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Towards stochastic forecasts of the Italian population: An experiment with conditional expert elicitations

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(Preliminary Draft – Please do not quote)

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Abstract

In this work we report on the whole process developed to produce expert-based stochastic forecast of the Italian population for the period 2011-2065.

In recent years several methods of stochastic forecasts have been developed. Three main approaches can be identified (Keilman et al., 2002). The first is based on time series models. The second approach is based on the extrapolation of empirical errors, with observed errors from historical forecasts used in the assessment of uncertainty in forecasts (e.g., Stoto, 1983). Finally, the third approach, referred to as random scenario, defines the probabilistic distribution of each vital rate on the basis of expert opinions.

We follow the latest approach and apply the method proposed by Billari et al. (2012), where the full probability distribution of forecasts is specified on the basis of expert opinions on future developments of the main components of the demographic change.

In particular, we derive the joint forecast distribution of the pair Total Fertility Rate and Immigration, on one side, and of the pair Male and Female Life Expectancies at Birth on the other side. The forecast distribution of Emigration and Mean Age at Birth are derived separately.

The conditional elicitation procedure makes it possible to elicit from experts information on the marginal behavior of a single indicator in terms of expected value and variability, but also on the across time correlation of each indicator and on the correlation between any two indicators at a given year or across time.

We designed a questionnaire according to such elicitation procedure and submitted it online to thirty Italian demographers. In particular, the forecast interval is divided in two subintervals: [2011 2030] and [2030 2065]. Considering the case of two indicators, at the beginning the expert is asked to provide central scenarios of indicators at 2030 and 2065 and a high scenario of one of them. The following questions elicit central and high scenarios of one indicator conditional on the values taken by the same indicator at a previous or by the other indicator at the same or previous time. Two typical questions are:

- if Immigration at 2030 is 100000, provide a central scenario for the Total Fertility Rate at 2030;
- if Immigration at 2030 is 100000, provide a central and high scenario for Immigration at 2065;

Any demographic indicator is then prorated in term of age specific values, the distribution of which is obtained resorting to well-known and widely used models.

Collected all the necessary information for running the probabilistic projections, in this paper we show the main results of this experimental project, covering the projected distribution for indicators of the demographic behaviour, for the total population and for age related population indicators.

In specifying the probabilistic distribution of the proposed indicators, we discuss the problems that can arise in the collection of expert opinions and the solutions that can be implemented in order to avoid inconsistencies in the calculation of the parameters.

1. Introduction

Aim of this work is to report the whole process developed to produce an experimental expert-based stochastic forecast of the Italian population, for the period 2011-2065.

This activity was carried out in the framework of a national research project (PRIN) coordinated by Bocconi University of Milano.

Istat has a longstanding tradition in the regular production of population projections. Since the mid-80's Istat is in charge of official deterministic projections in Italy. This approach includes plausible variants based on different assumptions regarding the future evolution of each demographic factor, in the more general framework of the cohort-component model (Rogers, 1985).

In the last years both national and international statistical offices have started to develop strategies for the implementation of probabilistic forecasts. May be mentioned the works carried out by *UN Population division* (Heilig et al., 2010), *Eurostat* (Bertino et al., 2010), *UK-Office for national statistics* (Rowan and Wright, 2010), the Uncertainty Population of Europe project, which was funded by the European Commission in 2005 and aimed at producing stochastic population forecasts of the population of 18 countries of the European Economic Area from 2004 up to 2050 and, above all, the work done at *Statistics Netherlands*, the first statistical office making available probabilistic projections on the website (CBS, 2011).

The main goal of probabilistic population projections is to obtain prediction intervals of demographic variables and thus to measure projection uncertainty. With variant projections, on the other hand, the user has no idea how likely they are. He has to trust what the experts have provided him with scenarios representing the "most likely" variant and its plausible borders (Abel et al., 2010).

Stochastic forecasts have the advantage of providing to user the level of likelihood that a particular future population value will occur given a set of assumptions about the underlying probability distributions.

