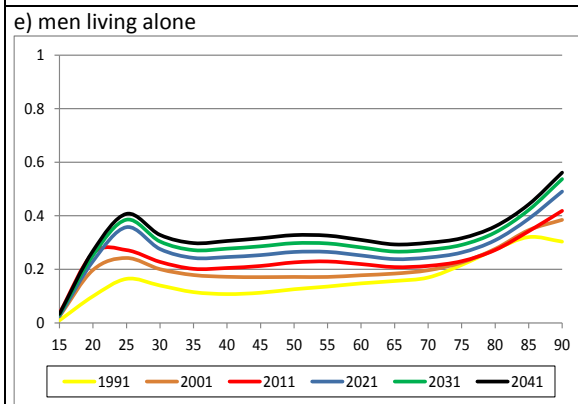
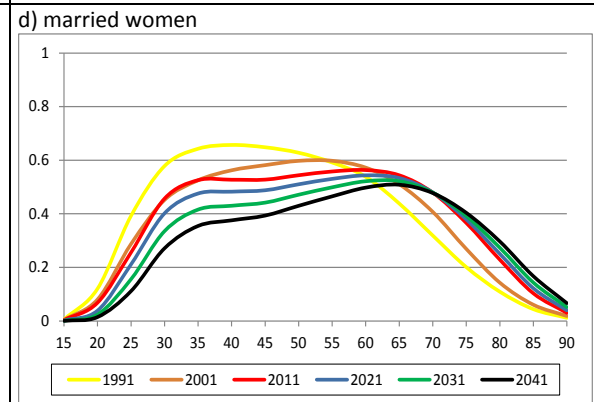
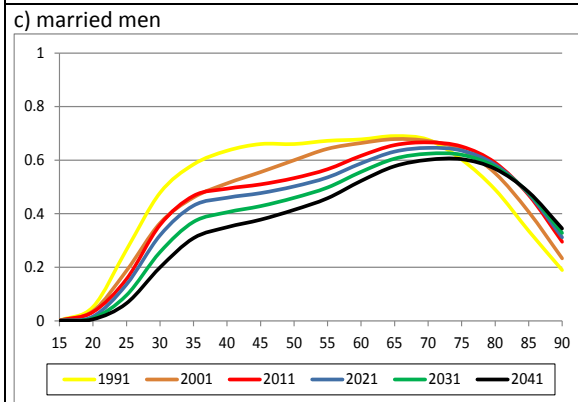
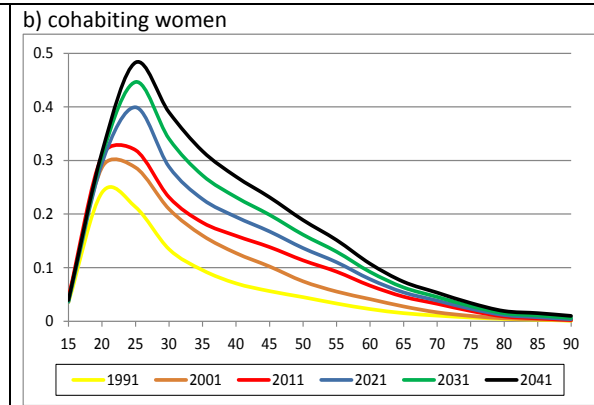
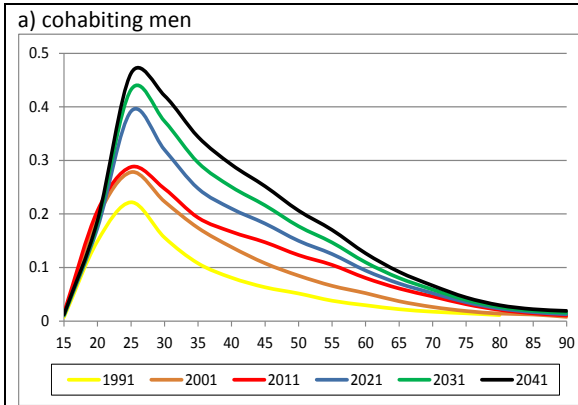
Use the model to predict shares to 2041, starting from 2011



