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Evaluation of Korean Mortality Forecasting Models

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Abstract

The purposes of this study are to examine a method to overcome the shortage of historical data on mortality of the elderly and to find the best model to forecast Korean mortality rates overall. To extend the mortality for ages 75 and over, we test two methods of estimating death probabilities: the 2-parameter logistic model and the Brass-Logit model. Based on the Mean Absolute Percent Error (MAPE), the logistic model has better performance than the Brass-Logit model. Four stochastic forecasting models (the Lee-Carter Model, the adjusted Lee-Carter Model, the Lee-Miller Model, and the Coherent Lee-Carter Model) are fitted to the period 1970-2010. The forecasts are compared to actual mortality for that period. The results of this evaluation show that the Coherent Lee-Carter Model is consistently more accurate in forecasting Korean mortality rates than other compared models. The Coherent Lee-Carter model yields a higher life expectancy at birth for both sexes and a larger difference between sexes than other models in which sex differentials diminish rapidly.

Keywords: Mortality forecasting method, the Coherent Lee-Carter Model, Korean mortality, Extending mortality rates at the older ages

1. Introduction

Koreans on average are living longer than ever before. Life expectancy at birth has increased by 19 years over the past 40 years. Between 1970 and 2010, life expectancy at birth in Korea increased from 59 to 77 for males and from 66 to 84 for females.

This excessive gain has seen decreases in mortality at all ages, but primarily from the improved survival of children and a decline in mortality for the middle aged and older populations. According to Korean Life Tables (Statistics Korea, 2012), in 2010, there was a 99 percent or higher probability that new-born baby girls will reach her 15th birthday (see Figure 6). About 93 percent of them will live to see their 65th birthday. While Forty years ago, the probability for new-born girls to reach their 15th birthday was about 92 percent and only 57 percent probability to be alive at their 65th birthday.

Population ageing is common to all modern societies and the growth in life expectancy seems to be an irreversible phenomenon in the near future. The future of mortality is of interest in the context of population projections as well as of economic, social and health

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planning.

Population projections are based on future rates of three demographic variables that assumed to follow historical patterns and trends. Determining the rate of growth requires a closer look at the performance of the mortality forecasting models.

Researchers also argue mortality reduction of seniors will have a growing contribution to future gains in life expectancy. The detailed age specific data for advanced ages, referred to as seniors, mortality would be the essential element to clarify understanding of the mortality pattern of Korean elderly.

However, until 2000, only aggregated Korean death rates at the oldest ages were available: above the age of 75 in the 1970s, above 85 in the 1980s and above age 95 in the 1990s. Because of the availability and quality of data in Korea on senior mortality, the 2006 Population Projection for Korea was based on a combination of the Lee-Carter model and the Brass Logit model.

The purposes of this study are to examine a method to overcome the shortage of historical data on the age limit for senior mortality and to find the best model for forecasting Korean mortality. To examine the mortality forecasting ability for all age groups, we decided to extend the senior mortality rates first, and then fitting and forecasting models. To fill in missing detail in senior mortality, we tested two methods of estimating death probabilities for above the age of 75: 2-parameter logistic model and the Brass-Logit model.

Next, using Korean mortality data, this study evaluates three mortality forecasting models (Lee-Carter, the adjusted Lee-Carter, Lee-Miller, and Coherent LC Model). The evaluation procedure involves fitting the different models with data supplied up to 2010, forecasting for the period of 1970-2010 and comparing the forecasts with actual mortality within these periods.

Finally, this paper presents the results of forecasting life expectancies from 2011 to 2060 through the use of four different models.

2. Estimating Methods for Mortality Rates for Advanced Ages

In this section, we briefly review the two estimating methods of mortality rates at advanced ages: 2-parameter logistic model and the Brass-Logit model.