In recent years several methods of stochastic forecasts have been developed. They can be grouped under three widely recognised approaches (Keilman et al., 2002), each one of them giving the probability distributions for fertility, mortality and migration:

- probabilistic projections based on time series analysis, where the future development of any demographic indicator is projected by extrapolation from given time series models fitted to past data;
- probabilistic projections based on the extrapolation of empirical errors, with observed errors from historical forecasts used in the assessment of uncertainty in forecasts (e.g., Stoto, 1983). In particular, Alho and Spencer (1997) proposed in this framework the so-called Scaled Model of Error. Forecasts are obtained by adding shocks to age-specific indicators, with variance and correlation across age and time estimated on the basis of past forecast error times series;
- probabilistic projections referred to as random scenario, where probabilistic distribution of each vital rate is defined on the basis of expert opinions.

ln our study, belonging to the latter approach, we apply the method proposed by Billari et al. (2012), where the full probability distribution of forecasts is specified on the basis of expert opinions on future developments of the main components of the population change. This can be seen as a follow-up of previous

studies carried out by Bocconi University and Istat (Graziani, Keilman, 2011; Corsetti, Marsili, 2012) on the effectiveness of the expert-based method.

In particular, the experts evaluations are elicited in a "conditional" way, through a questionnaire submitted to seventeen Italian demographers out of a total of thirty initially invited. Collecting such information we derive opinions on the predicted value of any indicator, its expected variability and its across time correlation; secondly, we derive its correlation (at the same time and across time) with respect another indicator.

The paper is structured as follows. Section 2 briefly recalls the stochastic population forecasting method suggested in Billari et al. (2012) and in particular the conditional elicitation procedure.

In sections three and four we focus on the description of the submitted questionnaire to the experts, on the definition of the group of experts and finally on the data collection.

In section five we deal with the data processing requested for preparing the input of our stochastic forecasts.

Section five shows the main results of the projections for Italy in the period 2011-2065.

Section six describes our main findings and conclusions.

2. Methodology

The attention is focused on summary indicators of the three components of the population change, namely Total Fertility Rate (TFR) and Mean Age at Childbearing (MAC) for fertility, Female and Male Life Expectancy at Birth (LE) for mortality, Number of Immigrants (IMM) and Emigrants (EMI) for migration with foreign countries. For each component the age-schedules are derived resorting to specific models, then we run population forecasts with the cohort-component method. The forecasting method makes it possible to work out the joint forecasting distribution of all summary indicators.

We allow for correlation between TFR and IMM, because we believe that the large number of people with a foreign background that will come to Italy may have an impact on the total level of fertility. We allow also for correlation between sexes for mortality, as we aim at exploring the future evolution of the gender gap. Last, to reduce the complexity of the problem, some simplifying assumptions are made. In particular, we assume that the pair TFR and IMM is independent on the mortality indicators and, moreover, we assume EMI and MAC being independent on the all components.

The method works by deriving the joint forecasting distribution of the indicators at 2030 and 2065, while the complete distribution of the process over the forecasting interval (from 2011 to 2065) is obtained by interpolation.

The forecasting distributions of the indicators at the two time points are assumed to be multivariate Gaussian, and the parameters are specified on the basis of the information elicited from experts. We rely on the conditional elicitation procedure suggested by Billari et al. (2012), where the term conditional refers to the fact that conditional indexes, namely means and quantiles, are elicited. In a first step of the questionnaire experts are asked to report on the mean of one indicator given the value taken by the same indicator at the

previous time point. In this case a single forecasting distribution for that indicator has to be derived. In a second step, given (1) the values taken by a couple of indicators at a previous time point and given (2) the value taken by one of them at the second time point, experts are asked to report on the mean of the second indicator at the second time point. In this latter case, a joint forecasting distribution of a pair of indicators has to be worked out. In the same way, conditional quantiles of given order are elicited to obtain means and variances of all conditional distributions. The joint distribution of the indicators across time is worked out using well known standard results on the Gaussian distribution.

3. The expert's questionnaire

The described elicitation procedure was implemented in a questionnaire to be submitted online. Thirty Italian demographers were invited to participate in the survey through a letter of presentation of the project and a description of the underlying methodology. Data collection took place in late Spring 2012.

The questionnaire is set out in two parts. The first part elicits the opinions of the experts on the drivers that might affect the development of the three main components of the demographic change: Fertility, Mortality and Migration. The second part, more quantitative, collects the opinions of the experts in terms of scenarios. More precisely it is broken down into four sections: the first on TFR and IMM, the second on Male and Female LE, the third on EMM and the fourth on MAC, the evaluations on this latter being used to derive the fertility age-schedules. The first two sections aim at eliciting opinions on a pair of indicators and have the same structure. At the beginning the expert is asked to provide central scenarios of the two indicators at 2030 and 2065 and a high scenario of one of them in a not conditional way. Then, in a second step, the expert is requested to elicit conditional central and high scenarios. For example, typical conditional questions in section 1 are:

a. if IMM is 100,000 at 2030, provide a central scenario for the TFR at 2030.
b. if IMM is 100,000 at 2030 and 80,000 in 2065 and the TFR is 1.5 at 2030, provide a central scenario for the TFR at 2065.
c. if IMM is 100,000 at 2030 and 80,000 in 2065 and the TFR is 1.5 at 2030, provide a high scenario (the quantile of order 0.10) for the TFR at 2065.