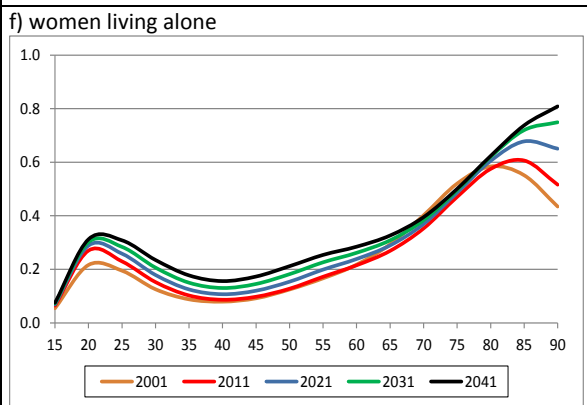
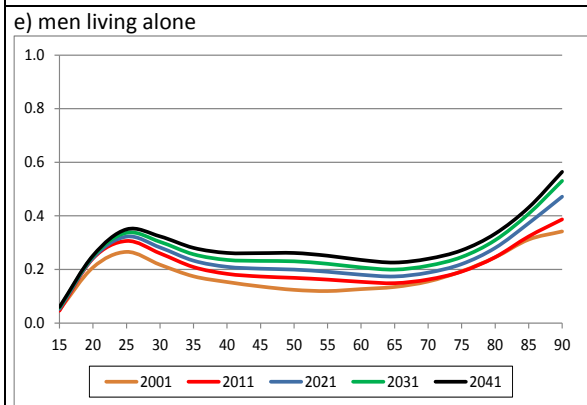
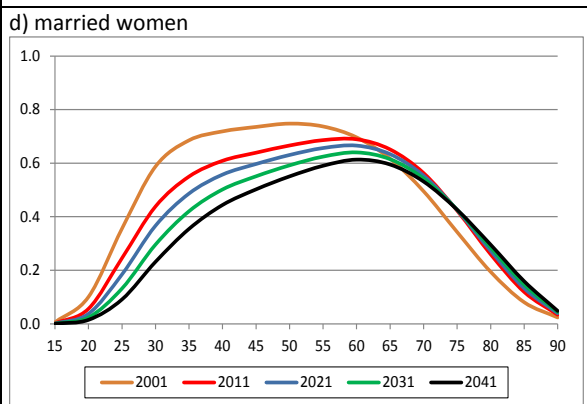
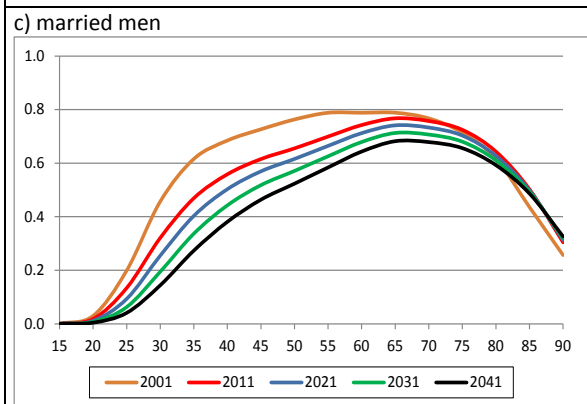
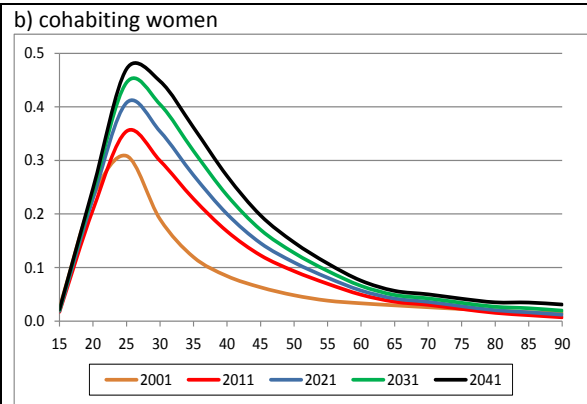
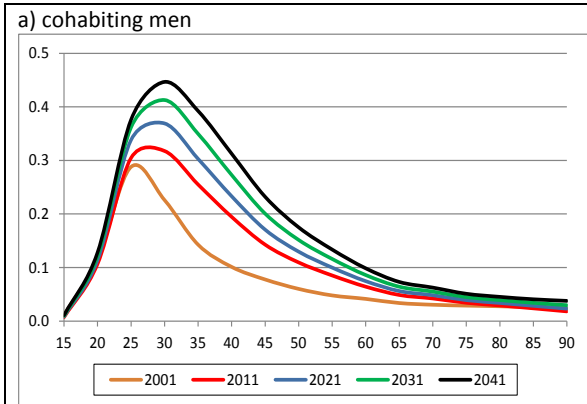
Denmark



Finland



Netherlands



A h years ahead forecast, starting year T, is

$$E[\xi_k(x, T+h)] = \xi_k(x, T) + h \cdot (\hat{a}_k + \hat{b}_k \cdot \xi_k^S(x))$$

The forecast error h years ahead is  $E[\xi_k(x, T+h)] - \xi_k(x, T+h)$ .

