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Item 9 – Stochastic methods in population projections

Measuring uncertainty in population forecasts: A new approach

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Abstract

Two basic approaches have been used to assess population forecast uncertainty: (1) a range of projections based on alternative scenarios; and (2) statistical forecast intervals. In terms of the latter, there are two complementary approaches: (1) model-based intervals; and (2) empirically-based intervals. We evaluate a model-based approach in this paper, but enhance it by using it the information in historical data, a feature found in the empirically-based approach. We describe and test in this paper a regression-based approach for developing 66% forecast intervals for age-group forecasts made using the Hamilton-Perry Method. We use a sample of four states (one from each census region in the United States) with nine ex post facto tests, one for each census from 1930 to 2010, which yields 576 observations. The four states and the nine test points provide a wide range of characteristics in regard to population size, growth, and age-composition, factors that affect forecast accuracy. The tests reveal that the 66% intervals contain the census age-groups in 397 of the 576 observations (69 percent). We discuss the results, to

include a summary by age group, and make some observations regarding the limitations of our study. We conclude that the results are encouraging, however, and offer suggestions for further work.

Introduction

Although they are widely used, population forecasts entail a tremendous amount of uncertainty, especially for long time horizons and for places with small or rapidly changing populations (Alho 1984; Alho and Spencer, 1984, 1985, 1990, 1997, 2005; Lutz, Sanderson and Scherbov 1999; Smith, Tayman and Swanson 2001: 340; Tayman, Smith, and Lin 2007; Tayman, Smith and Rayer 2011; Wilson 2012). As such, virtually every forecast is wrong, making the task of an accurate forecast impossible, but the task is unavoidable (Keyfitz 1987: 236). It is impossible in that the forecasted numbers turn out to be different from what actually occurs, but unavoidable in that forecasts must be done in the modern world. Swanson and Tayman (1995) describe this irony as the "rock" and the "hard place." As they observed, demographers have developed several strategies for dealing with the "irony" of forecasting. They include the use of the term "projection" rather than "forecast," (Keyfitz 1972; Pittenger 1978; Smith and Bayya 1992; Smith, Tayman, and Swanson 2001: 301), "normative" forecasting (Moen 1984), and providing measures of forecast uncertainty. One way to assess uncertainty is to produce several alternative projections or scenarios based on different sets of assumptions (Campbell 1996; Cheeseman-Day 1992; Spencer 1989; Tayman 2011; Thompson and Whelpton 1933; U. S. Census Bureau 1966). Another approach is to develop statistical forecast intervals based on historical data and stochastic models (Alho and Spencer 2005; Stoto 1983; Swanson and Beck 1994). It is the latter that we explore in this paper.

Forecast intervals based on statistical theory and data on error distributions provide an explicit estimate of the probability that a given range will contain the future population. These intervals are sometimes called prediction intervals, probability intervals, confidence intervals, or confidence limits. We call them forecast intervals to distinguish them from traditional confidence intervals, which—strictly speaking—apply only to sample data.

Two types of forecast intervals have been used most frequently for population forecasts. One is based on the development of statistical models of population growth and the other is based on empirical analyses of errors from past population projections. Both rely on the assumption that historical or simulated error distributions can be used to predict future error distributions. To a large extent, the two approaches complement one another, but neither is fully satisfactory. On the one hand, model-based intervals exploit the theories and underlying inferential statistics, but fall short in utilizing the information available in historical data. On the other hand, empirically-based intervals utilize the information from historical data, but fall short in exploiting the theories underlying inferential statistics. We evaluate a model-based approach in this paper, but enhance it by using it the information in historical data.

We begin with a discussion the model-based and empirically-based approaches. This is followed by a description of our model-based approach, which employs simple regression models applied to a forecasting method known as the Hamilton-Perry Method (Hamilton and Perry 1962).[1] We next evaluate the efficacy of the forecast intervals by examining population forecasts by age for four states covering target years for each decade from 1930 to 2010. We conclude with a discussion and suggestions for future research.

Model-Based Intervals

Model-based forecast intervals capitalize on the stochastic (or random) nature of population processes. They can be developed in a number of ways. Past applications have included maximum likelihood estimators of population growth rates (Cohen 1986), Monte Carlo simulations of fertility and migration rates (Pflaumer 1988), regression-based projection models (Swanson and Beck 1994), Bayesian projection models (Alkema et al. 2011), models based on the opinions of a group of experts (Lutz, Sanderson, and Scherbov 1999; San Diego County Water Authority 2002), and time series models covering mortality rates (Lee and Carter 1992), life expectancy (Torri and Vaupel 2012), fertility rates (Lee 1993), net migration (De Beer 1993), and total population size (Alders, Keilman, and Cruijsen 2007; Hyndman and Booth 2008). Although much of the research on model-based intervals has focused on national or regional projections, recent research has extended the analysis to subnational projections as well (Cameron and Poot 2011; Tayman et al. 2007; Wilson and Bell 2004).

Time series models (especially ARIMA models) are most commonly used for developing forecast intervals for population. These models assume that the pattern (structure) of the data does not change over time, that errors are normally distributed with a mean of zero and a constant variance, and that errors are independent of each other (Makridakis et al. 1987). Time series models require a fairly long series of historical observations and can be difficult to apply, especially when attempting to combine forecast intervals for all three components of growth and developing intervals for various subgroups of the population.

Providing a detailed description of model-based forecast intervals is beyond the scope of this paper, but we can give several examples of the intervals produced by these models and compare them to the high and low projection series produced using the traditional approach. Lee and Tuljapurkar (1994) projected a population of 398 million for the United States in 2065, with a 95% forecast interval of 259-609 million. This range is considerably wider than the spread between the low and high projections produced by the Census Bureau at about the same time; those projections ranged from 276-507 million in 2050, with a medium projection of 383 million (Cheeseman-Day 1992). The previous set of Census Bureau projections reported much lower numbers and a slightly smaller range, with a medium projection of 300 million and a range of 230-414 million for 2050 (Spencer 1989).

Pflaumer (1992) made two time series projections of the U.S. population, one based on population size and the other based on the natural logarithm of population size. The first model produced a medium projection of 402 million in 2050, with a 95% forecast interval of 277-527 million. These numbers are similar to the Census Bureau's projections from the same time. The second model produced a medium projection of 557 million, with a 95% forecast interval of 465-666 million. These numbers are much higher and provide a narrower range than the Census Bureau's projections.

McNown et al. (1995) made time series projections of the components of growth for the U.S. population, as well as total population size. For 2050, they projected a total population of 373 million, with a 95% forecast interval ranging from 243 million to 736 million. The total fertility rate was projected to be 2.46 in 2050, with a 95% forecast interval ranging from 0.91 to 5.53. Life expectancy at birth for males was projected to be 75.5, with a 95% forecast interval ranging from 68.5 to 82.8. For fertility these intervals are much larger than those found in the Census Bureau projections, which assumed that the total fertility rate would range only from 1.83 to 2.52 in 2050 (Cheeseman-Day 1992). For mortality the intervals are not much different than those reported by the Census Bureau, in which life expectancy at birth was projected to range between 75.3 and 87.6 in 2050.

Swanson and Beck (1994) developed a regression-based model for making short-term county population projections in the state of Washington. They compared the 2/3 forecast intervals associated with this model to census counts of Washington's 39 counties in 1970, 1980, and 1990. They found the forecast intervals to contain the 1970 census count in 30 counties (77%), the 1980 census count in 24 counties (62%), and the 1990 census count in 31 counties (79%). These results suggest that Swanson and Beck's 2/3 forecast intervals provided a reasonably accurate view of forecast uncertainty.

Model-based forecast intervals are valid only to the extent that the assumptions underlying the models are valid. In spite of their objective appearance, they are strongly influenced by the analyst's judgment. The models themselves are often complex and require a substantial amount of base data. They are subject to errors in the base data, errors in specifying the model, errors in estimating the model's parameters, and future structural changes invalidating the model's parameter estimates (Lee 1992). In addition, it is the case that many alternative forecasting models can be specified, each providing different (perhaps dramatically different) forecast intervals (Cohen 1986; Lee 1974; Tayman, Smith and Lin 2007).

In spite of these problems, model-based forecast intervals offer one important benefit: they provide explicit probability statements to accompany point forecasts. The intervals are often wide, exceeding the low and high projections produced by official statistical agencies. Given that many data users (and producers) tend to overestimate the accuracy of population projections, model-based forecast intervals provide an important reality check.

Empirically-Based Intervals

The second type of forecast interval is based on empirical analyses of errors from past projections (Keyfitz 1981; Smith and Sincich 1988; Stoto 1983; Smith and Rayer 2012; Tayman et al. 1998). Keyfitz (1981) took approximately 1,100 national projections made between 1939 and 1968 and calculated the difference between the projected annual growth rate and the rate actually occurring over time. He found this difference to be largely independent of the length of horizon over which the projections were made. He calculated the RMSE for the entire sample to be approximately 0.4 percentage points and developed 2/3 forecast intervals by applying that error to the growth rates projected for each country. For example, if a country were projected to grow by 2% per year for the next 20 years, the probability would be approximately 2/3 that the actual growth rate would be somewhere between 1.6% and 2.4%.

