Hagstofa Íslands

Population projections when using time series with extreme values

Violeta Calian, Statistics Iceland

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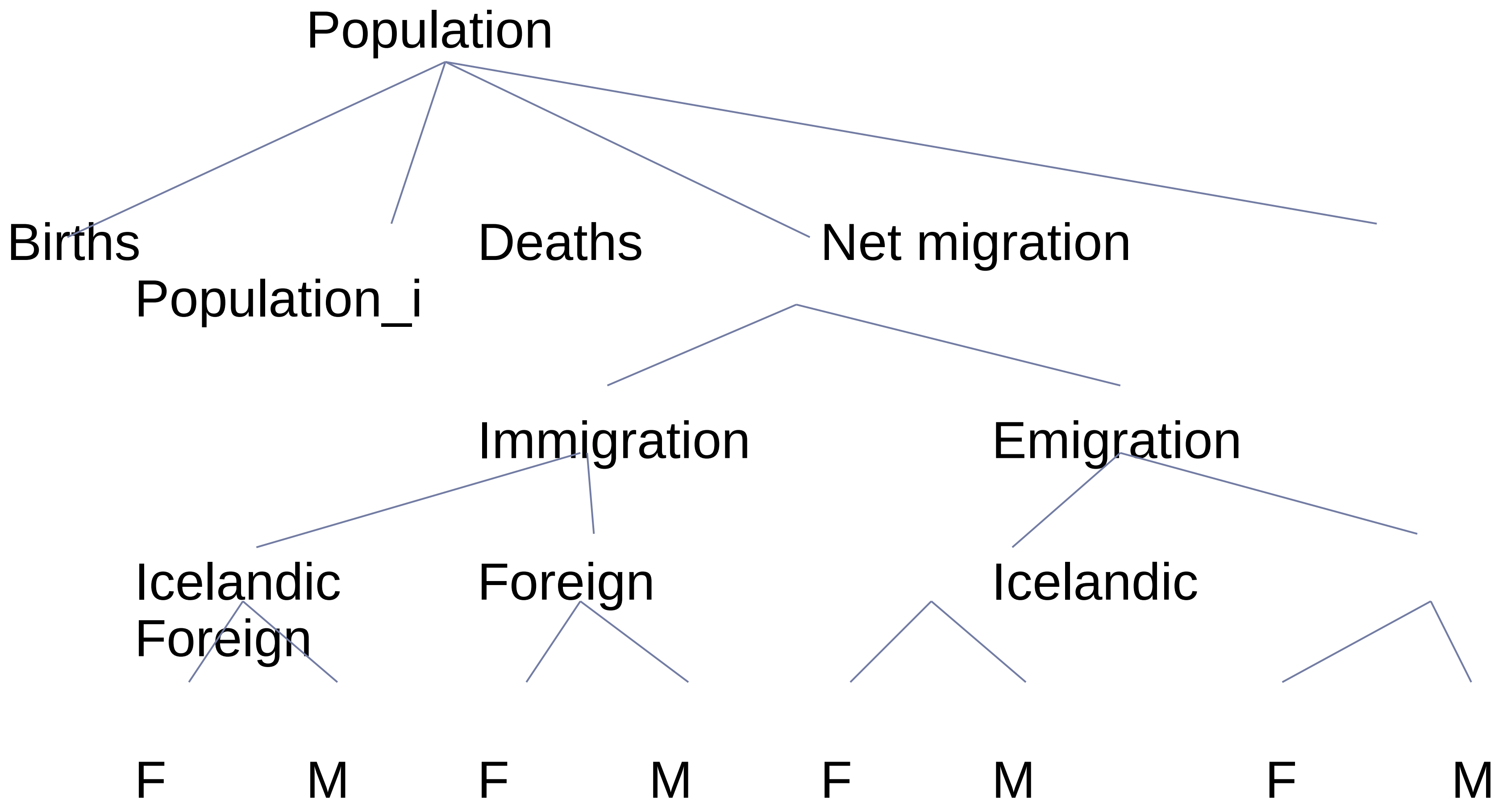
Outline

- Data and formulation of the mathematical problem
- Hypotheses testing and probability of rare events for time series: outliers and extreme values.
- Methods and models:
  - Vector dynamical (ARDL) models, vector arima models
  - functional data modelling with time series coefficient functions of orthonormal function expansions

Hierarchical time series data structure



The mathematical problem

Notation $y(t, x)$:

- the log of the observed mortality or fertility rate for age x and year t
- migration components by age, at time t
- a vector of several/all migration components $(y_1, y_2, \ldots)$

Observations $\{t_i, x_a, ; y_{ia}\}$

$i = 1, \ldots, N$ (years) and $a = M_0, \ldots, M$ (ages),

at discrete points (t_i, x_a) of a two-dimensional domain

Predictions $y(t_i, x_a)$ with prediction intervals/densities

for same set of age values and for years t_i ($i = N+1, \ldots, N+h$),
where h = the length of the forecasting horizon

Principles

- *joint modeling* the (non-stationary) vector time series data (and its autocorrelations) so that the residuals are independently, identically and preferably normally distributed

- using (multiple) hypotheses testing (ergodicity, stationarity, outliers)

- scale independence

- data exchangeability when needed

- explaining and predicting

- data models depending on goal

- $g(x, y) = 0 \rightarrow y = f(x)$ when it exists!

Outliers and extreme values

- Outlier

small probability of being generated by the same statistical distribution / stochastic process as the other observations in the data set