Its variance is

$$h \cdot \sigma_k^2 + h^2 \cdot \text{Var}[\hat{a}_k] + h^2 \cdot \left( \xi_k^S(x) \right)^2 \cdot \text{Var}[\hat{b}_k] - 2 \cdot h \cdot \xi_k^S(x) \cdot \text{Cov}[\hat{a}_k, \hat{b}_k],$$

where  $\sigma_k^2 = \text{Var}[d_k(x, t)]$  is the innovation variance of the RWD

Correlations between men and women from model residuals

- **Lone parents**: assumed **independence** between men and women (estimated correlation 0.065)
- all other hh pos: 0.623 (median)

Correlations across ages (assumed AR1):

- **COH** and **MAR** assumed **perfect** correlation between neighbouring age groups (estimate 0.982)
- all other hh pos: 0.756 (median)

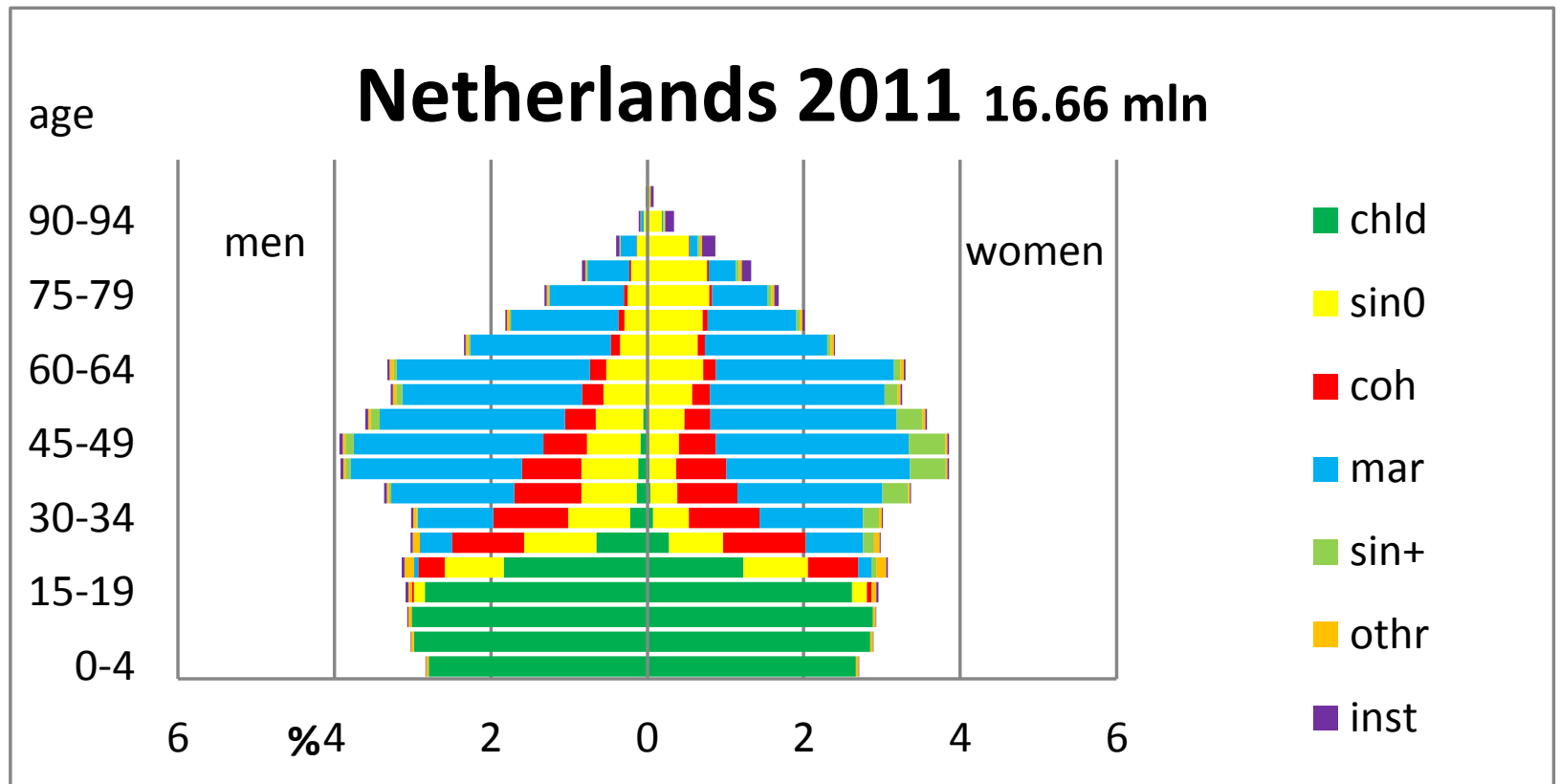
Result: point predictions (expected values)  $\mu$  for the shares