Keyfitz (1981) refined his analysis by separating countries according to their population growth rates, finding a RMSE of 0.60 for rapidly growing countries, 0.48 for moderately growing countries, and 0.29 for slowly growing countries. He illustrated this refinement by applying the 0.29 RMSE to the U.S. growth rate of 0.79% per year projected by the Census Bureau, yielding annual growth rates of 0.50% and 1.08%. Applying those growth rates to the 1980 population of 260 million produced a range of 245-275 million in 2000. He concluded that the odds were about 2 to 1 that this range would contain the U.S. population in that year.

Stoto (1983) followed a similar approach, but analyzed projections containing more temporal and geographic diversity. Like Keyfitz (1981), he calculated forecast error as the difference between the projected annual growth rate and the rate actually realized over time. He differentiated between two components of error, one related to the launch year of the projection and the other to seemingly random events (the residual). For more developed countries he found the launch-year component to have a distribution that was stable over time and centered on zero, implying that the projections were unbiased. For less developed countries he found the variance of the launch-year component to be stable, but that earlier sets of projections had a strong downward bias (although recent sets had little bias). The second component (the residual) was found to have a stable distribution but to have occasional outliers. For both components the variance was larger for less developed countries than more developed countries.

Stoto (1983) calculated the standard deviations for these two components of error and constructed forecast intervals in a manner similar to that used by Keyfitz (1981). He applied those intervals to projections of the U.S. population and estimated that there was about a 2/3 probability that an interval of 241-280 million would contain the actual population in 2000, and a 95% probability that an interval of 224-302 million would contain that population. He compared his results to projections produced by the Census Bureau, concluding that the Census Bureau's low and high series were very similar to a 2/3 forecast interval. Keyfitz (1981) had reached the same conclusion.

Smith and Sincich (1988) also used the distribution of past forecast errors to construct forecast intervals, but followed a different approach. They modified a technique developed by Williams and Goodman (1971), in which the predicted distribution of future forecast errors was based directly on the distribution of past forecast errors. An important characteristic of this technique is that it can accommodate any error distribution, including the asymmetric and truncated distributions typically found for absolute percent errors.

Using population data for states from 1900 to 1980, they used four simple extrapolation methods to make a series of projections covering 10- and 20-year horizons. They calculated absolute percent errors for each target year by comparing projections with census counts, focusing on the 90% intervals for each set of projections (i.e., the absolute percent error larger than exactly 90% of all absolute percent errors). They investigated two approaches to constructing 90% forecast intervals, one using the 90% interval from the previous set of projections and the other using the 90% interval from all other sets of projections. They found both approaches to provide relatively accurate forecast intervals. For most individual target years, 88%-94% of state forecast errors fell within the forecasted 90% interval. Summing over all target years, 91% of all forecast errors fell within the forecasted 90% interval. They concluded that stability in the distribution of absolute percent errors over time made it possible to construct useful forecast intervals for state projections.

Rayer, Smith, and Tayman (2009) constructed and tested forecast intervals for a large sample of counties in the United States using the Williams and Goodman approach. They constructed county forecasts covering 10-, 20-, and 30-year horizons and calculated forecast errors for target years covering decades from 1900 to 2000. Although the center of the algebraic error distributions shifted considerably from one decade to the next, their shape remained relatively constant over time. They evaluated the performance of 90% forecast intervals based on the distribution of absolute percent errors and found over all decades errors for 91% of the counties fell within the forecast intervals for all three horizons. Although there was some decade to decade variation, the proportion of errors falling within the intervals was usually between 88% and 93% and never varied by more than 10 percentage points.

Smith and Rayer (2012) used the Williams and Goodman approach to construct and test forecast intervals for county projections in Florida. Using forecast errors for target years 1985, 1990, and 1995, they constructed 2/3 forecast intervals for projections with launch years 1995, 2000, and 2005 and counted the number of counties in which the subsequent population counts or estimates fell within the forecasted intervals. They found that 43 counties (64%) fell within the forecasted range for 5-year horizons and 49 counties (73%) for both 10- and 15-year horizons. These numbers were fairly close to the 45 counties implied by the forecast intervals. Given the year-to-year volatility of Florida's population growth, this reflects a reasonably good forecasting performance.

Tayman et al. (1998) developed statistically-based forecast intervals for subcounty population forecasts in San Diego County. They started by projecting the population residing in 2,000 ft. by 2,000 ft. grid cells. These projections had 1980 as a launch year and 1990 as a target year. Using repeated sampling techniques and randomly selected grid cells, they developed projections for a large number of areas varying in size from 500 to 50,000. Forecast errors were calculated by comparing the 1990 projections with 1990 census counts.

Rather than constructing forecast intervals for the population forecasts per se, they developed forecast intervals for the mean errors implied by those forecasts. Empirical forecast intervals for MAPEs and MALPEs were developed using an approach similar to that used by Williams and Goodman (1971) and Smith and Sincich (1988). For areas with 500 persons, they found a 95% forecast interval of 67.4%-80.3% for the MAPE. For areas with 50,000 or more, the interval was 9.7%-11.5%. For MALPE, the intervals were wider but centered closer to zero.

The Hamilton-Perry Method

Before describing the Hamilton-Perry method, it is useful to recall that any quantitative approach to forecasting is constrained to satisfy various mathematical identities (Land 1986). In regard to population forecasting, an approach should ideally satisfy demographic accounting identities, which is summarized in the fundamental demographic equation:

$$P_t = P_0 + \text{Births} - \text{Deaths} + \text{Inmigrants} - \text{Outmigrants}. \qquad [1]$$

That is, the population at some time in the future, P_t, must be equal to the population at an earlier time, P_0, plus the births and in-migrants and less the deaths and out-migrants that occur between time *0* and time *t*. The most commonly used approach to population forecasting, cohort-component method, satisfies the fundamental equation, but it is data-intensive (George et al. 2004, Smith Tayman and Swanson 2001:159; Murdock and Ellis 1991, Pittenger 1976).

As we show at the end of this section, the Hamilton-Perry Method also satisfies the fundamental demographic equation. However, it has far less intensive input data requirements than does the cohort-component method. Instead of mortality, fertility, migration, and total population data, which are required by the full-blown cohort-component method, the Hamilton-Perry method requires data only from the two most recent censuses (Hamilton and Perry 1962; Smith, Tayman, and Swanson 2001: 153-158), Swanson Schlottmann, and Schmidt 2010, Swanson and Tedrow 2012).

The Hamilton-Perry method moves a population by age (and sex) from time *t* to time *t+k* using cohort-change ratios (CCR) computed from data in the two most recent censuses. It consists of two steps. The first uses existing data to develop CCRs and the second applies the CCRs to the cohorts of the launch year population to move them into the future. As shown by Swanson, Schlottmann, and Schmidt (2010), the formula for developing a CCR is:

$${}_nCCR_{x,i,t} = {}_nP_{x,i,t} \,/\, {}_nP_{x-k,i,t-k}. \qquad [2]$$

where,

${}_nP_{x,i,t}$ is the population aged x to $x+n$ in area i at the most recent census (t),

${}_nP_{x-k,i,t-k}$ is the population aged $x-k$ to $x-k+n$ in area i at the 2nd most recent census ($t-k$), and

k is the number of years between the most recent census at time t and the one preceding it at time t-k.

The basic formula for the second step, moving the cohorts of a population into the future is:

$${}_nP_{x+k,i,t+k} = {}_nCCR_{x,i,t} \times {}_nP_{x,i,t} \qquad [3]$$

where,

${}_nP_{x+k,i,t+k}$ is the population aged $x+k$ to $x+k+n$ in area i at time $t+k$, and

${}_nCCR_{x,i,t}$ and ${}_nP_{x,i,t}$ are as defined in equation [2].

Given the nature of the CCRs, 10-14 is the youngest five-year age group for which projections can be made if there are 10 years between censuses. To project the population aged 0-4 and 5-9 one can use the Child Woman Ratio (CWR) or more generally a “Child Adult Ratio” (CAR). These ratios do not require any data beyond what is available in the decennial census. For projecting the population aged 0-4, the CAR is defined as the population aged 0-4 divided by the population aged 20-34. For projecting the population aged 5-9, the CAR is defined as the population aged 5-9 divided by the population aged 25-39The CAR equations for projecting the population aged 0-4 and 5-9 are:

Population 0-4: $${}_5P_{0,t+k} = ({}_5P_{0,t} / {}_{15}P_{20,t}) \times {}_{15}P_{20,t+k} \qquad [4a]$$

Population 5-9: $${}_5P_{5,t+k} = ({}_5P_{5,t} / {}_{15}P_{25,t}) \times {}_{15}P_{25,t+k} \qquad [4b]$$

where

P is the population,

t is the year of the most recent census, and

$t+k$ is the projection year.

There are other “adult” age groups that could be used to define CAR (Smith, Tayman, and Swanson 2001: 156-157). The definitions shown in the two preceding equations are designed for a population in which fertility is at or below replacement, (i.e., the TFR is less than 2.1 or so), which correlates with the fact that first births tend to be postponed.