Section 2, dealing with the correlations between male and female LE, has similar settings and questions. Last two sections of the questionnaire, respectively about EMM (section 3) and MAC (section 4), concern one single indicator, so that central and high scenarios are elicited conditioned on the values taken by the same indicator at the previous time.

4. Data collection

Seventeen experts, out of a total of thirty, answered to the survey by filling out the questionnaire completely or partially. Fourteen out of seventeen experts is the minimum number of opinions collected for any question, once we controlled for missing or misreported answers. Some results are summarized below. Figure 1 shows the opinions about the future of the demographic indicators at 2030 and 2065, values that

have been collected from non-conditional questions on central scenarios. As expected any indicator shows an increasing variability from 2030 to 2065. Furthermore, the simple average of the opinions is in line with the assumptions produced by Istat in latest deterministic projections (base 1.1.2011):

- about male LE, average value increases from 82.8 years at 2030 to 86.8 at 2065, whereas the width of the confidence interval at 95% moves from 1.7 to 2.7 years;
- about female LE, average value increases from 87.1 years at 2030 to 90.3 at 2065, whereas the width of the confidence interval at 95% moves from 1.1 to 2.3 years;
- about TFR, average value increases from 1.53 child per woman at 2030 to 1.65 at 2065, whereas the width of the confidence interval at 95% moves from 0.12 to 0.15 child per woman;
- about MAC, average value increases from 31.7 years at 2030 to 32.2 at 2065, whereas the width of the confidence interval at 95% moves from 1.3 to 1.9 years;
- about IMM, average value decreases from 253 thousands at 2030 to 212 at 2065, whereas the width of the confidence interval at 95% moves from 100 to 152 thousands;
- about EMM, average value increases from 130 thousands at 2030 to 140 at 2065, whereas the width of the confidence interval at 95% moves from 120 to 170 thousands.