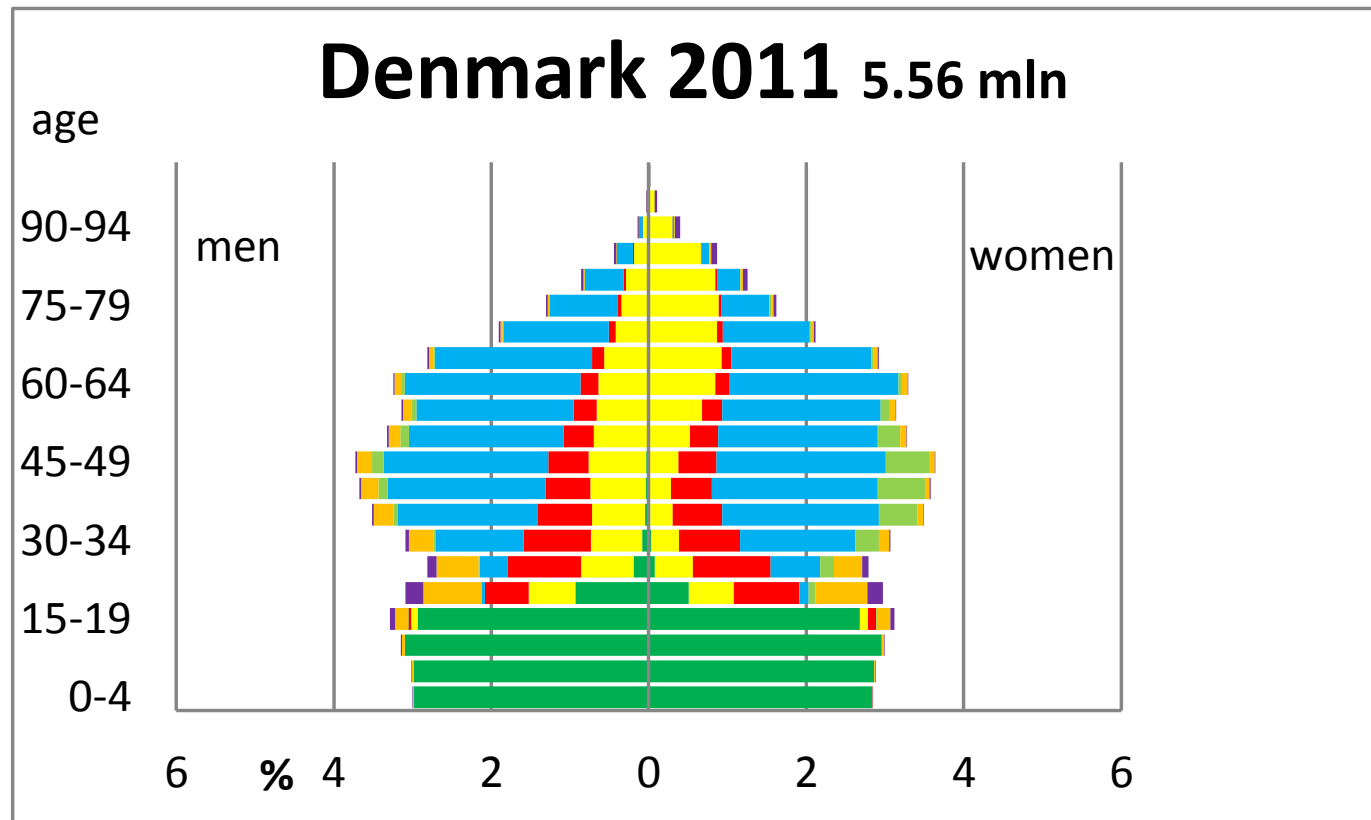
Variances and covariances  $\sigma^2$  added

→ Predictive distributions (Normal( $\mu, \sigma^2$ ) logit scale) for shares  
2021, 2031, 2041