Another way to project the youngest age groups is to take their ratios at two points in time and apply that ratio to the launch year age group (*t*). In the first step, the ratios are as follows:

Population 0-4: ${}_5R_{0,t} = {}_5P_{0,t} / {}_5P_{0,t-k}$ [5a]

Population 5-9: ${}_5R_{5,t} = {}_5P_{5,t} / {}_5P_{5,t-k}$. [5b]

In the second step, the projected population at $t+k$ is found as follows:

Population 0-4: ${}_5P_{0,t+k} = {}_5P_{0,t} \times {}_5R_{0,t}$ [6a]

Population 5-9: ${}_5P_{5,t+k} = {}_5P_{5,t} \times {}_5R_{5,t}$. [6b]

We use the ratio method in this paper since it is better suited for the regression-based method for creating intervals around forecasts for the two youngest age groups discussed later in the paper. One reason that it is better suited with the regression-based method is that the CAR values are substantially different than the CCRs, whereas the ratios are not. This means that the CAR values are potential outliers that could serve as influential observations that deleteriously affect model construction (Fox 1991).

Projections of the oldest open-ended age group also differ slightly from the projections for the age groups beyond age 10 up to the oldest open-ended age group. If for example the final closed age group is 70-74, with 75+ as the terminal open-ended age group, then calculations for the $CCR_{i,x+,t}$ require the summation of the three oldest age groups to get the population age 65+ at time t-k:

${}_\infty CCR_{75,i,t} = {}_\infty P_{75,i,t} / {}_\infty P_{65,i,t-k}$. [7a]

The formula for projecting the population 75+ of area *i* for the year $t+k$ is:

$$_{\infty}P_{75+,i,t+k} = {}_{\infty}CCR_{75,i,t} \times {}_{\infty}P_{65,i,t}. \qquad [7b]$$

To show the Hamilton-Perry Method satisfies the fundamental demographic equation, we restate equation [2] using the terms in equation [1]:

$$P_{i,t+k} = P_{i,t} + B_i - D_i + I_i - O_i$$

where,

$P_{i,t}$ = Population of area i at time t (the launch year),

$P_{i,t+k}$ = Population of area i at time $t+k$ (the projection year),

B_i = Births in area i between time t and $t+k$,

D_i = Deaths in area i between time t and $t+k$,

I_i = In-migrants in area i between time t and $t+k$, and

O_i = Out-migrants in area i between time t and $t+k$,

then,

$$_{n}CCR_{x,i,t} = ({}_{n}P_{x-k,i,t-k} + B_i - D_i + I_i - O_i\ )/\ {}_{n}P_{x-k,i,t-k} \qquad [8]$$

Since we can also express equation [3] in terms of equation [1]:

$$_{n}P_{x+k,i,t+k} = (({}_{n}P_{x-k,i,t-k} + B_i - D_i + I_i - O_i) / ({}_{n}P_{x-k,i,t-k})) \times (\ {}_{n}P_{x,i,t}) \qquad [9]$$

where $x+k$ >= 10, then,

$$_{n}CCR_{x,i,t} = ({}_{n}P_{x-k,i,t-k} - D_i + I_i - O_i) / {}_{n}P_{x-k,i,t-k}, \text{ and}$$

since $N_i = I_i - Oi$, where $x+k \geq 10$, we have

$$_{n}CCR_{x,i,t} = ({}_{n}P_{x-k,i,t-k} - D_i + N_i) /({}_{n}P_{x-k,i,t-k}. \qquad [10]$$

Equations [8], [9], and [10] show that the Hamilton-Perry Method is not only consistent with the fundamental demographic equation, but also closely related to the cohort-component method. The Hamilton-Perry Method simply expresses the individual components of change—births, deaths, and migration—in terms of CCRs. As such, it satisfies the fundamental demographic equation. As we will see in the following section, this way of expressing the components of population change can be exploited. An important reason for a demographic forecasting method to be consistent with the fundamental demographic equation is to minimize the potential errors associated with hidden heterogeneity (Vaupel and Yaushin 1985).

Developing Forecast Intervals: A Regression Approach to Estimating CCRs

As should be clear from the preceding discussion, the Hamilton-Perry Method is deterministic. This is not surprising given its consistency with the fundamental demographic equation, which by its nature is an accounting method. However, we also know that population forecasting is subject to uncertainty since we do not precisely know the future components making up the fundamental equation. So, the question is how to introduce an element of statistical uncertainty into a method that is inherently deterministic. One answer to this question is found by employing a simple regression method to estimate CCRs and then applying the regression-estimated CCRs to the launch-year age groups to obtain forecasts of these age groups.

Recall from equation [2] that ${}_nCCR_{x,i,t} = {}_nP_{x,i,t} / {}_nP_{x-k,i,t-k}$. From this, we can define the CCR for the preceding census period as ${}_nCCR_{x,i,t-k} = {}_nP_{x,i,t-k} / {}_nP_{x-k,i,t-2k}$. We can then construct a regression model with ${}_nCCR_{x,i,t}$ as the dependent variable and ${}_nCCR_{x,i,t-k}$ as the independent variable. We note that for age groups 0-4, 5-9, and the terminal open-ended age group that the dependent and independent observations follow the equations provided earlier. Given this adjustment, we can generally describe the estimated CCRs at time t as follows:

$$ {}_nECCR_{x,i,t} = a + b \times {}_nCCR_{x,i,t-k}. \qquad [11] $$

We can then multiply the regression-estimated CCR and the corresponding population by age at time *t* to forecast the CCR at time *t+k*:

$$ {}_nCCR_{x,i,t+k} = {}_nECCR_{x,i,t} \times {}_nP_{x,i,t}. $$

Utilizing the regression measure of statistical uncertainty (the standard error of estimate) for the model along with the sample size and other characteristics of the data, we can generate forecast intervals around ${}_nCCR_{x,i,t+k}$. The forecast intervals are based on equation 4.2 found in Hyndman and Athanasopoulos, Chapter 4 (2012). These intervals can then be translated directly to the actual population numbers forecasted for each age group (Espenshade and Tayman 1982; Swanson and Beck 1994). Appendix 2 contains the derivation of the 2010 forecast intervals for Minnesota.[2]

Empirical Evaluation

To empirically examine the regression-based method for developing intervals around population forecasts by age generated from the Hamilton-Perry Method, we selected a sample made up of one state from each of the four census regions in the United States. The states selected are Georgia (the South Region), Minnesota (the Midwest Region), New Jersey (The Northeast Region) and Washington (The West Region). We then assembled census data for these four states for each census year from 1900 to 2010. The data provide nine points in time at which the forecast intervals can be evaluated, 1930, 1940, 1950, 1960, 1970, 1980, 1990, 2000, and 2010. This sample provides a wide range of demographic characteristics in terms of variation in population size, age-composition, and rates of change. Table 1 provides an overview of this range by displaying the population of each of the four states in 1900 and in 2010 and decennial rates of population change from 1900 to 2010. Although we do not show a summary of the changes in age composition by state and census year, they are extensive as seen in Appendix 1, which provide the age data by state and census year.

(Table 1 about here)

Because of the way data for the terminal open-ended age group are reported differently over the period for which we assemble census data, we used “75 years and over” for the entire period since it was the common denominator. This means there are 16 age groups (0-4, 5-9,…, 70-74, and 75+) used in the empirical evaluation.

We proceed by constructing CCRs over two successive decennial periods (e.g., 1910-1920/1900-1910) over the entire period, using regression to estimate the CCR in the numerator from the CCR in the denominator. We then use the regression-based estimate of the CCR of the “current period” (e.g., 1910-1920) to forecast the CCRs to the next period, the “launch year” (e.g., 1920-1930) and develop forecast intervals around the forecasted CCRs, which are then translated into the forecasted age groups for the “target year” (e.g., 1930). The forecast intervals are then examined to see if they contain the census age groups for the target year.

How well does the regression approach based on the Hamilton Perry method perform in its ability to predict the uncertainty of population forecasts? One way to address this question is to determine the number of population counts that fall inside the forecast intervals (Tayman, Smith, and Lin 2007). In terms of the forecast interval probability, we selected 0.66 or 66 percent because of prior research indicating that “low” and “high” scenarios constructed for the cohort-component method corresponded empirically to 66% confidence intervals (Stoto 1983) as well as findings by Swanson and Beck (1994). Table 2 provides a summary of the results for all four states at each of the nine census test points. The table shows the number of times (out of 16) that the 66% forecast interval contained the corresponding census number for a given age group. If the forecast intervals provide a valid measure of uncertainty, they will contain approximately 11 of the 16 observed population counts. The table also shows percent of the counts falling within the forecast intervals for all target years for each state (144 intervals), the percent falling within all states for each target year (64 intervals), and the single percent falling within all states for all target years (576 intervals).