Figure 1 – Expert opinions on male life expectancy at birth at 2030 and 2065

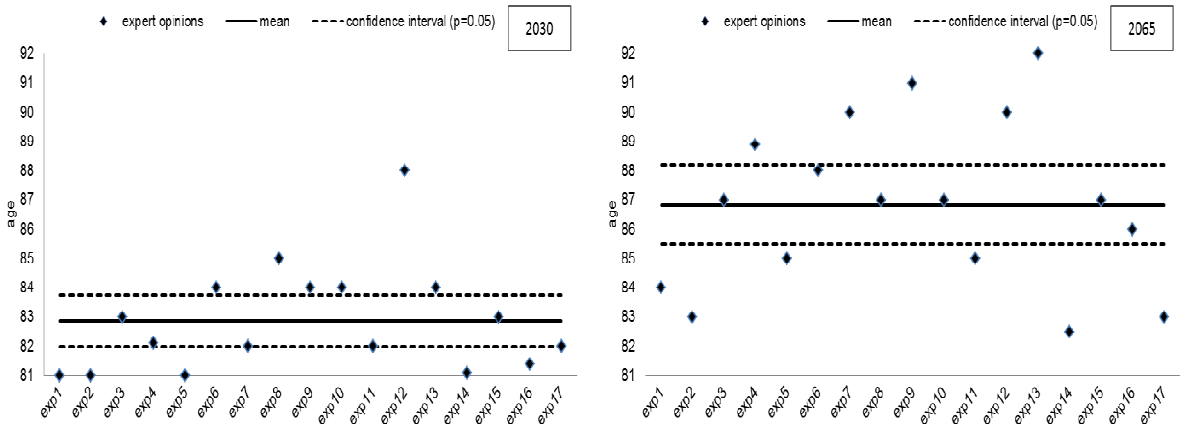


Figure 2 – Expert opinions on female life expectancy at birth at 2030 and 2065

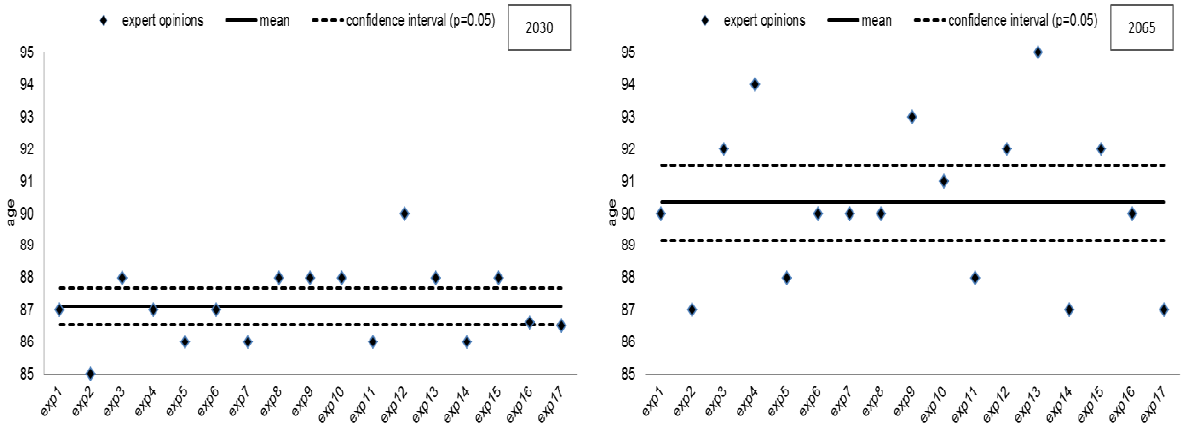


Figure 3 – Expert opinions on total fertility rate at 2030 and 2065

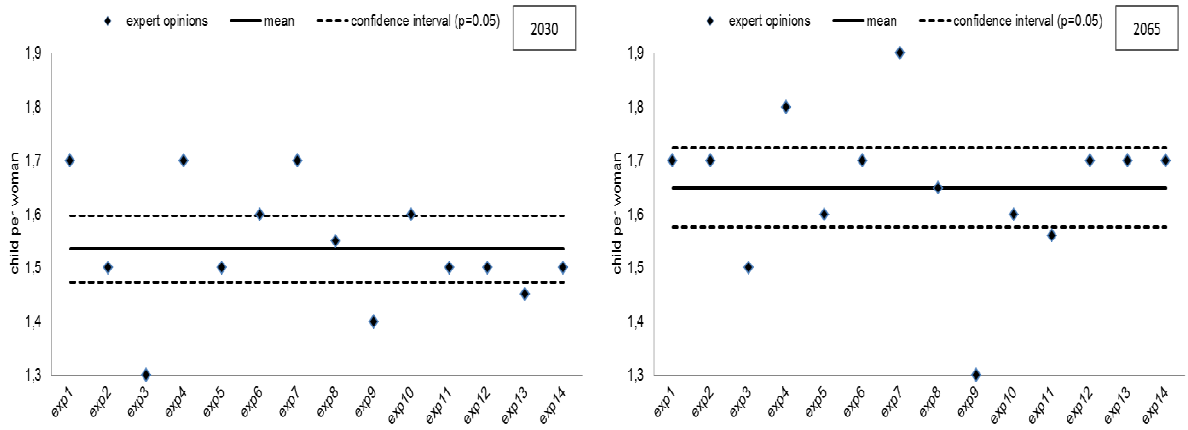


Figure 4 – Expert opinions on mean age at childbearing at 2030 and 2065