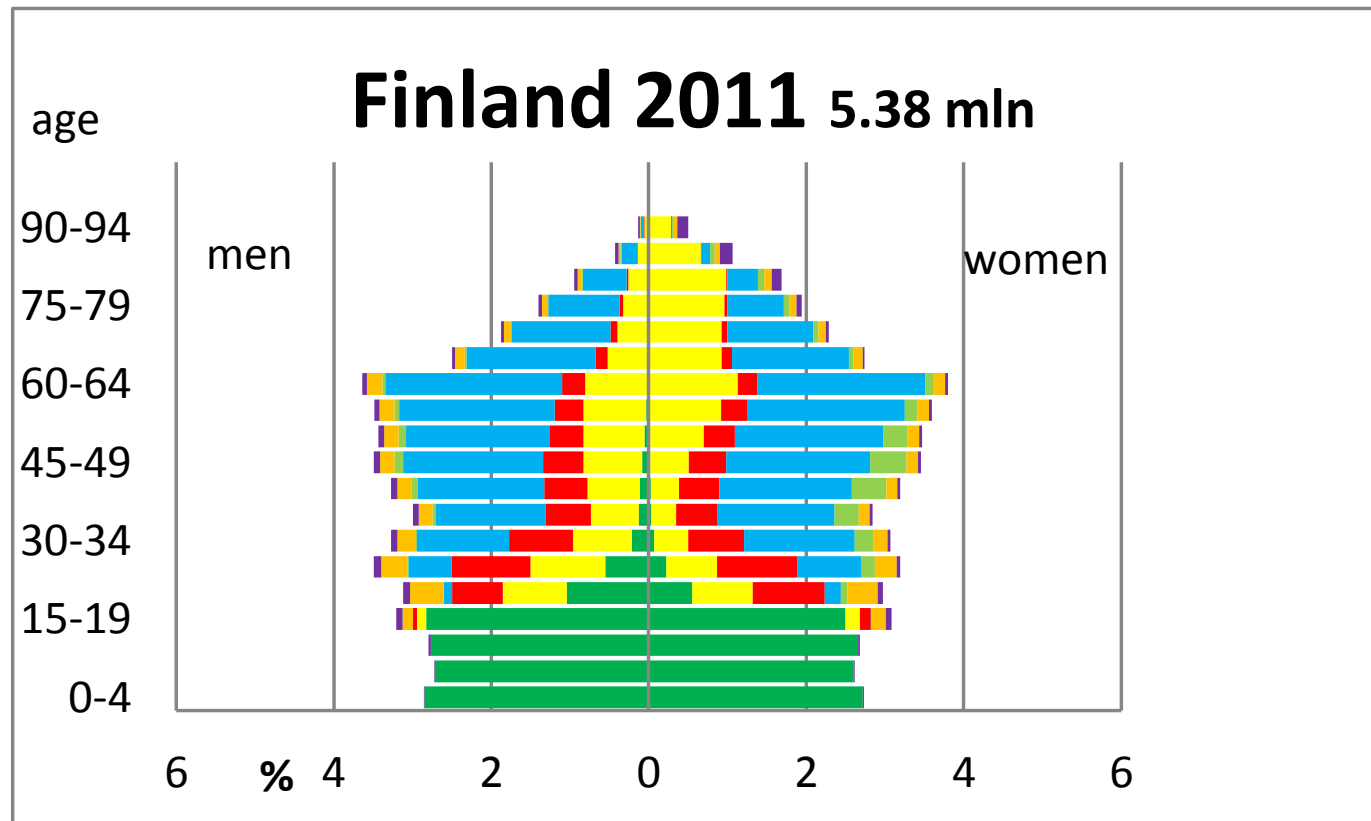
Combined with updates of stochastic population forecasts (age, sex) from UPE

1000 simulations for each country









# Results

- All households
- Persons aged 80+ who live alone

Selected results for three countries (+ Germany and Norway) at <http://mopact.group.shef.ac.uk/research-posts/householdforecasts/>

If current trends continue ...

## Denmark: private households and population. Point predictions (millions) and Coefficients of variation (% , in parentheses)

		One person households	Cohabiting couples	Married couples	Lone fathers	Lone mothers	All private households (incl. other private households)	Population size
2011		0.95	0.30	1.03	0.03	0.15	2.54	5.56
2021	Average	1.19	0.40	1.08	0.03	0.12	2.85	5.83
	CV	9.8	17.1	7.4	48.4	29.0	2.5	1.0
2031	Average	1.39	0.479	1.02	0.02	0.11	3.04	6.10
	CV	13.0	21.3	11.8	52.2	35.2	3.9	2.8
2041	Average	1.54	0.53	0.96	0.02	0.10	3.19	6.32
	CV	15.7	24.8	14.7	59.4	39.4	5.5	5.3

Households increase faster than population (by 26% vs. 14% 2011-2041)

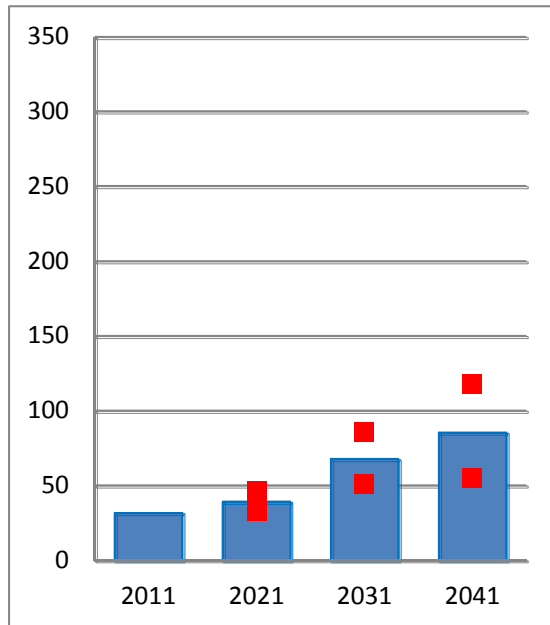
Married cpls decline (41% in 2011 → 30% in 2041)