After observing modern empirical data, Kannisto (1992) suggests that the mortality at the older ages is close to one of the logistic curves rather than the Gomperz exponential curve. It grows quickly at first and then growth slows, presenting a convenient asymptotic pattern. Note that this asymptote is equal to 1 for 2-parameter model (Coelho, Magalhães, and Bravo). 2-parameter logistic model is expressed as follows:

$$m_x = \frac{Be^{\beta x}}{1 + Be^{\beta x}}$$

Where m_x is the mortality rate at age x .

Another model is the Brass Logit Model used as the extending method for senior mortality in the 2006 Korean Population Projection. Brass (1971) proposed that there is a relationship between the two survivorship functions in logits. Survivorship values $m_{x,t}$ and m_x, s where the later is the standard, it is possible to find constant α and β.

$$Logit(m_{x,t}) = \alpha + \beta \cdot Logit(m_{x,s})$$

Where α represents the mortality level of the standard and β denotes the slope of the standard. In this study, 2010 Life Tables (preliminary data) is used as the standard life table.

The models described were used to fit death probabilities of ages 65 to 75 from the Korean life tables between 2001 and 2010. The models were then used to forecast death probabilities from the ages of 75 to 100 during that period. Results were evaluated by comparing forecasting figures with actual probabilities during that period. To compare the goodness-of-fittness for the two models, we use Mean Absolute Percent Error (MAPE) and Root Mean-Squared Error(RMSE) statistics.

Based on RMSE and MAPE, the 2-parameter logistic model fitted with ages 65 to 75 exhibits a better fit in forecasting the old mortality than the Brass-Logit model. Regardless of models, male mortality forecasting is more accurate than female. The 2-parameter logistic model fitted with ages 65 to 75 is chosen as an appropriate model for filing in missing detail in the mortality rate of 75years old and over.

Table 1. The Results of RMSE and MAPE between estimated and observed probabilities of death for ages above 75

	Male			Female		
	2-Parameter Logistic		Brass-Logit	2-Parameter Logistic		Brass-Logit
	Fitting ages 55 to 75	Fitting ages 65 to 75		Fitting ages 55 to 75	Fitting ages 65 to 75	
MAPE	0.16124	0.09812	0.10006	0.28468	0.14249	0.12844
RMSE	0.02933	0.02325	0.03078	0.04124	0.02727	0.03357

3. Evaluation of Korean Mortality Forecasting Methods

In this section, we investigate the forecasting performance of the four kinds of Lee-Carter models : an original Lee-Carter model, Lee-Carter model with adjusted jump off bias, Lee-Miller Model and the Coherent Lee-Carter model.

Lee and Carter(1992) propose a two-factor(age and time) model. They use a matrix decomposition to extract a single time-varying index of the level of mortality. The Lee-Carter method is as follows:

$$\log[m_{x,t}] = a_x + b_x \cdot k_t$$

Where a_x is the age pattern of mortality, b_x is the age pattern of mortality change, and k_t represents a time-varying index of the level of mortality. The a_x, b_x, and k_t were obtained by a singular value decomposition(SVD) method to find a least squares solution (Wilmoth, 1993). To forecast mortality index of k_t, Random Walk with Drift(ARIMA (0,1,0)) is used in this study.

The second model is the LC model with adjustment of jump off bias. The LC model did not fit well in the jump-off year between fitted rates for the last year of the fitting period and actual rates in that year. This study used the actual 2010 death probability (preliminary data) to solve this jump-off bias.

The Lee-Miller Model (2001) used in this study differs from the basic Lee-Carter Model in terms that the adjustment of k_t involves fitting to life expectancy at birth in year *t* rather than death probability.

The final model is the Coherent Lee-Carter model with adjustment of jump off bias. Li and Lee (2005) argue that sex and geographic differential in mortality are unlikely to increase in the long run. To avoid divergent forecasting for members of a group, they suggested a common factor for group mortality and a specific factor for each member within this group. The Coherent Lee-Carter Model is as follows:

$$\log(m_{x,t,i}) = a_{(x,i)} + B_{(x)}K_{(t)} + b_{(x,i)}k_{(t,i)} + \epsilon_{(x,t,i)},\quad 0 \le t \le T$$

Where Bx Kt is the pattern of combined mortality and b(x,i)k(t, i) is the pattern of separate mortality. The Coherent LC model is applied to forecast two-sex combined mortality in this study.