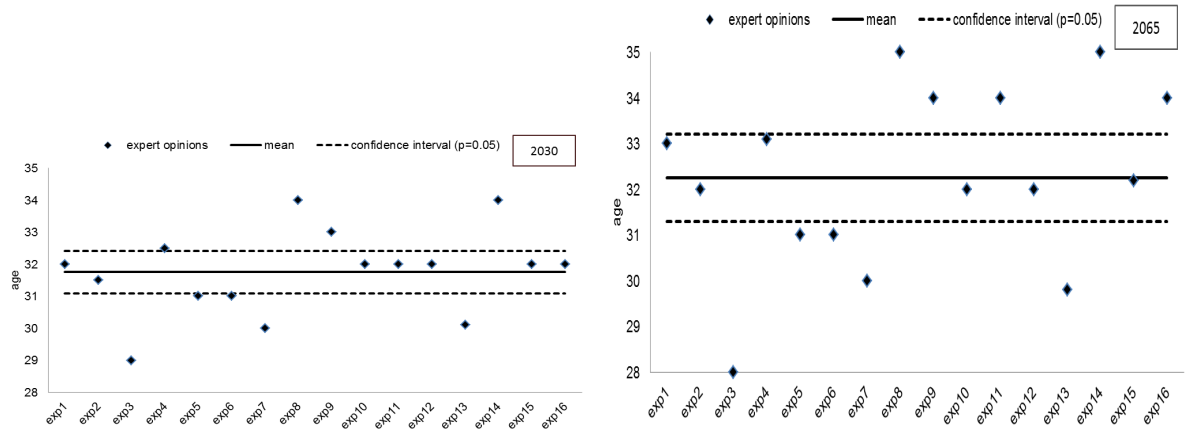


Figure 5 – Expert opinions on number of immigrants at 2030 and 2065

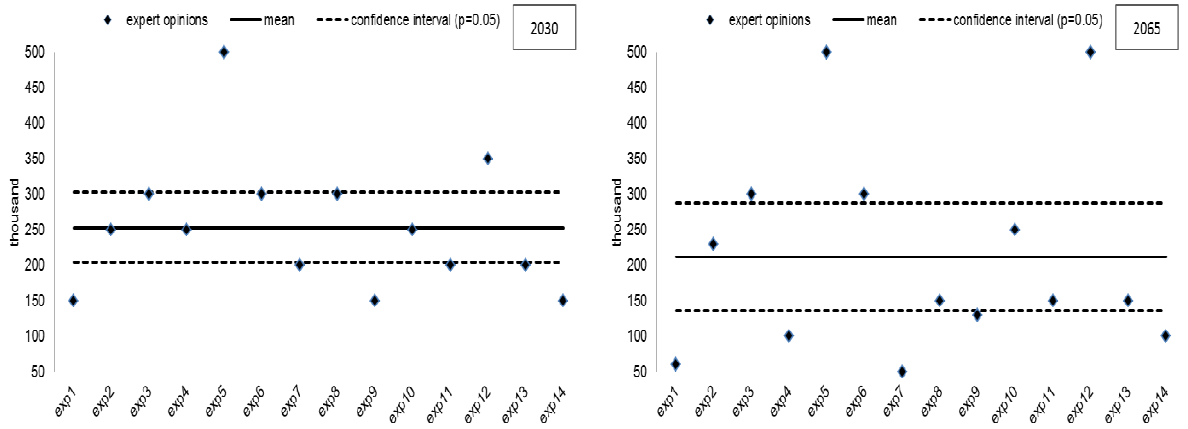
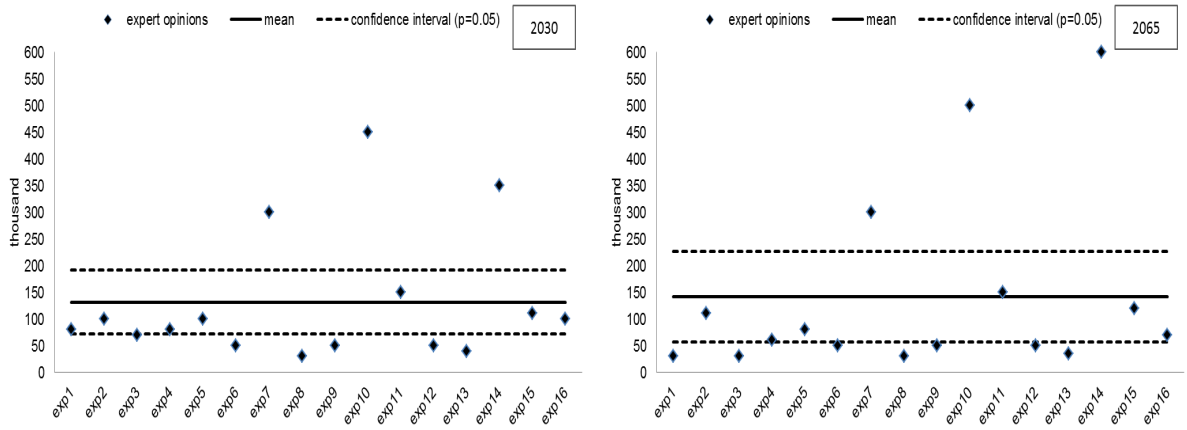


Figure 6 – Expert opinions on number of emigrants at 2030 and 2065



5. Data processing

The expert opinions, synthesized with an arithmetic mean, produce the average values we assume for our stochastic distributions of the indicators.