Strong growth in 1-p hhds (by 62%), cohabiting cpls (by 77%)

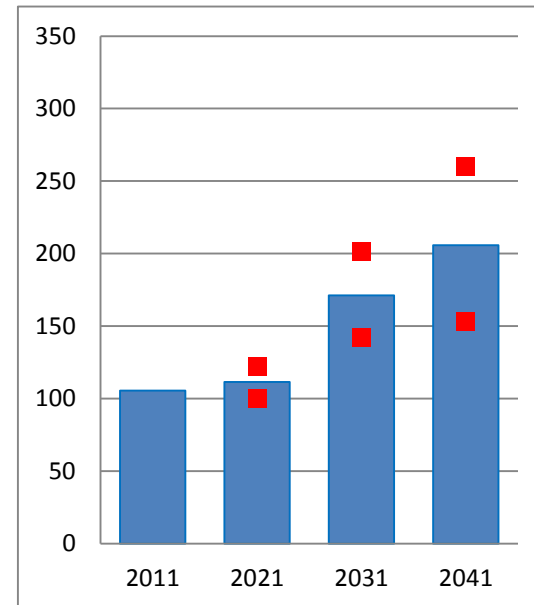
Numerous hhds (all, 1-p, mar) much more certain than less numerous hhds (lone par)

## Denmark: numbers of men and women aged 80+ living alone (in 1000s). Census (2011) and predicted (2021, 2031, 2041)

men



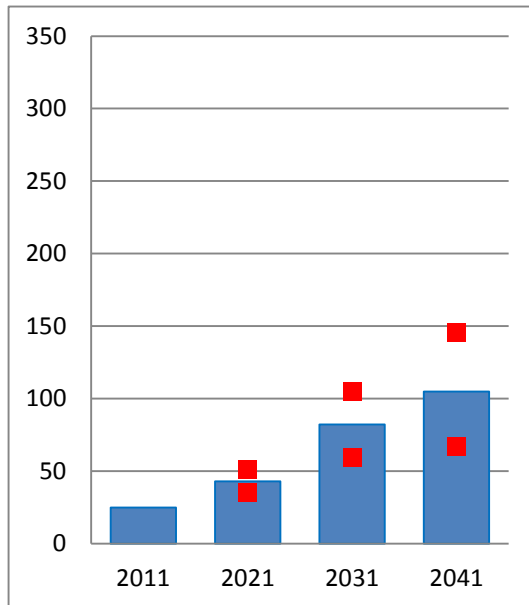
women



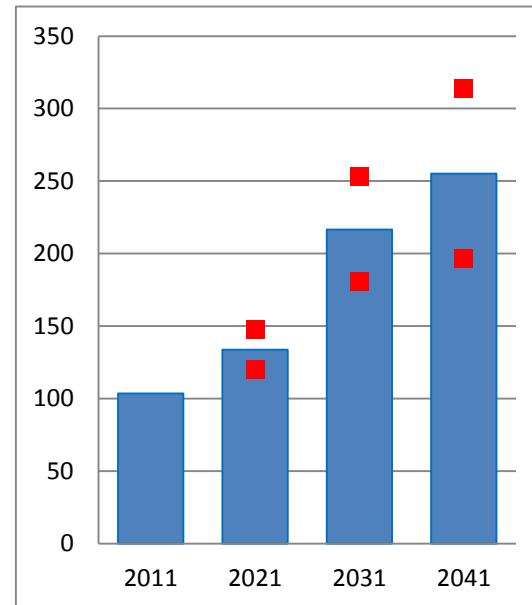
Red dots represent upper and lower bounds of 80% prediction intervals