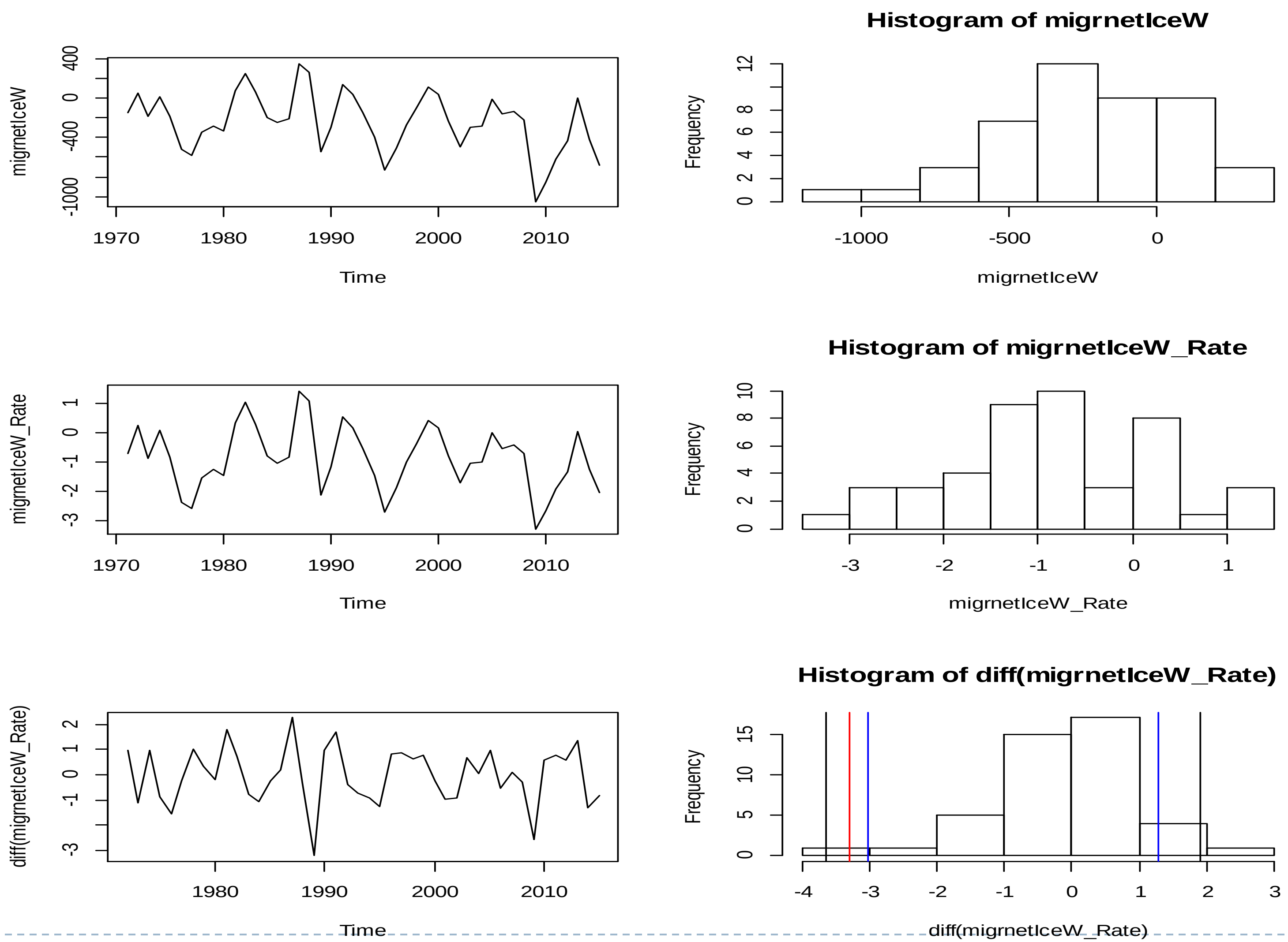
- Time scales of outlier effects in time series data

- Extreme value

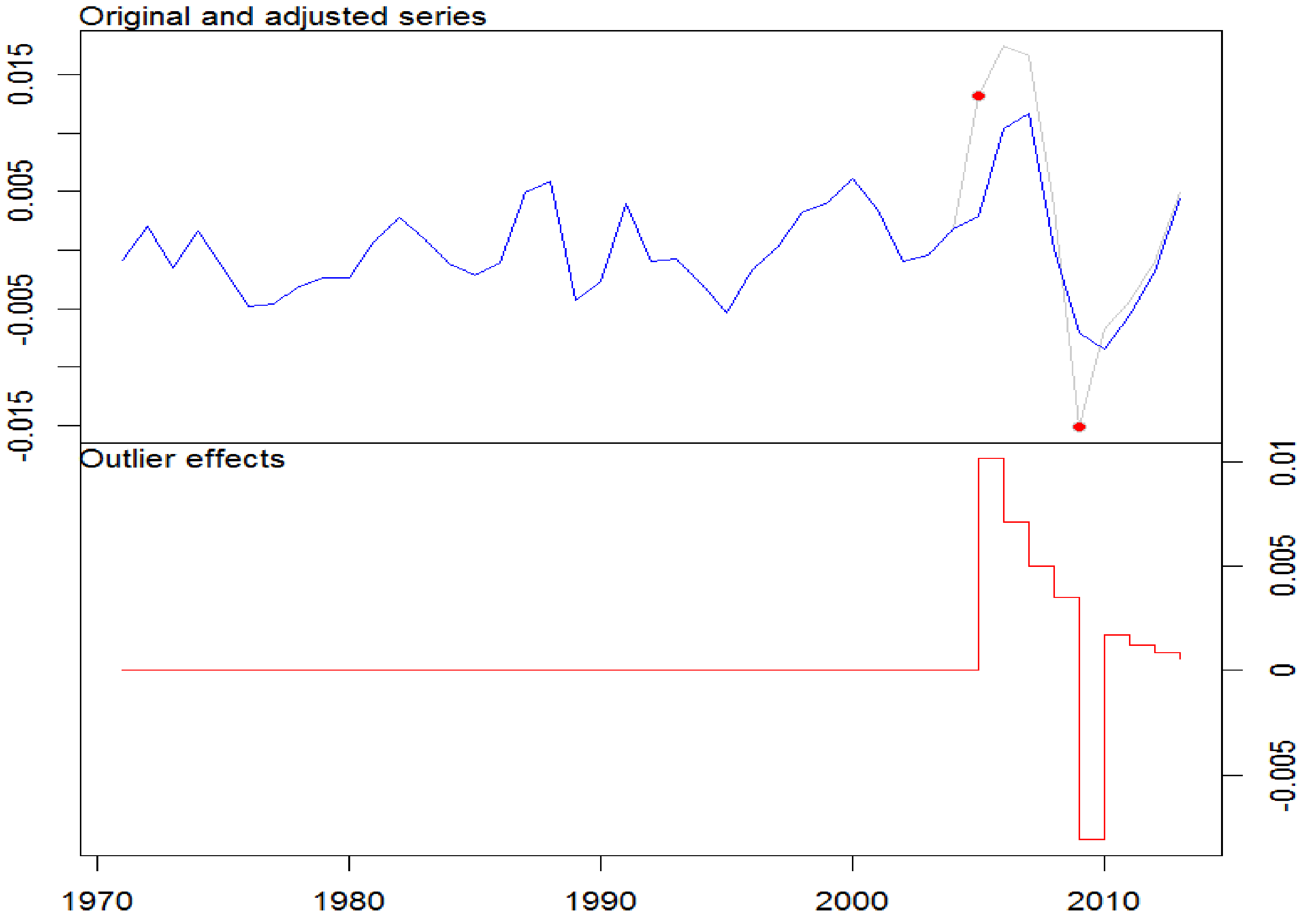
observation with low probability of occurence but generated from the same statistical distribution / stochastic process as the other observations in the data set