(Table 2 about here)

In examining the state of Georgia (South Census Region), we find that its population increased by almost five-fold between 1900 and 2010. In 1900 it had the largest population of any of the four sample states it retained that position in 2010. Its annual average growth rates (by decade) ranged from 0.05% between 1920 and 1930 to 2.34% between 1990 and 2000. Changes in its age composition are extensive (See Appendix Table A1.1), with large impacts associated with the great depression, World War II, the baby boom, and immigration to the sunbelt states more recently. The 66 percent forecast intervals contain their corresponding age groups 76 times out of 144 observations, or 53 percent. Overall, Georgia has the lowest percent of census age groups within the 66 percent forecast intervals.

The population of Minnesota tripled from 1900 to 2010. Its average annual growth rates ranged from a low of 0.66% between 1940 and 1950 to a high of 1.70% between 1900 and 1910, a period when the state was still receiving a large number of immigrants from Europe. As is the case for Georgia, changes in its age composition are extensive (See Appendix Table A1.2), with big impacts associated with the restrictions placed on immigration by World War I and by the great depression, World War II, the baby boom, and outmigration to sunbelt states in more recent decades. The 66 percent forecast intervals contain their corresponding age groups 113 times out of 144 observations, or 78 percent. Overall, Minnesota has the highest percent of census age groups within the 66 percent forecast intervals.

For New Jersey, we see that its population grew from 1,879,890 in 1900 to 8,791,894 in 2010. New Jersey had the second highest population in 1900 and again in 2010. Its average annual growth rates ranged from a low of 0.27% between 1970 and 1980 to a high of 2.99% between 1900 and 1910. As is the case for Georgia and Minnesota, changes in its age composition are extensive (See Appendix Table A1.3) , with big impacts associated with the restrictions placed on immigration by World War I and the great depression, World War II, the baby boom, and outmigration to sunbelt states in more recent decades. The 66 percent forecast intervals contain their corresponding census age groups 106 times out of 144 observations, or 74 percent. Overall, New Jersey has the second-highest percent of census age groups within the 66 percent forecast interval.

In 1900, Washington was largely a frontier state. It had the smallest population (511,844) of any of the four states in the sample. However, by 2010, it had grown to 6,724,540 which surpassed the population of Minnesota in 2010. Its annual rates of population change are somewhat more dramatic than the other states between the 1900-1910 period and the 2000-2010 period. Between 1900 and 1910 it posted an annual rate of 7.97%, the highest of any of the decennial growth rates in the sample. It also posted the second highest rate. Between 1940 and 1950 the state grew at an annual rate of 3.14%. The lowest rate of annual population change (1.06%) is found between 1930 and 1940. The 66 percent forecast intervals contain their corresponding census age groups 102 times, which represents 71 percent of the 144 observations.

Discussion

Overall, the 66 percent intervals contain their corresponding census age groups in 397 cases, which represents 69 percent of the 576 total observations.[3] In terms of the nine census target years, the overall results show that in five of them (1960, 1970, 1990, 2000, and 2010) the forecast intervals contain the census age groups substantially more than 66 percent of the time. In two target years (1930 and 1980), the intervals contain the census age groups 67 percent of the time. In the remaining two target years, 1940 and 1950, the intervals contain the census age groups 48 percent and 47 percent of the time, respectively. We note that the 1940 test point encompasses the economic boom experienced in the 1920s and the economic depression during the 1930s and the large scale "baby bust" associated with it. The 1950 point encompasses the depression and baby bust period of the 1930s and the economic recovery stimulated by World War II and the initial part of the large scale "baby boom" from 1946 to 1950.

Table 3 contains a summary of the results by age group across all of the nine census target years and the four states. The table shows the number of times (out of 36) that the 66% forecast interval contained the corresponding census number for a given age group. If the forecast intervals provide a valid measure of uncertainty, they will contain approximately 24 of the 36 observed population counts. In general, Table 3 shows that forecast intervals capture the population count at least 66 percent of the time for age groups 10-14, 15-19, 20-24 and 40-44 through 75+. For age groups 0-4 and 5-9, the forecast intervals only encompass the population counts 25 percent of time. For age group 30-34, the count is encompassed 53 percent of the time while for age group 25-29, it is 58 percent of the time. The population counts are captured by the forecast intervals 61 percent of the time for age group 35-39.

Perhaps it should not be surprising that the cohort change method is better able to capture older age groups than the very youngest since births are not part of a cohort change ratio. In addition, migration likely comes into play in that the population in the two youngest age groups (0-4 and 5-9) would be moving with their parents, who are likely to be in age groups 25-29, 30-34, and 35-39, the other age groups for which the forecast intervals encompassed the population counts less than 66 percent of the time. Overall, we find that these effects are consistent with theory regarding migration in that those who tend to move are less socially integrated into communities than those who tend not to move and that as adults age, community social integration tends to increase (Goldscheider 1978). Finally, as shown at the bottom of Table 3, the intervals capture the population count 69 percent of the time (397 out of 576), which matches the summary for Table 2.

(Table 3 about here)

Although they are not shown here, the average width of the forecast intervals appears to us to be reasonable at the 66 percent level in that they are neither so wide as to be meaningless nor too narrow to be overly-restrictive. This is largely consistent with prior work by Swanson and Beck (1994) on confidence intervals derived from regression-based forecasts. Also consistent with the work by Swanson and Beck (1994), is the fact that the regression-based forecast intervals contain the actual numbers by age in 71 percent of the 576 observations provide further support that 66 percent forecast intervals based on the regression-estimated CCR approach are both useful and feasible. We find these results encouraging.

At this point, we suggest caution using this method beyond a ten-year forecast horizon. This is consistent with observations about the use of the Hamilton-Perry method in general (Smith, Tayman, and Swanson 2001; Swanson, Schlottmann, and Schmidt 2010) and as such is not a major limitation.[4] We also suggest that this approach to developing uncertainty measures be used with care when applied to small populations, such as those found at the county and sub-county levels. While our sample provides a wide range of demographic behavior in terms of size, age composition, and population changes, it is a sample of states, which means that greater variability in demographic characteristics found at sub-state levels is not present (Swanson, Schlottmann, and Schmidt 2010). We suggest that further research using this approach would be useful by examining both longer forecast horizons and smaller populations (i.e., the sub-state populations) and different probability intervals. Another area for further research would be to utilize Keyfitz's (1981) approach using root mean square errors in conjunction with the Hamilton-Perry Method.

The fact that the forecast intervals do not contain the population counts at least 66 percent of the time for neither the two youngest age groups (0-4 and 5-9) nor the age groups associated with those most likely to be the parents of these children (25-29, 30-34 and 35-39) should not be surprising: The dynamics of birth and migration are difficult to capture in a full-blown cohort-component method forecast and the Hamilton-Perry Method is a variant of the full- blown method (Smith, Tayman and Swanson 2001; Smith and Tayman 2003). Thus, work on these issues in regard to one of these two methods should be of use to the other.

Endnotes

1. Although the name “Hamilton-Perry Method” is virtually universal today, the first published instance of cohort change ratios being used for purposes of projecting a population is found in Hardy and Wyatt (1911), who built cohort change ratios from the 1901 and 1906 census counts of England and applied them to the 1906 census to generate a forecast for 1911. Hamilton and Perry acknowledge that they learned about this method from a general description found in Wolfenden (1954) who cited the Hardy and Wyatt article. However, they were unable to secure a copy of the 1911 article and were, therefore, not exactly certain what was done by Hardy and Wyatt. In any event, Hamilton and Perry deserve credit for providing a clear and detailed description of this approach to population projection in a journal (*Social Forces*) that was read by many demographers in the United States and elsewhere prior to the founding of demographic journals such as *Canadian Studies in Population* (first published in 1973) *Demography* (first published in 1966) and *Population Research and Policy Review* (first published in 1982).

2. Space considerations prevent us all of the regression, forecast intervals, and evaluation results here.. The authors will be pleased to provide them upon request.

3. It should be clear that we are primarily interested in measuring uncertainty in forecasts of age groups. This is an important topic due to the role that the absolute and relative sizes of age groups have in regard both to commerce (Gauthier, Chu and Tuljapurkar 2006; Martins Yusuf and Swanson 2012, Murdock et al. 1997) and public policy (Bongaarts and Bulatao 2000, Murdock et al. 1997, Smith Tayman and Swanson 2001, Tuljapurkar, Pool, and Prachuabmoh 2005, Gauthier, Chu and Tuljapurkar 2006). We are aware that levels of uncertainty in regard to forecasts of the total population are important as well. In this regard, we note that technically the forecast intervals we generated here apply only to the age groups. There are two ways in which

they can be used to place intervals around the total population forecast, one is informal while the other is formal. In the informal approach, we obtain 66 percent forecast intervals for the total population by adding the lower and upper boundaries of the intervals for each age group. We found that in 28 of the 36 forecasts (four states at each of nine time points) the summed lower and upper boundaries contained the actual total population, or 78 percent. By state, we find: Georgia's total population is contained in 5 of the 9 time points (56%); Minnesota's is in 9 of the 9 time points; New Jersey's is in 6 of the 9 time points (67%); and Washington's is in 8 of the 9 time points (89%). By target year, we find: 4 of 4 were contained in the 1960, 1970, and 1990 years; 3 of 4 were contained in the 1930, 1980, 2000, and 2010 years; and 2 of 4 in the 1940 and 1950 years.