## Finland: numbers of men and women aged 80+ living alone (in 1000s). Census (2011) and predicted (2021, 2031, 2041)

men



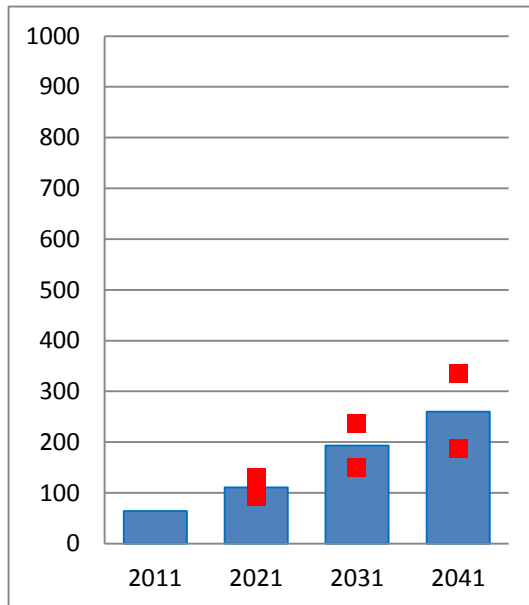
women



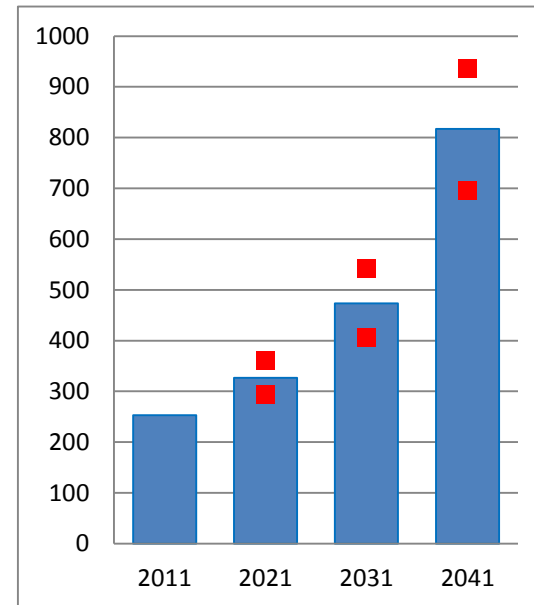
Red dots represent upper and lower bounds of 80% prediction intervals

# The Netherlands: numbers of men and women aged 80+ living alone (in 1000s). Census (2011) and predicted (2021, 2031, 2041)

men



women



Red dots represent upper and lower bounds of 80% prediction intervals

## **Conclusion elderly who live alone:**

Two- to four-fold increase between 2011 and 2041

Sex ratio's (W/M): 2011  $\approx$  3-4; 2041  $\approx$  2-3

Quite certain – narrow prediction intervals

Persons who live in an institution. Observed 2011 (Census numbers), and predictions 2021-2041. Average across 1000 stochastic simulations, in thousands. Coefficient of variation (CV), in per cent. Lower and upper bounds of 80% prediction interval in thousands

	2011	2021	2031	2041
Denmark				
Average	81.5	69.0	64.2	55.2
CV		21.2	33.2	48.4
Interval		[51.5, 87.7]	[40.6, 93.8]	[28.3, 87.3]
Finland				
Average	110.7	171.5	287.5	436.1
CV		21.2	29.7	32.7
Interval		[128.2, 223.0]	[184.5, 403.1]	[259.6, 629.0]
Netherlands				
Average	219.3	224.9	260.8	326.8
CV		21.3	32.4	42.0
Interval		[165.8, 287.2]	[165.8, 376.2]	[180.8, 492.8]

Diverging trends (DK down, Fi & NL up), very uncertain



Oldest old sensitive for assumption on institutional capacity

Exogenous?

# Conclusions

When time series of stock data exist: time-series model for household shares is feasible

Brass appropriate for modelling age patterns, cf. Lee-Carter

2041: 2-4 times as many elderly who live alone compared to 2011  
relatively certain ; sex gap narrows

Relative uncertainty (CV) in numerous hhds (married couples, one-person households) smaller than in less numerous hhds (lone parents)

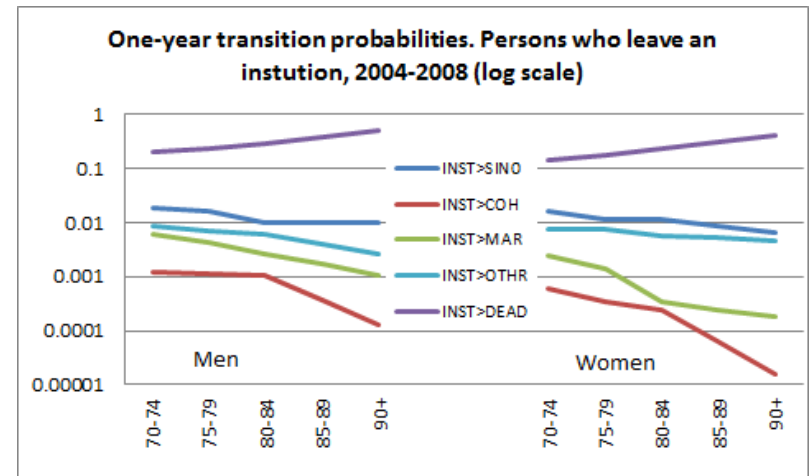
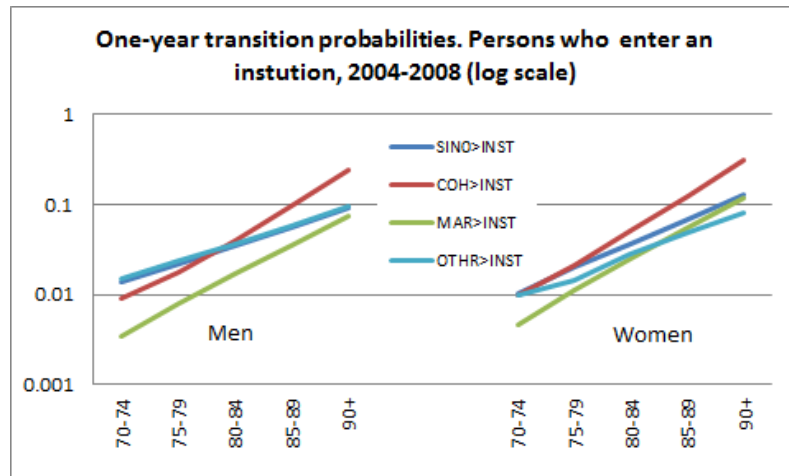
# Issue