- Predicting extreme values occurrence for time series data

Example: Outliers and extreme values in migration data



Net migration rate



Model $f(t,x)$ types

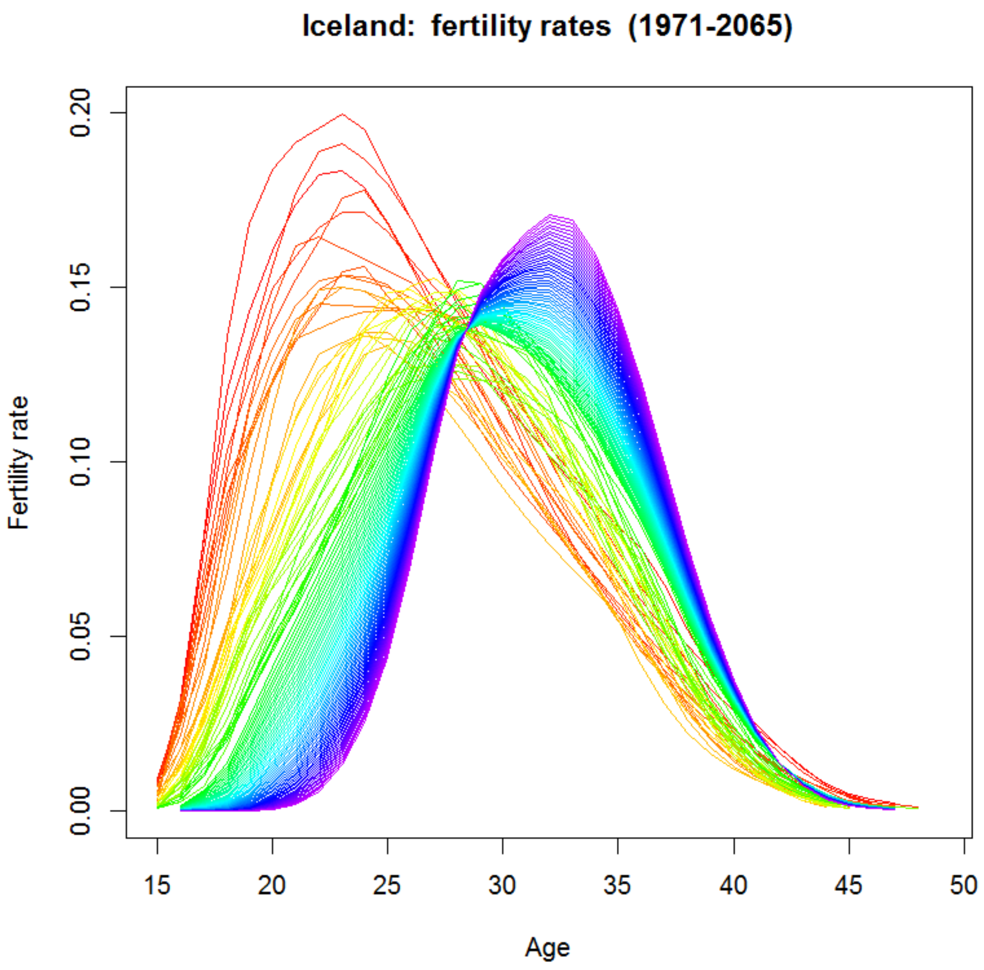
- a model $f(t,x)$ of time and age dependent rates (trick: which can be written as an expansion with time-series coefficient functions which are independent by construction)*
- a multivariate model $(f_{x_{M_0}}(t), \dots, f_{x_M}(t))$ of time dependent rates
- a multivariate model $(f_{t_1}(x), \dots, f_{t_N}(x))$ of age dependent rates
- a (multivariate-) model $f(t)$ for $\sum_x f(t,x)$ of time dependent (vector of-) rates for the sums over all ages (trick: when age distribution stable in time, disaggregation is possible) as functions of exogenous variables $Z_1, \ldots$and their lags.*
- a multivariate model of „everything" $(f_1, f_2, \dots, Z_1, \ldots)$ as functions of $(\boldsymbol{t}, \boldsymbol{x})$ ☺

Models for fertility and mortality rates

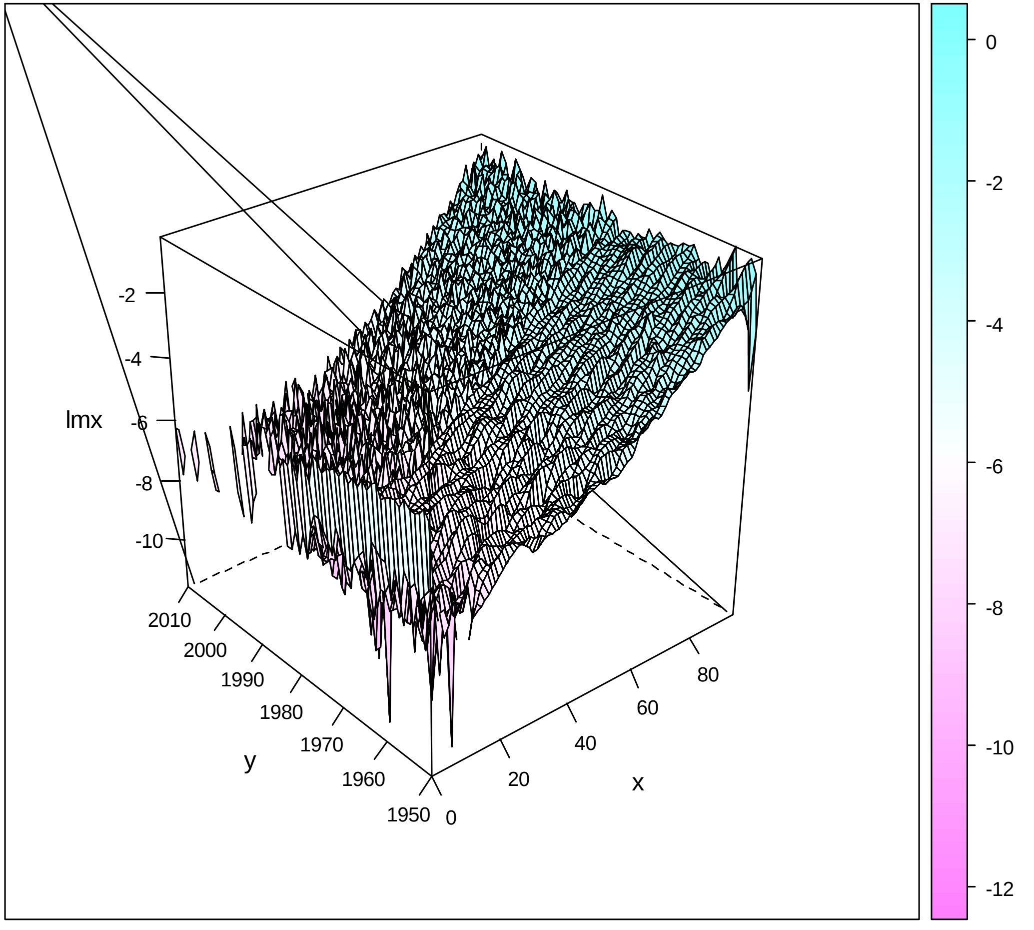
Functional data approach – Hyndman et al (2007, 2008)

- $y_{ia} = \mu(x_a) + f_0(t_i, x_a) + e_{t_i}(x_a) + \alpha_{t_i}(x_a)\varepsilon_{t_i,x_a}$
- $\mu(x)$ - the mean of $f_0(t, x)$ across time (years)
- $e_t(x)$ is the residual modelling error (assumed serially uncorrelated)
- the coefficient functions $\beta_k(t)$ of the expansion $f_0(t, x) = \sum_{k=1,\ldots K} \beta_k(t)\varphi_k(x)$ are independent (by construction)
- $\varepsilon_{t,x}$ - the random variation in birth or death rates
- $\alpha_t(x)$ allow the variance to change with age and time.