The formal approach is called the "error propagation method" by Deming (1950: 127-134). In different forms it has been used by Alho and Spencer (2005), Espenshade and Tayman (1982), and Hansen, Hurwitz, and Madow (1953), among others. In this application, the error propagation method involves summing the squared values of the forecast intervals by age, finding the square root of the summed forecast interval values and dividing this square root of the sample size (n=16) to obtain an estimate of the standard error for the total population forecast. This standard error is then multiplied by the total population forecast (found by summing the point forecast for each age group) to obtain the margin of error. The margin of error is added to and subtracted from the total population forecast to obtain its 66% forecast interval. . This approach assumes that the 16 age groups are independent, which is not an unreasonable assumption in that the age group forecasts are not forced to sum to any specified total (i.e., they are not "controlled" to an externally produced population total). In following this approach, we found that in 29 of the 36 forecasts (four states at each of nine time points) the error propagation intervals contained the actual total population, or 81 percent. By state, we find: Georgia's total population is contained in 6 of the 9 time points (67%); Minnesota's is in 9 of the 9 time points; New Jersey's is in 6 of the 9 (67%), and Washington's is in 8 of the 9 time points (89%). By time point, we find: 4 of 4 were contained in the 1960, 1970, 1990, and 2010 target years; 3 of 4 were contained in the 1930, 1980, and 2000 target years; and 2 of 4 in the 1940 and 1950 target years.

Both the informal and formal approaches can be used to construct forecast intervals for any desired aggregations of the five-year age groups such age group 25-34, the working age population (e.g., ages 25-64), and so forth.

4. The ten-year horizon is also consistent with accuracy evaluations of the Hamilton-Perry Method, which show that the method performs well for ten year forecasts (Smith and Tayman 2003, Swanson and Tayman, 2013) and even 20 year forecasts (Smith and Tayman 2003).

References

Alders, M., Keilman, N., and Cruijsen, H. (2007). Assumptions for long-term stochastic population forecasts in 18 European countries. *European Journal of Population* 23: 33-69.

Alho, J. (1984). Probabilistic forecasts: The case of population projections. *Scandinavian Housing and Planning Research* 1: 99-105.

Alho, J. and Spencer, B. (1985). Uncertain population forecasting. *Journal of the American Statistical Association* 80 (390): 306-314.

Alho, J. and Spencer, B. (1990). Error models for official mortality forecasts. *Journal of the American Statistical Associatio*n 85 (411): 609-616.

Alho, J. and Spencer, B. (1997). The practical specification of the expected error of population forecasts. *Journal of Official Statistics* 13: 203-225.

Alho, J. and Spencer, B. (2005). *Statistical demography and forecasting*. Dordrecht, The Netherlands: Springer.

Alkema, L., Raftery, A.E., Gerland, P., Clark, S.J., Pelletier, F., Buettner T., and Heilig, G.K (2011). Probabilistic projections of the total fertility rate for all countries. *Demography* 48: 815-839.

Bongaarts, J. and Bulatao, R (Eds.). (2000). *Beyond six billion: forecasting the world's population*. Washington, D.C.: National Academy Press

Cameron, M.P. and Poot, J. (2011). Lessons from stochastic small-area population projections: The case of Waikato subregions in New Zealand. *Journal of Population Research* 28: 245-265.

Campbell, P.R. (1996). Population projections for states by age, sex, race, and Hispanic Origin: 1995 to 2050. PPL-47. Washington, DC: U.S. Bureau of the Census.

Cheeseman-Day, J. (1992). Population Projections of the United States, By Age, Sex, Race, and Hispanic Origin: 1992-2050. Current Population Reports P-25, No. 1092. Washington, D.C.: U.S. Bureau of the Census.

Cochran, W. (1977). *Sampling Techniques, 3rd Edition*. Wiley: New York.

Cohen, J. (1986). Population forecasts and confidence intervals for Sweden: A comparison of model-based and empirical approaches. *Demography* 23 (1): 105-123.

De Beer, J. (1993). Forecast intervals of net migration: The case of the Netherlands. *Journal of Forecasting* 12: 585-599.

Deming, W.E. (1950). *Some theory of sampling*. New York, NY: Dover Publications.

Espenshade, T. and Tayman, J. (1982). Confidence intervals for postcensal population estimates. *Demography* 19 (2): 191-210.

Fox, J. (1991). *Regression Diagnostics*. Sage Quantitative Applications in the Social Sciences, no. 79. London, England: Sage Publications.

Gauthier, A., Chu, C.Y.C. and Tuljapurkar, S. (Eds.) (2006). *Allocating public and private resources across generations: Riding the age waves, Vol. 2*. Dordrecht, The Netherlands: Springer.

George, M.V., Swanson ,D.A., and Tayman, J. (2004). Population projections. pp. 561-601 in J. Siegel and D.A. Swanson (eds.) *The Methods and Materials of Demography, 2nd Edition*. New York, NY: Elsevier Academic Press.

Goldscheider, C. (1978). *Modernization, migration, and urbanization*. Paris, France: International Union for the Scientific Study of Population.

Hamilton, C.H. and Perry, J. (1962). A short method for projecting population by age from one decennial census to another. Social Forces, 41: 163-170.

Hansen, N., Hurwitz, W., and Madow, W. (1953). *Sample survey methods and theory, Volume I, methods and applications*. New York, NY: John Wiley and Sons (re-published in 1993).

Hardy, G.F. and Wyatt, F.B. (1911). Report of the actuaries in relation to the scheme of insurance against sickness, disablement, etc. *Journal of the Institute of Actuaries* XLV: 406-443.

Hyndman, R.J., and Athanasopoulos, G. (2012) *Forecasting: Principles and Practice* (online at http://otexts.com/fpp/)

Hyndman, R.J. and Booth, H. (2008). Stochastic population forecasts using functional data models for mortality, fertility and migration. *International Journal of Forecasting* 24: 323-342.

Keyfitz, N. (1972). On future population. *Journal of the American Statistical Association* 67: 347-363.

Keyfitz, N. (1981). The limits of population forecasting. *Population and Development Review* 7: 579-593.

Keyfitz, N. (1987). The social and political context of population forecasting. pp. 235-258 in W. Alonso and P. Starr (Eds.) *The Politics of Numbers*. New York, NY: Russell Sage Foundation.

Land, K. (1986). Methods for national population forecasts: A review. *Journal of the American Statistical Association* 81: 888-901.

Lee, R. (1974). Forecasting births in post-transition populations: Stochastic renewal with serially correlated fertility. *Journal of the American Statistical Association* 69: 607-617.

Lee, R. (1992). Stochastic demographic forecasting. *International Journal of Forecasting* 8: 315-327.

Lee, R. (1993). Modeling and forecasting the time series of U.S. fertility: Age distribution, range, and ultimate level. *International Journal of Forecasting* 9: 187-212.

Lee, R. and Carter, L. (1992). Modeling and forecasting U.S. mortality. *Journal of the American Statistical Association* 87: 659-675.

Lee, R. and Tuljapurkar ,S.. (1994). Stochastic population projections for the United States: Beyond high, medium, and low. *Journal of the American Statistical Association* 89: 1175-1189.

Lutz, W., Sanderson, W., and Scherbov, S. (1999). Expert-based probabilistic population projections. pp. 139-155 in W. Lutz, J. Vaupel, and D. Ahlburg (Eds.) *Frontiers of Population Forecasting*. New York, NY: The Population Council. (A supplement to *Population and Development Review*, 24).

Makridakis, S., Hibon, M., Lusk, E., and Belhadjali, M. (1987). Confidence intervals: An empirical investigation of the series in the M-competition. *International Journal of Forecasting* 3: 489-508.

Martins, J., Yusuf, F., and Swanson, D.A. (2012). *Consumer demographics and behaviour: markets are people*. Dordrecht, The Netherlands: Springer.

McNown, R., Rogers, A., and Little, J. (1995). Simplicity and complexity in extrapolative population forecasting models. *Mathematical Population Studies* 5: 235-257.

Moen, E. (1984). Voodoo Forecasting: Technical, political, and ethical issues regarding the projection of local population growth. *Population Research and Policy Review* 3: 1-25.

Murdock, S. and Ellis, D. (1991). *Applied demography: An introduction to basic concepts, methods, and data*. Denver, Colorado: Westview Press.

Murdock, S., Hoque, M.N., Michael, M., White, S., and Pecotte, B. (1997). *The Texas challenge: population change and the future of Texas*. College Station, TX: Texas A&M University Press.

Pflaumer, P. (1988). Confidence intervals for population projections based on Monte Carlo methods. *International Journal of Forecasting* 4: 135-142.

Pflaumer, P. (1992). Forecasting U.S. population totals with the Box-Jenkins approach. *International Journal of Forecasting* 8: 329-338.

Pittenger, D. (1976). *Projecting state and local populations*. Cambridge, MA: Ballinger.

Pittenger, D. (1978). The role of judgment, assumptions, techniques, and confidence limits in forecasting population. *Socio-Economic Planning Sciences* 12: 271-276.

Rayer, S., Smith, S.K., and Tayman, J. (2009). Empirical prediction intervals for county population forecasts. *Population Research and Policy Review* 28, 773-793.