The evaluation procedure involves fitting four models with data up to 2010 and estimating death probabilities for the period of 1970-2010 by the fitted models and comparing the forecasts to the actual probabilities for the same period.

Table 2 provides forecast accuracy measures based on fitted period of 1970-2010. The fourth row in the table 2 shows averaged forecasting error in death probability (forecast – actual) over the forecasting years.

Based on MAPE, the Coherent LC model has the best performance for both sexes. The original LC method provides more accurate results than the LM and the LC adjusted. It is also notable that mortality for males is more difficult to forecast than mortality for female for Korea.

Table 2. MAP E of death probability by sex, 1970-2010

Sex	Male				Female			
Method	Lee-Carter	LC with adj.	Lee-Miller	Coherent LC	Lee-Carter	LC with adj.	Lee-Miller	Coherent LC
Overall	0.0889	0.1850	0.1401	0.0607	0.0655	0.1047	0.0755	0.0548
1970	0.0783	0.1386	0.0855	0.0739	0.1007	0.0673	0.1082	0.0497
1980	0.0527	0.2023	0.0613	0.0719	0.1030	0.1692	0.1071	0.1312
1990	0.0967	0.2622	0.1303	0.0381	0.0503	0.1121	0.0512	0.0296
2000	0.0628	0.2205	0.1243	0.0566	0.0437	0.1164	0.0462	0.0369
2010	0.1821	0.0006	0.3050	0.0980	0.0795	0.0016	0.0881	0.0613

Tables 3 shows estimated future life expectancy at birth between 2010 and 2060. Among the four methods, the Coherent LC has the highest life expectancy at birth for females. For males, there is a 1.05 year gap between the LC model (86.45) and the LM model(87.50). For females, the LC model shows the lowest life expectancy (88.14) while the Coherent LC has the highest figure (90.48). The sex differential in life expectancy at birth is smaller in the LC (1.69 years) while bigger in the Coherent LC (3.74 years).

Table 3. Forecasted life expectancy at birth using four methods, 2010-2060

(years)

Sex	Male				Female			
Method	Lee-Carter	LC with adj.	Lee-Miller	Coherent LC	Lee-Carter	LC with adj.	Lee-Miller	Coherent LC
2010	76.34	77.19	77.18	76.91	82.24	84.10	84.06	83.78
2020	78.90	79.76	79.79	79.53	83.95	85.75	85.62	85.56
2030	81.16	81.98	82.12	81.75	85.29	87.07	86.93	87.06

2040	83.14	83.91	84.14	83.65	86.40	88.14	87.99	88.35
2050	84.89	85.62	85.92	85.30	87.34	89.04	88.88	89.48
2060	86.45	87.14	87.50	86.73	88.14	89.81	89.65	90.48

Table 4. Forecasted sex differentials in life expectancy at birth, 2010-2060

(years)

	Sex Differential			
	Lee-Carter	LC with adj.	Lee-Miller	Coherent LC
2010	5.90	6.91	6.88	6.87
2020	5.05	5.99	5.83	6.04
2030	4.14	5.09	4.81	5.31
2040	3.27	4.23	3.85	4.70
2050	2.45	3.42	2.96	4.18
2060	1.69	2.67	2.15	3.74

4. Concluding Remark

This study tried to examine a method to overcome the shortage of historical data on the limit age for the oldest of the old mortality and to find the best model for forecasting Korean mortality. To extend the senior mortality, the 2-parameter logistics model provided a better goodness-of-fit for Korean mortality data.

The results showed that the Coherent LC model was consistently more accurate in forecasting Korean death probability than the other models. The Coherent LC model yields the higher life expectancy at birth and the bigger sex differential in life expectancy than other models.

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