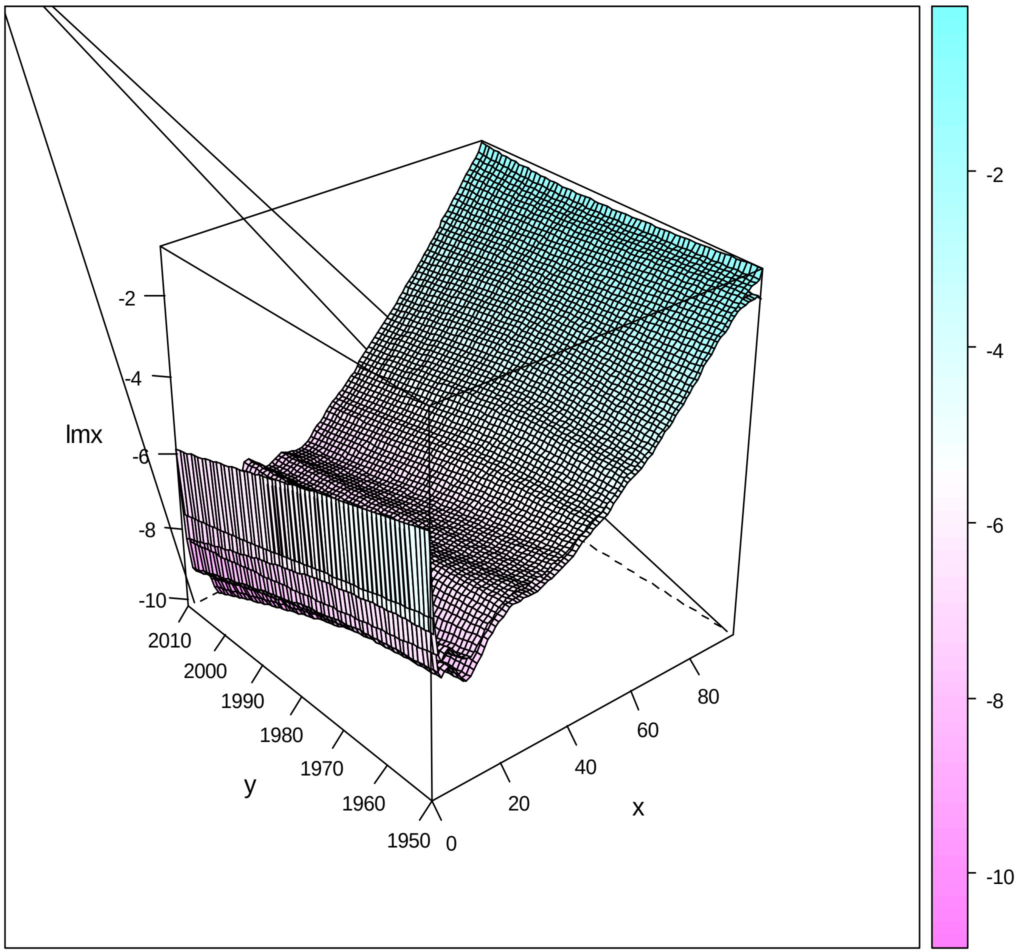
Fertility


Iceland: fertility rates (1971-2065)
Fertility rate
0.00
0.05
0.10
0.15
0.20
15
20
25
30
35
40
45
50
Age

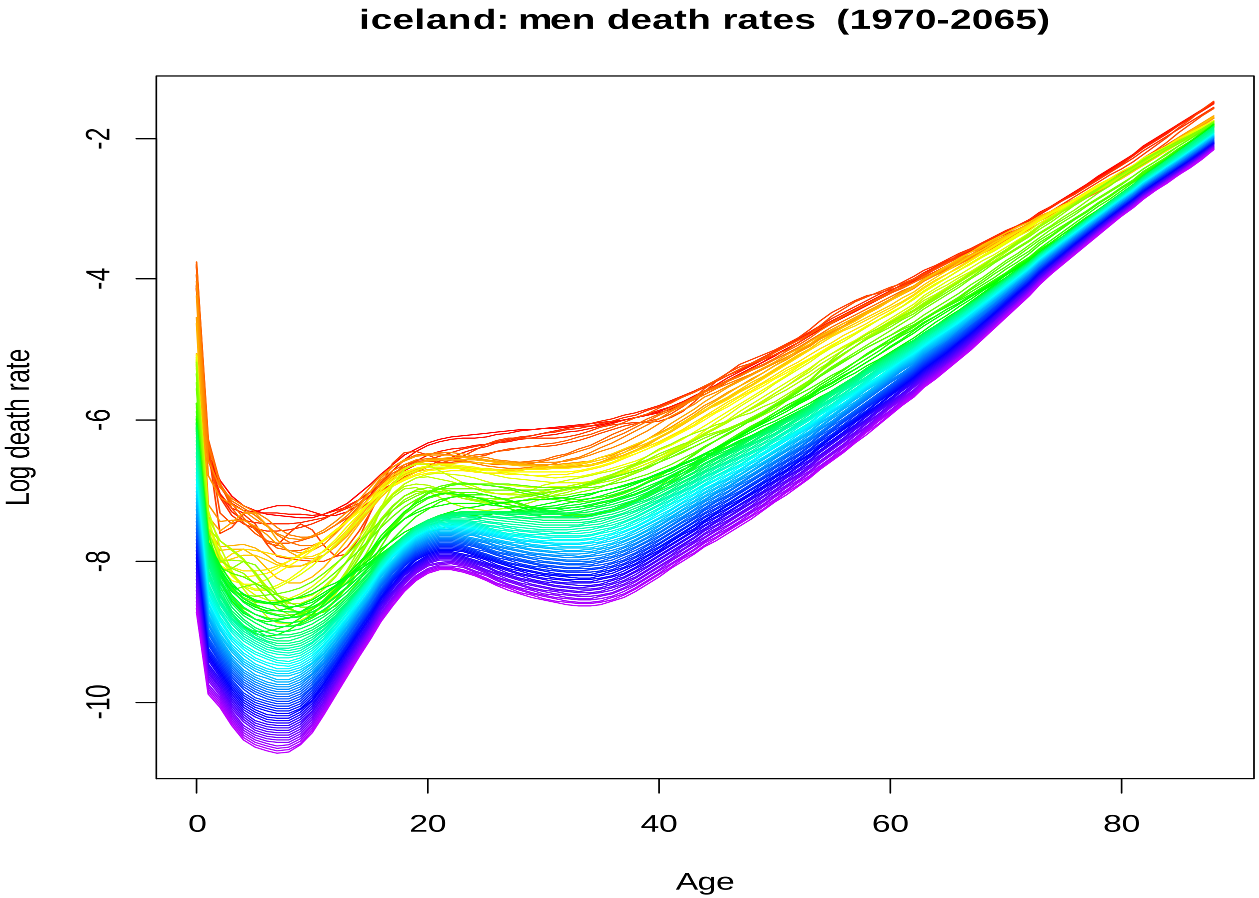
Raw data example and why extreme values matter