San Diego County Water Authority. 2002. Regional water facilities master plan. Appendix C: Development of probabilistic water demand forecasts. San Diego, CA: San Diego County Water Authority.

Smith, S.K. and Bayya, R. (1992). An evaluation of population forecast errors for Florida and its counties. *Applied Demography* 7: 1-5.

Smith, S.K. and Rayer. S. (2012). An evaluation of population forecast errors for Florida and its counties, 1980-2010. Paper presented at the Conference on Applied Demography, San Antonio, TX.

Smith, S.K. and Sincich. T. (1988). Stability over time in the distribution of population forecast errors. *Demography* 25: 461-474.

Smith, S.K., and Tayman, J. (2003). An evaluation of population projections by age. *Demography* 40 (4): 741-757.

Smith, S.K., Tayman, J., and Swanson D.A. 2001. *Population projections for state and local areas: Methodology and analysis*. New York, NY: Kluwer Academic/Plenum Press.

Spencer, G. 1989. Projection of the population of the United States by age, sex, and race: 1988 to 2008. Current Population Reports, P-25, No. 1018. Washington, DC: U.S. Bureau of the Census.

Stoto, M. (1983). The accuracy of population projections. *Journal of the American Statistical Association* 78: 13-20.

Swanson, D.A. and Beck, D. (1994). A new short-term county population projection method. *Journal of Economic and Social Measurement* 20: 1-26.

Swanson, D.A. and Tayman, J. (2013).The accuracy of the Hamilton-Perry method for forecasting state populations by age. Working paper 13-01, Center for Sustainable Suburban Development, University of California Riverside (http://cssd.ucr.edu/Papers/PDFs/THE%20ACCURACY%20OF%20THE%20HAMILTON-PERRY%20METHOD%20FOR%20FORECASTING%20STATE%20POPULATIONS%20BY%20AGE%20V6.pdf

Swanson, D.A. and Tayman, J. (1995). Between a rock and a hard place: The evaluation of demographic forecasts. *Population Research and Policy Review* 14: 233-249.

Swanson, D.A. and Tedrow, L. (2012). Using cohort change ratios to estimate life expectancy in populations with negligible migration: A new approach. *Canadian Studies in Population* 39: 83-90.

Swanson, D.A., Tayman, J., and Beck, D. (1995). On the utility of lagged ratio-correlation as a short-term population projection method: A case study of Washington State. *Journal of Economic and Social Measurement* 21 (1): 1-16.

Swanson, D.A., Schlottmann, A., and Schmidt, R. (2010). Forecasting the population of census tracts by age and sex: An example of the Hamilton–Perry method in action. *Population Research and Policy Review* 29: 47-63.

Tayman, J. (2011). Assessing uncertainty in small area forecasts: State of the practice and implementation strategy. *Population Research and Policy Review* 30:781-800.

Tayman, J., Schaefer, E., and Carter, L. (1998). The role of population size in the determination and prediction of population forecast errors: An evaluation using confidence intervals for subcounty Areas. *Population Research and Policy Review* 17 (1): 1-20.

Tayman, J., Smith, S.K., and Lin, J. (2007). Precision, bias, and uncertainty for state population forecasts: An exploratory analysis of time series models. *Population Research and Policy Review* 26 (3): 347-369.

Tayman, J., Smith, S.K., and Rayer, S. (2011). Evaluating population forecast accuracy: A regression approach using county data. *Population Research and Policy Review* 30: 235-262.

Thompson, W. and Whelpton, P. (1933). *Population Trends in the United States*. New York, NY: McGraw-Hill Book Company.

Torri, T. and Vaupel, J.W. (2012). Forecasting life expectancy in an international context. *International Journal of Forecasting* 28: 519-531.

Tuljapurkar, S., Pool, I. and Prachuabmoh, V. (Eds.) (2005). *Population, resources and development: Riding the age waves, Vol. 1*. Dordrecht, The Netherlands: Springer.

U.S. Census Bureau. (1966). Illustrative projections of the population of states: 1970 to 1985. Current Population Reports, P-25, No. 326. Washington, DC.

U.S. Census Bureau. (1982). 1980 Census of Population, General Population Characteristics, (by State). Washington, DC: Government Printing Office.

U. S. Census Bureau. (1992). 1990 Census of Population, General Population Characteristics (by State). Washington, DC: Government Printing Office.

U.S. Census Bureau (1973). 1970 Census of Population, General Population Characteristics (by State). Washington, D.C., Government Printing Office.

Vaupel, J. and Yaushin, A. (1985). Heterogeneity's ruses: Some surprising effects of selection on population dynamics. *The American Statistician* 39 (August): 176-185.

Williams, W., and Goodman, M. (1971). A simple method for the construction of empirical confidence limits for economic forecasts. *Journal of the American Statistical Association* 66: 752-754.

Wilson, T. (2012). Forecast accuracy and uncertainty of Australian Bureau of Statistics state and territory population projections. *International Journal of Population Research*. Article ID 419824, 16 pages, 2012. doi:10.1155/2012/419824

Wilson, T. and Bell, M. (2004). Probabilistic regional population forecasts: The example of Queensland, Australia. *Geographical Analysis* 39: 1-25.

Wolfenden, H.H. (1954). *Population statistics and their compilation, Revised Edition*. Chicago, IL: University of Chicago Press.

Table 1. Total Population 1900 and 2010 and
Annual Rate of Change by Decade, Sample States

Census Year	Georgia	Minnesota	New Jersey	Washington
1900[a]	2,209,974	1,747,292	1,879,890	511,844
1900-1910	1.64%	1.70%	2.99%	7.97%
1910-1920	1.05%	1.41%	2.19%	1.75%
1920-1930	0.05%	0.72%	2.47%	1.44%
1930-1940	0.72%	0.86%	0.30%	1.06%
1940-1950	0.98%	0.66%	1.50%	3.14%
1950-1960	1.35%	1.35%	2.27%	1.83%
1960-1970	1.52%	1.08%	1.67%	1.78%
1970-1980	1.74%	0.69%	0.27%	1.92%
1980-1990	1.70%	0.71%	0.48%	1.64%
1990-2000	2.34%	1.17%	0.85%	1.92%
2000-2010	1.68%	0.75%	0.44%	1.32%
2010	9,687,653	5,303,925	8,791,894	6,724,540

[a] The 1900 population totals exclude those for whom age was not reported.

Table 2. Number of Population Counts Falling within the 66% Forecast Intervals by State and Target Year

Target Year	Georgia	Minnesota	New Jersey	Washington	Total	Percent (N/64)
1930	9	12	8	13	42	67%
1940	3	5	11	12	31	48%
1950	10	14	4	3	31	47%
1960	13	14	14	8	49	86%
1970	6	12	14	13	45	77%
1980	7	12	12	10	41	67%
1990	13	14	14	14	55	83%
2000	8	15	14	15	52	81%
2010	7	15	15	14	51	81%
Total	76	113	106	102	397	
Percent	53%	78%	74%	71%	69%	
	Percent (N/144)	Percent (N/144)	Percent (N/144)	Percent (N/144)	Percent (N/576)	

Table 3. Number of Population Counts Falling within the 66% Forecast Interval by Age Group

Age	Number	Percent (N/36)
0 to 4	9	25%
5 to 9	9	25%
10 to 14	26	72%
15 to 19	27	75%
20 to 24	24	67%
25 to 29	21	58%
30 to 34	19	53%
35 to 39	22	61%
40 to 44	26	72%
45 to 49	28	78%
50 to 54	30	83%
55 to 59	31	86%
60 to 64	30	83%
65 to 69	31	86%
70 to 74	33	92%
75+	31	86%
Total	397	69%

Appendix 1
Population by Age, 1900 to 2010 by Decade

Table A1.1 Population by Age, 1900 to 2010, Georgia

Age	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000	2010
0 to 4	325,473	376,641	363,229	316,404	313,122	422,486	471,901	421,709	414,935	495,535	595,150	686,785
5 to 9	313,524	347,369	382,373	353,910	319,056	355,208	440,198	470,311	446,831	483,952	615,584	695,161
10 to 14	277,865	315,217	365,312	338,860	325,009	311,293	411,650	480,924	469,598	466,614	607,759	689,684
15 to 19	241,478	280,383	307,549	334,836	328,410	291,806	331,554	442,571	530,773	497,152	596,277	709,999
20 to 24	229,199	260,140	272,814	288,126	304,638	276,193	271,211	416,949	516,084	522,634	592,196	680,080
25 to 29	172,819	214,250	230,373	222,930	277,500	276,270	251,770	330,790	481,276	589,952	641,750	673,935
30 to 34	127,782	169,314	180,749	183,399	236,138	255,385	256,351	273,995	448,765	584,944	657,506	661,625
35 to 39	111,711	152,232	185,500	186,959	209,545	254,264	260,063	256,934	356,263	531,619	698,735	698,059
40 to 44	97,256	109,644	140,477	151,156	174,120	219,640	244,981	260,140	291,069	484,079	654,773	699,481
45 to 49	78,565	85,850	125,849	133,154	156,489	182,855	229,397	252,278	266,793	374,918	573,017	722,661
50 to 54	78,307	96,240	106,175	131,455	134,244	153,118	196,204	232,825	261,211	294,033	506,975	668,591
55 to 59	46,756	61,442	66,256	84,633	102,773	126,309	161,507	207,126	246,907	259,735	375,651	573,551
60 to 64	42,863	55,526	64,125	67,562	83,965	100,096	125,668	175,565	215,869	238,779	285,805	496,006
65 to 69	27,942	35,469	44,269	45,142	75,095	95,556	113,144	137,744	188,897	218,078	236,634	356,007
70 to 74	18,887	21,911	29,550	33,738	42,732	60,606	81,647	97,362	141,977	169,973	199,061	250,422
75+	19,547	23,349	28,292	34,398	40,887	63,493	95,870	132,352	185,857	266,219	349,580	425,606
Age Not Reported	6,357	4,144	2,940	1,844	0	0	0	0	0	0	0	0
Total	**2,216,331**	**2,609,121**	**2,895,832**	**2,908,506**	**3,123,723**	**3,444,578**	**3,943,116**	**4,589,575**	**5,463,105**	**6,478,216**	**8,186,453**	**9,687,653**

Sources:

U.S. Census Bureau, Table QT-P1, 2000 and 2010 census.