The variance of each indicator and the covariance among indicators are obtained from the conditional questions, summarized through the average values provided by the experts.

Calculation of the variances and covariance is more complex. In presence of a single indicator (MAC, EMM) variances and covariance of the bivariate random variables (MAC_{2030}, MAC_{2065}) and (EMM_{2030}, EMM_{2065}) are obtained by resorting to the rules of the standard bivariate normal variable.

In the case of a couple of indicators, as (TFR-IMM) for instance, variance and covariance of the multivariate normal random variable ($TFR_{2030,}IMM_{2030}$,$TFR_{2065,}IMM_{2065}$) are obtained by assuming conditional independence between TFR_{2065} and IMM_{2030} given the bivariate random variable (TFR_{2030},IMM_{2065}).

For example we show the calculation for two of the above mentioned parameters:

- variance of variable MAC_{2030} is equal to

$$VAR(MAC_{2030})=H_1-C_1 / z^2_{1-\alpha},$$

where C_1 and H_1 are the mean values provided by the experts for central and high scenario (with a confidence level α equal to 0.1) of MAC at 2030;

- covariance between MAC_{2030} and MAC_{2065} is indeed equal to

$$COVAR(MAC_{2030,}\ MAC_{2065})=(C_2//H_1-C_2)*\ VAR(MAC_{2030}) / (H_1-C_1),$$

where $C_2//H_1$ is the central scenario for the indicator at time 2065 given H_1, and C_2 is a not conditional central scenario at time 2065.

Table 1 shows means, standard deviations and correlations that represent our input data of the stochastic process that allows us to draw 2,000 samples from the corresponding multivariate distributions.

It is interesting to analyze the values of the correlations between indicators. Summary of expert opinions shows that the statistical associations between indicators are positive, except for the TFR_{2065}. This rate has always negative correlation with TFR_{2030} (- 0.19), IMM_{2030} (-0.38) and IMM_{2065} (-0.40).

Then we notice that:

- an increase of TFR in the 2011-2030 will cause a weak decrease of the same indicator in 2030-2065;
- in contrast, a decrease in the number of immigrants in both the first and the second forecast period will push the TFR to grow in the second period.

Next step is to obtain the values of demographic parameters for each forecast year by interpolation with linear (IMM, EMI) or quadratic functions (TFR, MAC, LE), the choice between the two functions depending on the best fit to observed past trends.

Table. 1 Means, variances and correlations obtained from expert elicitations

				Correlation											
Indicator	**Year**	**Mean**	**Variance**	**LEM**		**LEF**		**IMM**		**TFR**		**MAC**		**EMM**	
				2030	2065	2030	2065	2030	2065	2030	2065	2030	2065	2030	2065
LEM	2030	83.00	1.57	1.00	0.89	0.89	0.74								
	2065	87.00	1.75		1.00	0.80	0.76								
LEF	2030	87.00	1.75			1.00	0.76								
	2065	91.00	2.63				1.00								
IMM	2030	258.33	93.64					1.00	0.57	0.35	-0.38				
	2065	211.67	117.75						1.00	0.20	-0.40				
TFR	2030	1.54	0.11							1.00	-0.19				
	2065	1.68	0.18								1.00				
MAC	2030	31.80	0.781									1.00	0.71		
	2065	32.30	1.10										1.00		
EMM	2030	133,00	39.01											1.00	0.69
	2065	142.00	48.00												1.00

In order to run a cohort-component model for each simulation we derive the age-schedules of each component through the implementation of well-known demographic models, that are briefly resumed below.

Fertility. The fertility age-schedule have been modelled using a system of quadratic splines (QS model) developed by Schmertmann (2003). The Schmertmann model describes the shape of the age fertility rates (ASFR) using only three parameters: the starting age of fertility α; the age P at which fertility reaches its peak level; the youngest age H above P at which fertility falls to half of its peak level. In this exercise the estimates for H and P are obtained by fitting the model to the time series 1952-2010 of ASFR's. Then we explicated the functional relationship between the parameters H and P and the parameter MAC, relationship that is kept constant over time. The projected values for P and H are then derived from the projected value of MAC, while the parameter α is kept constant to 14 years of age.