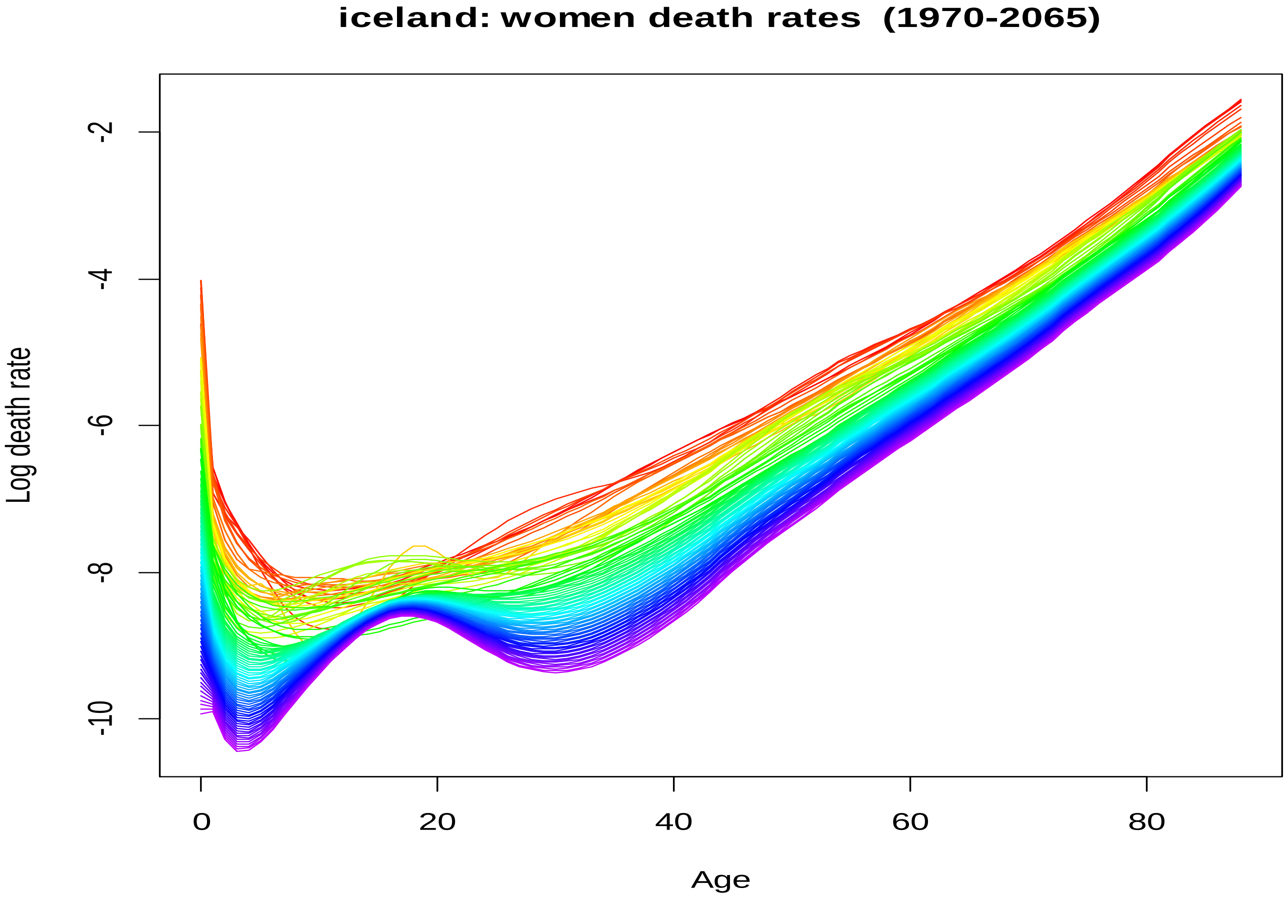
Smooth age-time mortality surface



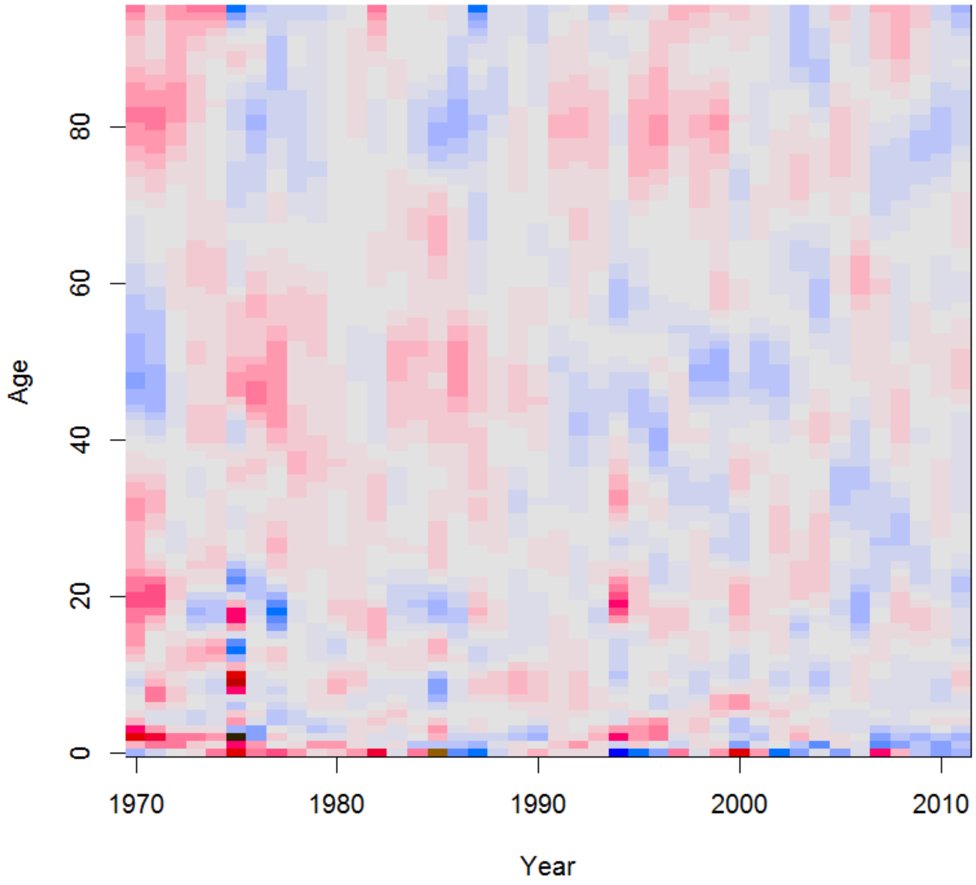
Mortality rates



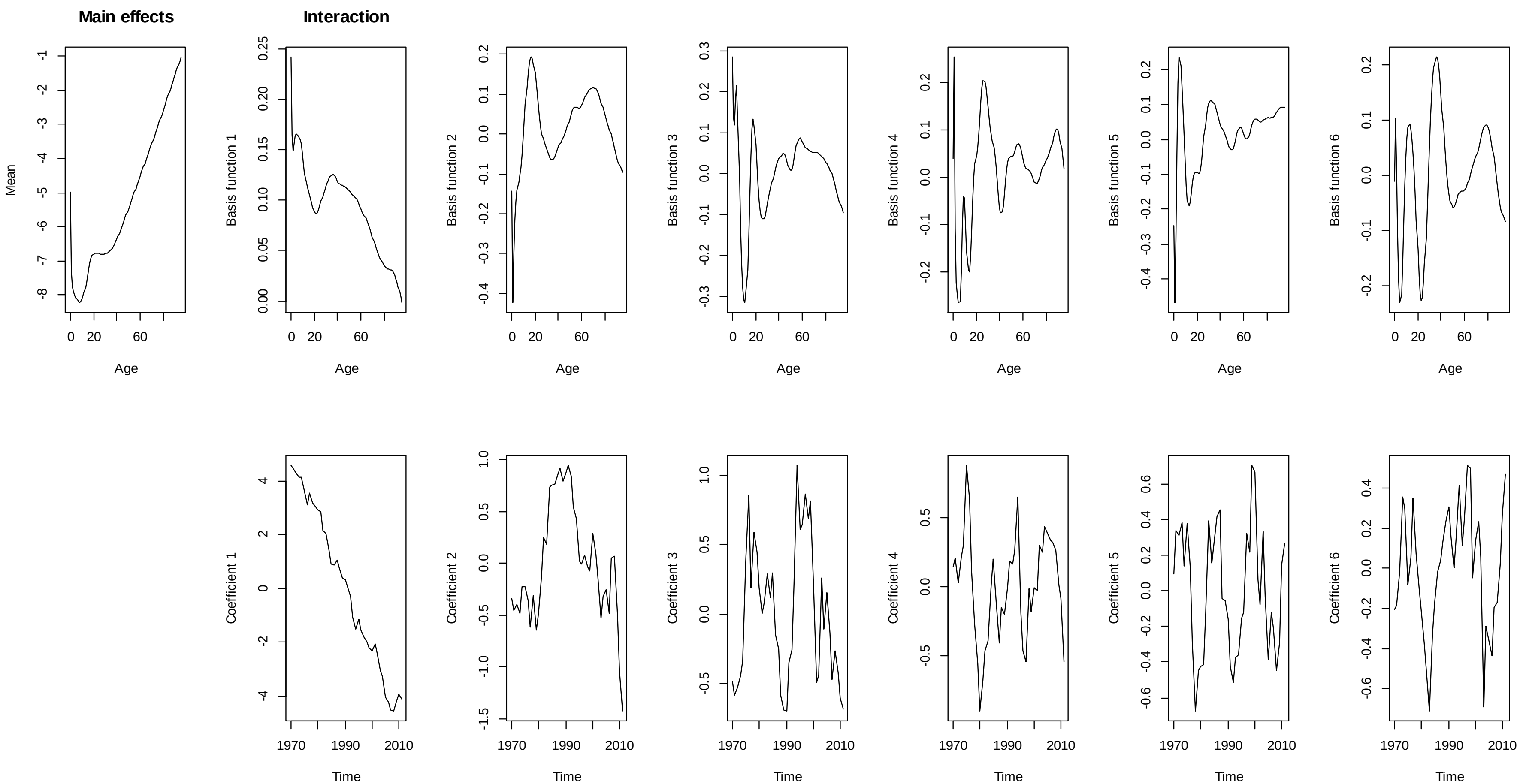
Mortality rates



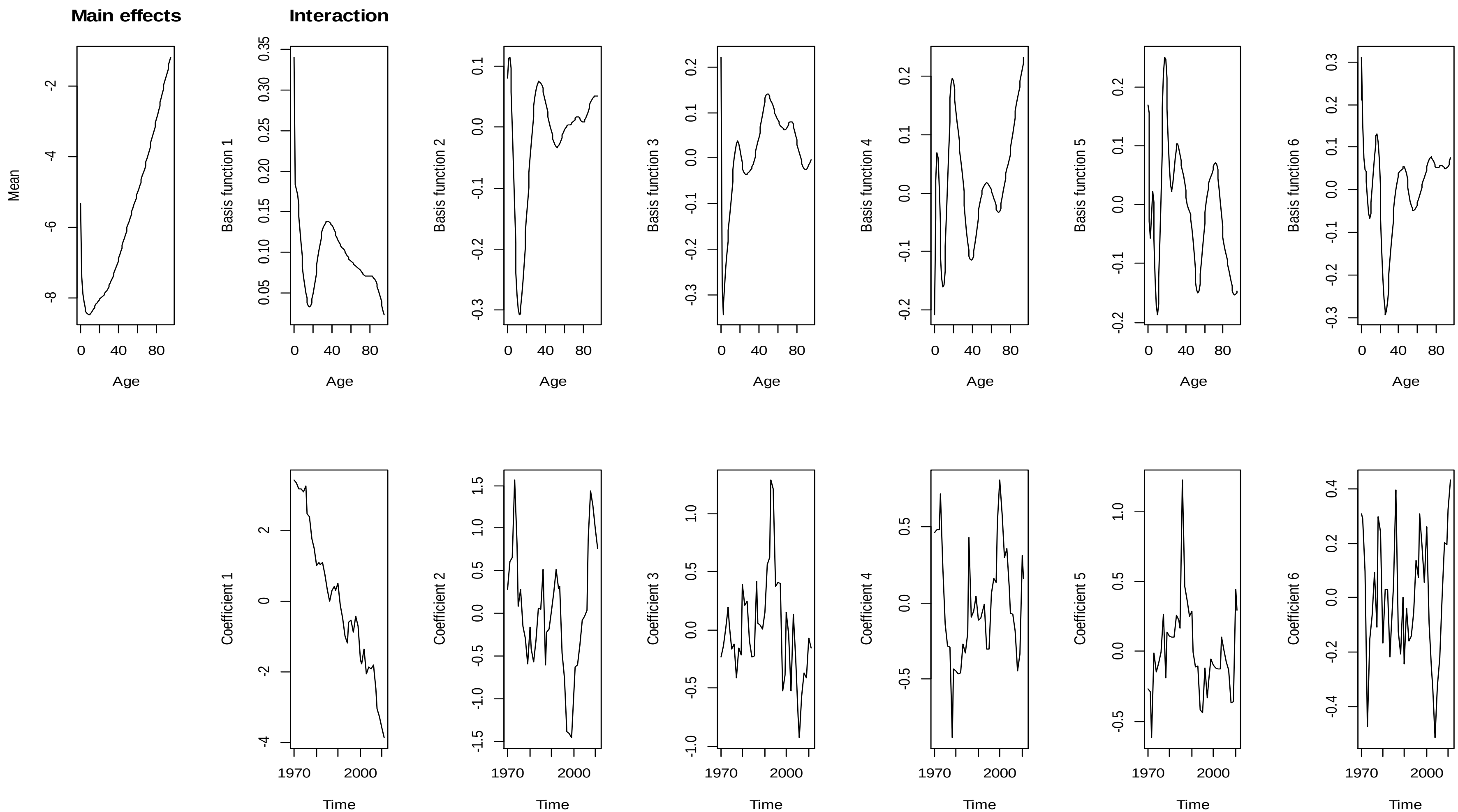
Residuals of mortality model



Functions of mortality model - men



Functions of mortality model - women



Migration models

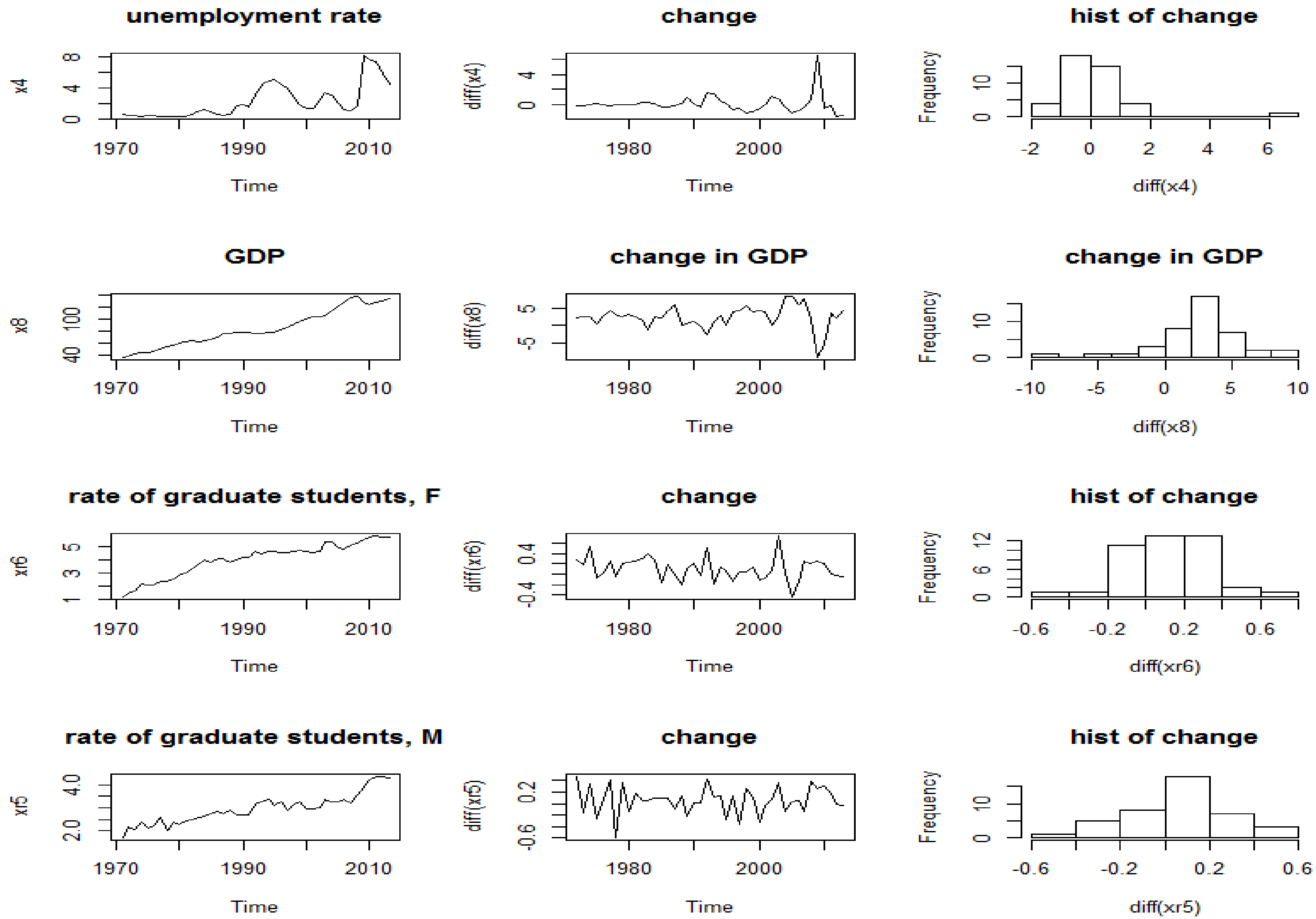
Vector ARDL for short term and *vector ARIMA for long term, i.e. (**S**)VAR*