Coherence between men and women who live as a couple

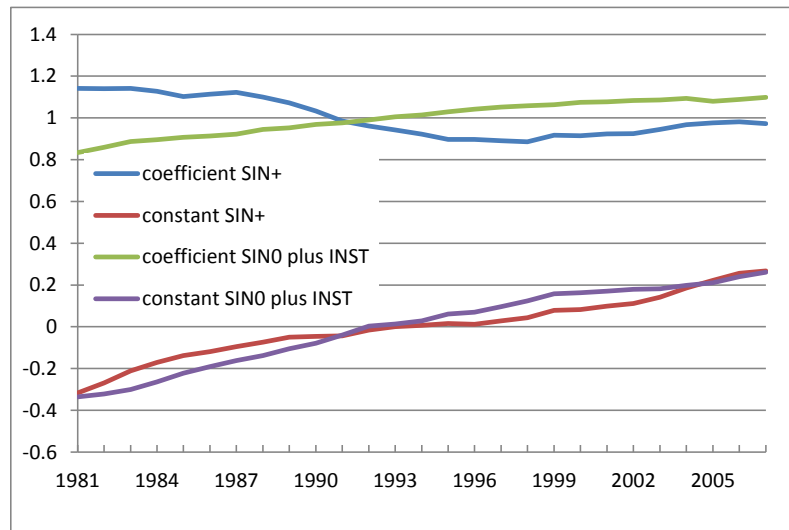
Cohort effects in age patterns of shares

# Elderly who live **alone** are more likely to enter and to leave an institution than those who live with **spouse**

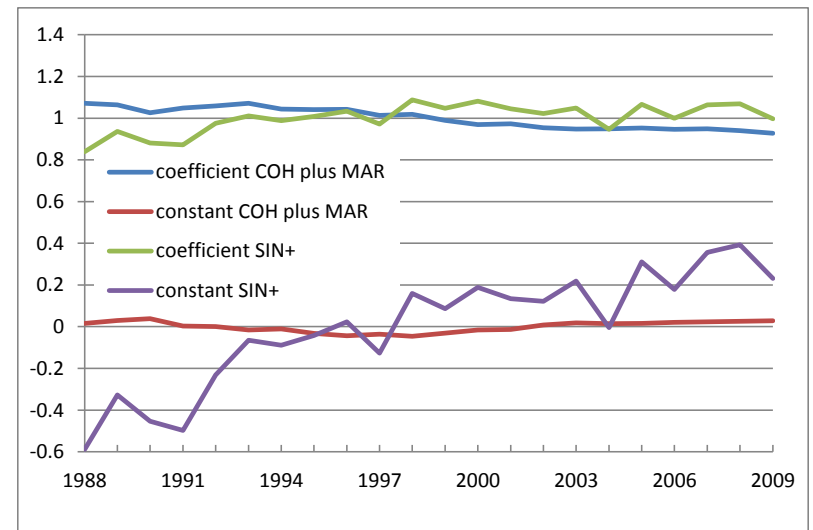
## Register data for Finland



# Estimates of Brass model parameters (constant and coefficient) when model is fitted to data from each year separately.



women in Denmark



men in Finland

k	$a_k$		$b_k$		$\text{Cov}(a_k, b_k)$
	estimate	t-value	estimate	t-value	estimate
2	-0.0005697	-0.7	-0.0076073	-7.6	-5.75e-7
3	-0.0432034	-11.8	0.0083405	5.2	-4.88e-6
4 (men)	0.0385686	27.6	-0.0033708	-1.8	-1.48e-6
4 (women)	0.0211024	18.1	0.0040122	4.4	-3.60e-7
5	0.0652686	14.2	-0.0109857	-6.3	-7.16e-6
6	0.0313597	1.1	0.0121778	0.3	0.0010577