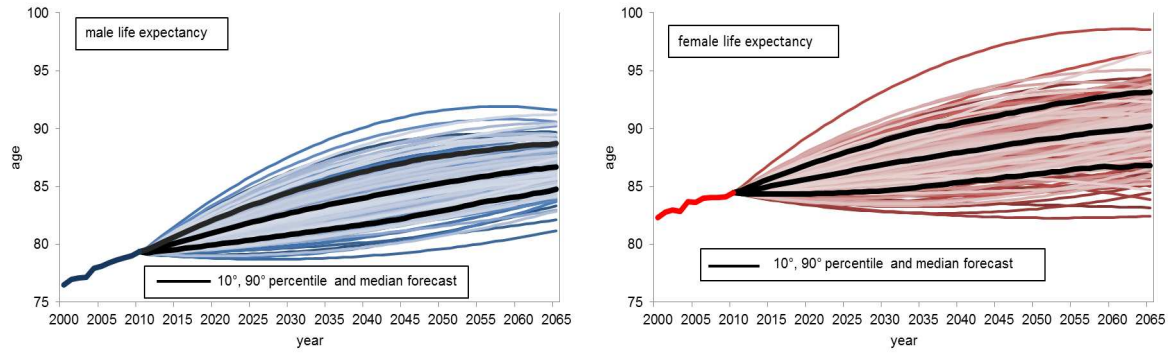
Mortality. In order to derive future age pattern of mortality the standard Lee-Carter model was performed. The model describes the shape of the log-mortality using the following three parameters: k(t), a(x) and b(x). The first is a general mortality index varying over time, while a(x) and b(x) are age-depending parameters. The three parameters are linked each other by almost precise relationships, so that it's quite simple to derive them by fitting the model to past time series of mortality rates. In particular, the age-parameters, a(x) and

b(x), are worked out fitting the model to the time series 1974-2010 of mortality rates, then they are kept constant over time. Last, the time-parameter k(t) is derived from the projected value of LE.

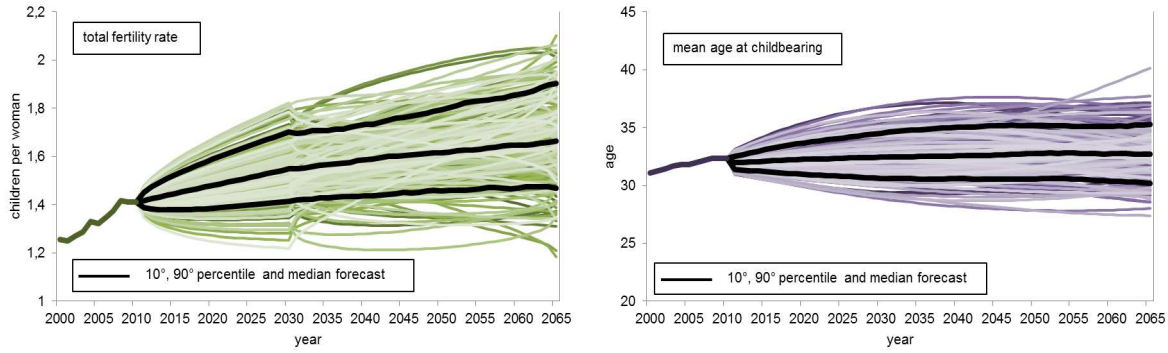
Migration. The distributions by age and sex for IMM and EMI are derived fitting a Rogers-Castro model to the 2005-2009 micro-data (Rogers and Castro, 1981). The age-pattern of these parameters is kept constant throughout the forecast period, constraining its sum over age being equal to the projected level of migration.

Figures 7-9 (representing only 250 simulations out of a total of 2,000) display future stochastic trends of each demographic parameter along time. Focusing on median values and confidence intervals, we can state that the trends of synthetic indicators follow a similar path to the assumptions set out in the latest deterministic projections released by Istat (base 1.1.2011): we observe a slight but steady increase for life expectancy at birth, a less pronounced one for fertility, a stationary trend for MAC, a decreasing trend for immigrants, especially in the first forecast period, and, finally, a constant growth of the number of emigrants.