Vector ARDL models $y(t) \sim y(t_k) + x(t_p)$

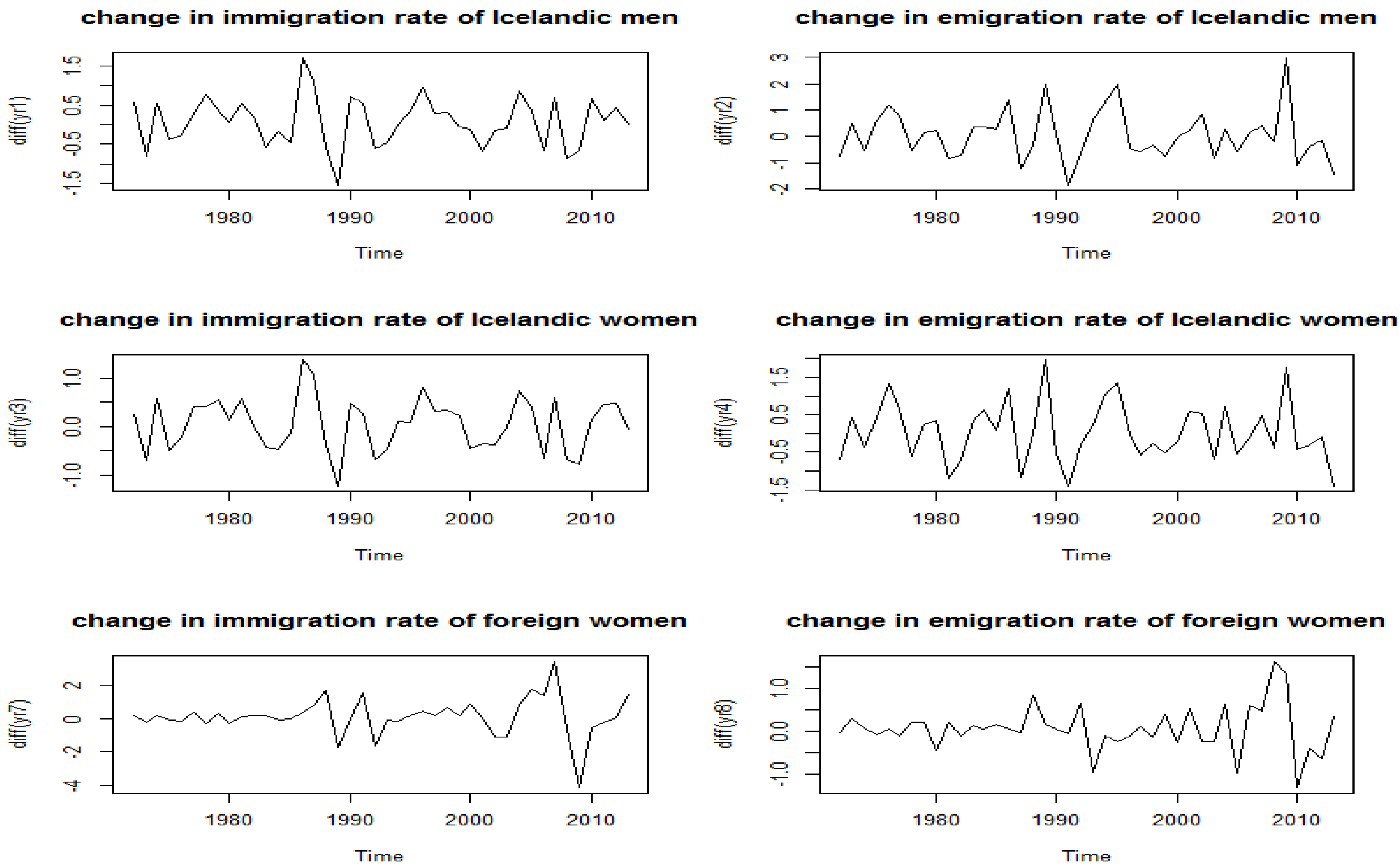
on components: $y_i(t) = \sum a_k y_k(t_k) + \sum b_p x_p(t_p) + \varepsilon(t)$

- $y_1 \sim ImIceM(t) \sim ImIceM(t-1) + UnEmpl(t) + UnEmpl(t-1) + EmIceM(t-1)$
- $y_2 \sim EmIceM(t) \sim EmIceM(t-1) + EmIceM(t-2) + GradM(t-2) + EmIceW(t)$
- $y_3 \sim ImIceW(t) \sim ImIceW(t-1) + UnEmpl(t) + GDP(t) + UnEmpl(t-1) + GDP(t-1)$
- $y_4 \sim EmIceW(t) \sim EmIceW(t-1) + EmIceW(t-2) + GradW(t-2)$
- $y_7 \sim ImForW(t) \sim ImForW(t-1) + UnEmpl(t) + GDP(t) + boom(t) + eea(t) + UnEmpl(t-1) + GDP(t-1)$
- $y_8 \sim EmForW(t) \sim EmForW(t-1) + ImForW(t) + ImForW(t-1) + UnEmpl(t) + GDP(t) + UnEmpl(t-1) + GDP(t-1)$
- $y_{net}(t) \sim y_{net}(t-1) + y_{net}(t-2) + UnEmpl(t) + GDP(t) + GDP(t-1) + UnEmpl(t-1) + GDP(t-2) + UnEmpl(t-2) + GradM(t-2) + GradF(t-2) + boom + bam + boom(t-1) + bam(t-1)$