U.S. Census Bureau, Table 19, General Population Characteristics, 1990 census.

U.S. Census Bureau, Table 19, General Population Characteristics, 1980 census.

U.S. Census Bureau, Characteristics of the Population (Georgia, Vol. 1, Part 12), March 1973 (years 1900 through 1970).

Table A1.2 Population by Age, 1900 to 2010, Minnesota

Age	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000	2010
0 to 4	228,290	226,840	261,394	231,001	230,057	332,460	416,005	331,771	307,249	336,800	329,594	355,504
5 to 9	217,447	220,233	248,599	256,751	220,176	267,652	380,650	402,635	296,295	345,840	355,894	355,536
10 to 14	192,064	214,402	233,961	253,788	238,918	223,787	324,710	415,021	333,378	313,297	374,995	352,342
15 to 19	170,177	215,148	219,609	239,946	257,349	207,460	251,352	373,405	399,818	297,609	374,362	367,829
20 to 24	160,674	216,670	217,919	214,432	245,592	213,712	194,883	292,037	393,566	316,046	322,483	355,651
25 to 29	148,607	187,438	213,646	193,469	225,097	220,780	193,160	249,516	363,435	381,759	319,826	372,686
30 to 34	131,055	153,195	189,778	189,705	204,311	212,765	206,487	206,769	313,104	397,984	353,312	342,900
35 to 39	121,193	135,612	168,540	192,934	192,452	205,447	211,163	192,863	246,356	361,274	412,490	328,190
40 to 44	100,646	117,256	135,353	172,980	187,196	189,729	204,868	202,710	202,860	304,810	411,692	352,904
45 to 49	72,042	105,289	122,435	147,143	182,525	176,212	194,149	202,904	187,051	237,050	364,247	406,203
50 to 54	57,896	88,110	105,208	122,171	162,931	170,805	176,190	193,956	193,199	191,410	301,449	401,695
55 to 59	45,293	59,272	87,437	100,813	129,941	157,690	159,840	177,011	189,457	173,066	226,857	349,589
60 to 64	35,137	45,188	69,827	84,372	103,137	134,854	146,056	155,454	170,638	171,220	178,012	279,775
65 to 69	28,251	34,825	45,827	69,079	82,635	105,188	131,315	130,155	149,114	160,036	153,169	202,570
70 to 74	19,424	23,536	30,188	48,256	60,455	73,705	102,086	110,251	121,034	134,486	142,656	151,857
75+	19,096	27,696	34,751	46,145	69,528	90,237	120,950	168,513	209,416	252,412	298,441	328,694
Age Not Reported	4,102	4,998	2,653	968	0	0	0	0	0	0	0	0
Total	**1,751,394**	**2,075,708**	**2,387,125**	**2,563,953**	**2,792,300**	**2,982,483**	**3,413,864**	**3,804,971**	**4,075,970**	**4,375,099**	**4,919,479**	**5,303,925**

Sources:

U.S. Census Bureau, Table QT-P1, 2000 and 2010 census.

U.S. Census Bureau, Table 19, General Population Characteristics, 1990 census.

U.S. Census Bureau, Table 19, General Population Characteristics, 1980 census.

U.S. Census Bureau, Characteristics of the Population (Minnesota, Vol. 1, Part 23), March 1973 (years 1900 through 1970).

Table A1.3 Population by Age, 1900 to 2010, New Jersey

Age	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000	2010
0 to 4	206,446	266,942	338,696	329,668	256,264	458,906	642,197	589,226	463,289	532,637	563,785	647,731
5 to 9	196,725	242,279	322,958	380,918	280,722	371,826	582,212	692,648	508,447	493,044	604,529	671,855
10 to 14	174,347	228,695	291,236	384,342	337,776	290,544	524,380	710,409	605,841	480,983	590,577	685,713
15 to 19	166,746	236,541	255,161	364,396	375,112	295,859	396,363	611,831	670,665	505,388	525,216	652,864
20 to 24	178,228	250,613	271,042	350,402	376,912	350,403	321,054	509,198	614,828	566,594	480,079	609,920
25 to 29	176,408	236,172	286,617	332,810	361,291	409,890	362,373	463,164	574,135	668,917	544,917	657,765
30 to 34	158,858	213,082	263,733	331,332	340,976	409,434	435,080	403,475	563,758	691,734	644,123	703,929
35 to 39	144,124	199,647	251,252	338,222	322,760	393,917	472,429	413,929	479,749	622,963	727,924	702,384
40 to 44	117,887	166,638	207,122	291,871	315,720	357,760	446,139	465,492	400,074	573,696	707,182	675,301
45 to 49	92,115	136,295	185,551	246,388	297,595	318,504	406,721	477,978	394,038	466,481	611,357	609,260
50 to 54	78,915	112,003	151,688	205,434	259,570	305,235	350,531	439,103	432,520	376,528	547,541	558,208
55 to 59	60,248	75,739	108,505	157,128	198,622	263,516	304,112	380,677	430,048	355,677	423,338	493,551
60 to 64	49,226	62,678	86,297	124,676	158,024	215,546	262,777	314,045	367,660	363,521	330,646	427,084
65 to 69	33,955	45,948	56,135	88,449	119,172	164,921	222,457	245,757	303,670	340,232	293,196	372,200
70 to 74	23,186	31,193	38,149	58,951	80,239	109,441	163,149	194,112	227,037	269,960	281,473	306,975
75+	22,476	29,946	39,197	53,643	79,410	119,627	174,808	257,120	329,064	421,833	538,467	488,842
Age Not Reported	1,128	662	792	244	0	0	0	0	0	0	0	0
Total	**1,881,018**	**2,535,073**	**3,154,131**	**4,038,874**	**4,160,165**	**4,835,329**	**6,066,782**	**7,168,164**	**7,364,823**	**7,730,188**	**8,414,350**	**9,263,582**

Sources:

U.S. Census Bureau, Table QT-P1, 2000 and 2010 census.

U.S. Census Bureau, Table 19, General Population Characteristics, 1990 census.

U.S. Census Bureau, Table 19, General Population Characteristics, 1980 census.

U.S. Census Bureau, Characteristics of the Population (New Jersey, Vol. 1, Part 32), March 1973 (years 1900 through 1970).

Table A1.4 Population by Age, 1900 to 2010, Washington

Age	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000	2010
0 to 4	53,243	108,756	126,434	114,854	121,918	263,326	315,633	280,442	306,123	366,780	394,306	439,657
5 to 9	56,423	99,678	128,258	136,013	116,762	203,786	301,051	328,397	296,011	371,093	425,909	429,877
10 to 14	48,233	92,802	117,553	138,393	127,842	159,695	275,510	348,892	321,995	337,662	434,836	438,233
15 to 19	44,104	99,647	106,485	137,922	146,725	157,695	208,575	329,903	369,023	322,711	427,968	462,128
20 to 24	46,403	122,058	111,014	130,401	148,867	175,619	173,804	295,964	400,542	351,680	390,185	461,512
25 to 29	46,093	126,074	120,421	120,651	146,594	195,087	166,376	238,704	389,997	411,822	403,652	480,398
30 to 34	47,118	106,963	119,446	115,448	134,757	188,636	179,899	193,398	354,645	443,366	437,478	453,383
35 to 39	46,368	90,149	117,587	122,833	124,990	180,749	198,495	181,020	273,382	427,690	483,950	448,607
40 to 44	37,863	77,286	95,805	118,105	118,525	159,090	189,191	192,828	213,832	376,073	491,137	459,698
45 to 49	26,027	64,992	81,764	108,280	117,709	136,714	176,071	203,880	193,473	284,674	454,223	492,909
50 to 54	20,754	52,413	69,451	90,223	112,915	125,939	150,495	188,774	198,548	216,869	391,749	495,296
55 to 59	14,127	33,661	55,053	69,260	96,698	115,306	129,003	166,878	203,986	191,602	285,505	453,078
60 to 64	10,407	24,144	42,352	57,530	77,569	103,916	110,066	138,028	179,037	189,382	211,075	382,087
65 to 69	7,195	16,585	27,298	44,440	57,963	86,551	98,659	107,008	151,324	186,679	176,225	270,474
70 to 74	4,161	10,374	16,647	30,075	41,943	59,655	80,938	84,335	112,023	149,355	160,941	186,746
75+	3,325	9,614	16,266	26,988	44,414	65,199	99,448	130,718	168,215	239,254	324,982	370,457
Age Not Reported	6,259	6,794	4,787	1,980	0	0	0	0	0	0	0	0
Total	**518,103**	**1,141,990**	**1,356,621**	**1,563,396**	**1,736,191**	**2,376,963**	**2,853,214**	**3,409,169**	**4,132,156**	**4,866,692**	**5,894,121**	**6,724,540**

Sources:

U.S. Census Bureau, Table QT-P1, 2000 and 2010 census.