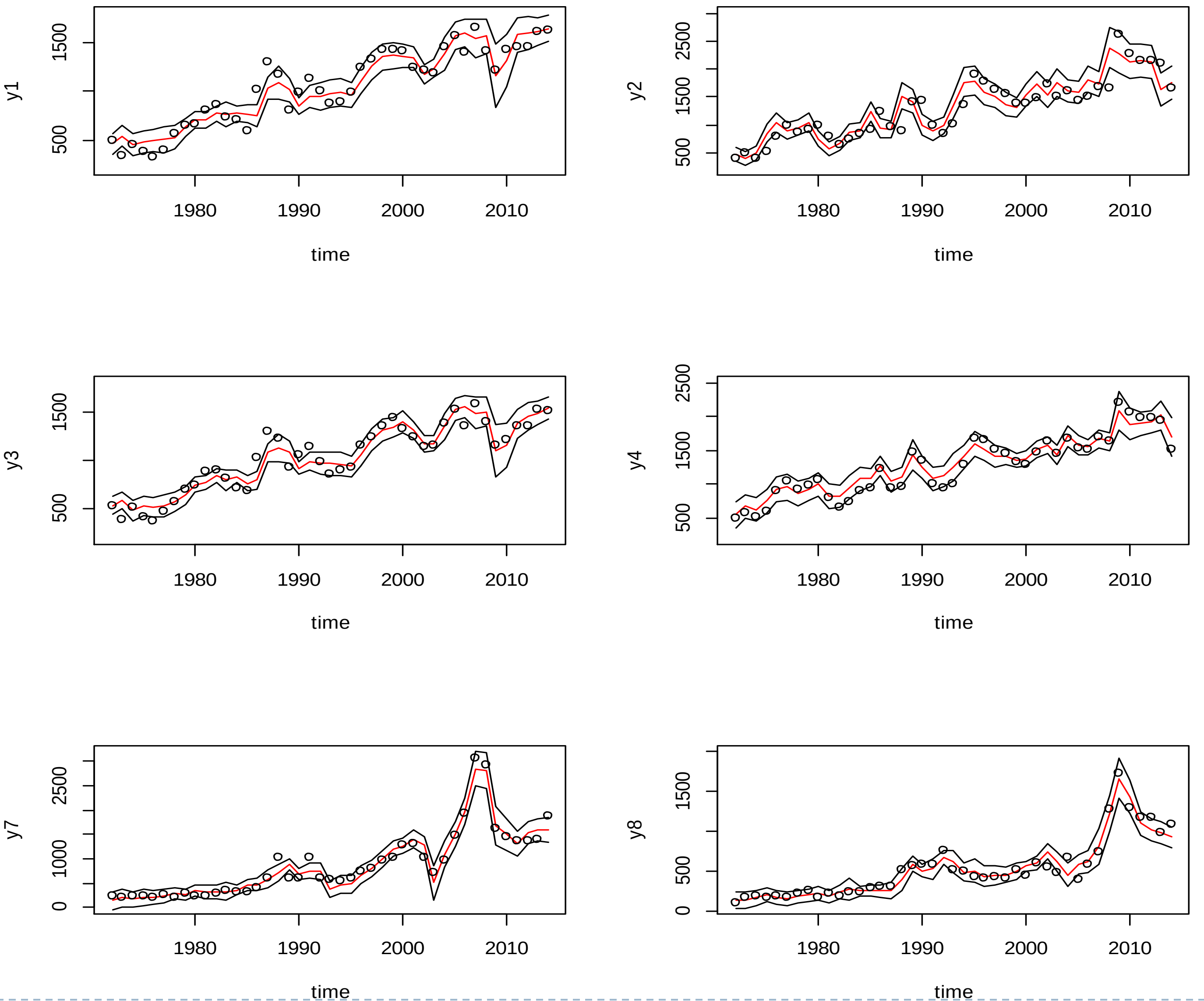
Factors


unemployment rate
x4
0
4
8
1970
1990
2010
Time
change
diff(x4)
0
4
1980
2000
Time
hist of change
Frequency
0
10
-2
0
2
4
6
diff(x4)
GDP
x8
40
100
1970
1990
2010
Time
change in GDP
diff(x8)
-5
5
1980
2000
Time
change in GDP
Frequency
0
10
-10
-5
0
5
10
diff(x8)
rate of graduate students, F
xr6
1
3
5
1970
1990
2010
Time
change
diff(xr6)
-0.4
0.4
1980
2000
Time
hist of change
Frequency
0
6
12
-0.6
-0.2
0.2
0.6
diff(xr6)
rate of graduate students, M
xr5
2.0
4.0
1970
1990
2010
Time
change
diff(xr5)
-0.6
0.2
1980
2000
Time
hist of change
Frequency
0
10
-0.6
-0.2
0.2
0.6
diff(xr5)

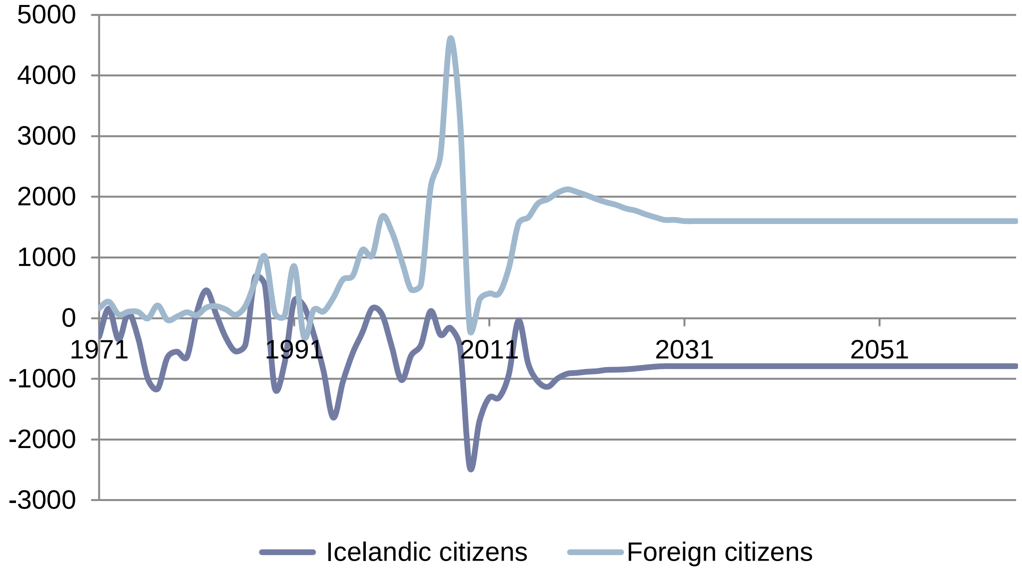
Migration rate components



Migration components modeled



Long term migration



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