U.S. Census Bureau, Table 19, General Population Characteristics, 1990 census.

U.S. Census Bureau, Table 19, General Population Characteristics, 1980 census.

U.S. Census Bureau, Characteristics of the Population (Washington, Vol. 1, Part 49), March 1973 (years 1900 through 1970).

Appendix 2
Calculation of Point and Interval Forecasts, 2010, Minnesota

We use data from 1980-2010for Minnesota to illustrate the derivation of point and interval forecasts using regression combined with the Hamilton Perry method. We begin by computing the CCRs and ratios for the two youngest age groups (Ratios) for 1980-1990 and 1990-2000 as shown in Table A2.1. We then estimate the Ratio for 1990-2000 by regressing the observed 1990-2000 Ratios against the observed 1980-1990 Ratios using the following:

$$ERatios_{x,1990\text{-}2000} = 0.1676667 + (0.8644256 \times Ratios_{x,1980\text{-}1990}) \qquad [1]$$

adj. $r^2 = 0.755$ and $s_e = 0.07124$.

Thus usual assumption in the Hamilton-Perry method is that the launch year ratios are held constant. Under this assumption, point forecasts in 2010 are computed by:

$$Pop_{x,\ 2010} = ERatios_{x,1990\text{-}2000} \times Pop_{x,2000}, \text{ where x (0-4 and 5-9)}, \qquad [2]$$

$$Pop_{x,\ 2010} = ERatios_{x,1990\text{-}2000} \times Pop_{x\text{-}10,2000}, \text{ where x (10 to 74), and} \qquad [3]$$

$$Pop_{x,\ 2010} = ERatios_{75+,1990\text{-}2000} \times Pop_{65+,2000}, \text{ where x (75+).} \qquad [4]$$

The 1990-2000 ERatios and point forecasts for population by age are shown in Table A2.1.

Table A2.2 shows the 66% forecast intervals for both the 1990-2000 ERatios and 2010 population. We first develop intervals around the 1990-2000 ERatios by:

$$ERatios_{x,1990\text{-}2000} \pm moe \qquad [5]$$

where,

moe is the margin of error at a given probability level.

Assuming that the regression errors are normally distributed, an approximate margin of error associated with a forecast is given by:

$$moe = t_{n-2} s_e \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{X})^2}{(n-1)s_x^{\,2}}} \quad [6]$$

where
n is the total number of observations,

t_{n-2} is the t-distribution value corresponding to the probability level,

$\bar{X}$ is the mean of the observed x values (0.940349),

s_x^2 is the variance of the observed x values (.020186), and

s_e is the standard error of the regression.

In this paper we show 66% forecast intervals, which are represented by a t_{n-2} value of 1.0. Equation [6] shows that the forecast interval is wider when x is farther from $\bar{X}$ (or the average of the 1980-1990 Ratios). That is, we are more certain about our forecasts when values of the predictor variable are close to its sample mean. For example, the largest margin of error is for ages 75+ (0.09091). The 1980-1990 ERatio for that group (0.52634) is 44% below the average ERatio.

We then translate the intervals around the 1990-2000ERatios into population forecast intervals by applying equations [2] through [4] to the lower and upper limits determined by equation [5]. Table A2.2 also compares the 2010 census population against the forecast intervals. In this example, only one interval for ages 5 to 9 does not contain the 2010 census population.

Table A2.1 Ratios, 1980-1990 and 1990-2000 and Projected Population 2010, Minnesota

	Population			Ratios[a]			2010
					1990-2000		
Age	1980	1990	2000	1980-1990	Observed	Estimated[b]	Population
0 to 4	307,249	336,800	329,594	1.09618	0.97860	1.11501	367,501
5 to 9	296,295	345,840	355,894	1.16722	1.02907	1.17641	418,677
10 to 14	333,378	313,297	374,995	1.01968	1.11341	1.04890	345,711
15 to 19	399,818	297,609	374,362	1.00443	1.08247	1.03572	368,607
20 to 24	393,566	316,046	322,483	0.94801	1.02932	0.98696	370,105
25 to 29	363,435	381,759	319,826	0.95483	1.07465	0.99286	371,689
30 to 34	313,104	397,984	353,312	1.01123	1.11791	1.04160	335,898
35 to 39	246,356	361,274	412,490	0.99405	1.08050	1.02675	328,381
40 to 44	202,860	304,810	411,692	0.97351	1.03444	1.00900	356,492
45 to 49	187,051	237,050	364,247	0.96223	1.00823	0.99925	412,181
50 to 54	193,199	191,410	301,449	0.94356	0.98897	0.98312	404,743
55 to 59	189,457	173,066	226,857	0.92523	0.95700	0.96727	352,325
60 to 64	170,638	171,220	178,012	0.88624	0.93000	0.93358	281,427
65 to 69	149,114	160,036	153,169	0.84471	0.88503	0.89769	203,647
70 to 74	121,034	134,486	142,656	0.78814	0.83317	0.84880	151,097
75+	209,416	252,412	298,441	0.52634	0.54566	0.62254	369,954
Total	4,075,970	4,375,099	4,919,479				5,438,435

[a] Ages 0-4 = $P_{0-4,t}$ / $P_{0-4,t-10}$.
Ages 5-9 = $P_{5-9,t}$ / $P_{5-9,t-10}$.
Ages 10-74 = $P_{x+10,t}$ / $P_{x,t-10}$.
Ages 75+ = $P_{75+,t}$ / $P_{65+,t-10}$.

[b] Based on the regression equation, 0.1676667 + (0.8644256 × $\text{Ratios}_{1980-1990}$)

[c] Ages 0-4 = Est.1990-2000 $\text{Ratio}_{0-4} \times P_{0-4,2000}$.
Ages 5-9 = Est.1990-2000 $\text{Ratio}_{5-9} \times P_{5-9,2000}$.
Ages 10-14 = Est.1990-2000 $CCR_x \times P_{x-10,2000}$.
Ages 75+ = Est.1990-2000 $CCR_{75+} \times P_{65+,2000}$.

Table A2.2 66% Forecast Intervals, 2010, Minnesota

	1990-2000 Ratios				2010 Population Forecast		2010 Census	
Age	Point Forecast[a]	Margin of Error[b]	Lower Limit[c]	Upper Limit[d]	Lower Limit[e]	Upper Limit[e]	Population	Inside Interval
0 to 4	1.11501	0.07615	1.03886	1.19116	342,402	392,599	355,504	X
5 to 9	1.17641	0.07909	1.09732	1.25550	390,530	446,825	355,536	
10 to 14	1.04890	0.07415	0.97475	1.12305	321,272	370,151	352,342	X
15 to 19	1.03572	0.07390	0.96182	1.10962	342,306	394,907	367,829	X
20 to 24	0.98696	0.07344	0.91352	1.06040	342,565	397,645	355,651	X
25 to 29	0.99286	0.07346	0.91940	1.06632	344,188	399,190	372,686	X
30 to 34	1.04160	0.07400	0.96760	1.11560	312,035	359,762	342,900	X
35 to 39	1.02675	0.07376	0.95299	1.10051	304,791	351,972	328,190	X
40 to 44	1.00900	0.07356	0.93544	1.08256	330,502	382,481	352,904	X
45 to 49	0.99925	0.07349	0.92576	1.07274	381,867	442,495	406,203	X
50 to 54	0.98312	0.07343	0.90969	1.05655	374,512	434,973	401,695	X
55 to 59	0.96727	0.07346	0.89381	1.04073	325,568	379,083	349,589	X
60 to 64	0.93358	0.07377	0.85981	1.00735	259,189	303,665	279,775	X
65 to 69	0.89769	0.07447	0.82322	0.97216	186,753	220,541	202,570	X
70 to 74	0.84880	0.07603	0.77277	0.92483	137,562	164,631	151,857	X
75+	0.62254	0.09091	0.53163	0.71345	315,930	423,979	328,694	X

[a] From Table A2.1.

[b] Based on equation 6, using a t-value of 1 for a 66% forecast interval.

[c] Point forecast – margin of error.

[d] Point forecast + margin of error.

[e] 2000 population × upper and lower limits of the 1990-2000 ERatios.