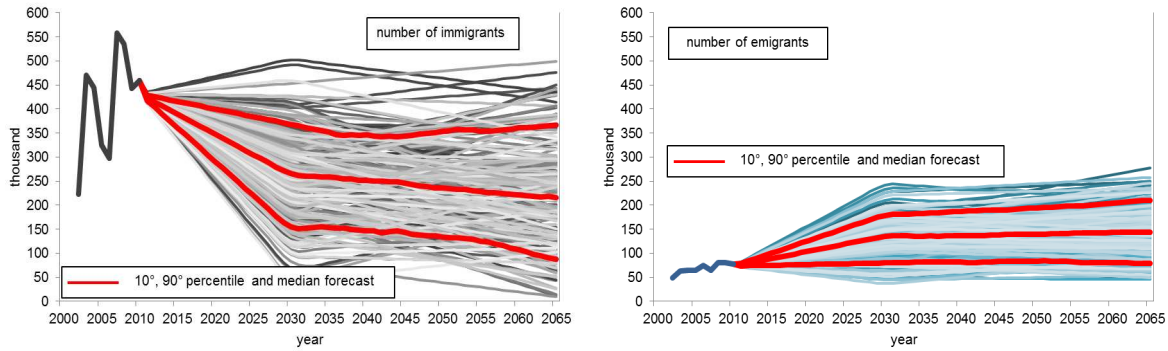
Figures 7 – Future stochastic trends for male and female life expectancy – 2011-2065



Figures 8 – Future stochastic trends for total fertility rate and mean age at childbearing – 2011-2065



Figures 9 – Future stochastic trends for the number of immigrant and emigrants – 2011-2065



6. Main results

Figure 10 displays our expert-based stochastic forecasts (EBM) of the total population for Italy in 2011-2065 in term of median values and confidence intervals. For comparison, the picture also shows the deterministic scenarios (low, main, high) of the latest Istat official projections (DET). The median EBM covers faithfully the trend of the main DET scenario until 2030, while in the second period it develops a faster decrease of the Italian population (56.9 million by 2065, against 61,1 million of the Istat assumption).

Variability measured in term of confidence intervals rapidly increases along time: at 75% of confidence level, the interval is large 3.7 million at 2030, 9.5 at 2050 and 14.2 at 2065. Looking at the figure we also see that the high DET scenario is external, with probability equal to 85% (darkest range), to the range provided by our stochastic forecast.

Figure 10 – Future stochastic trends for the total population – 2011-2065

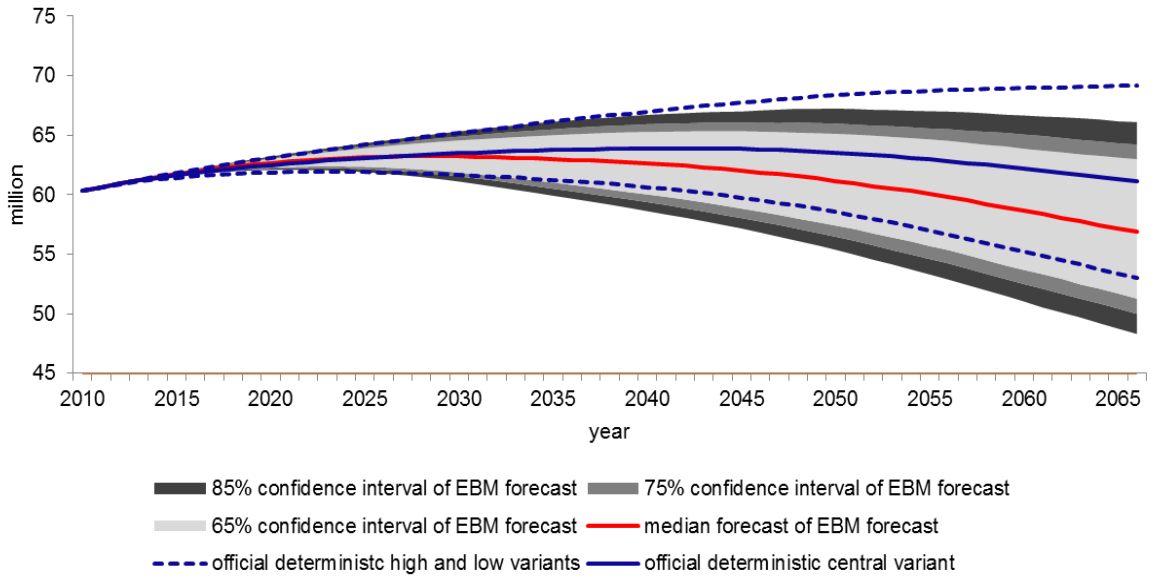


Figure 11 shows the evolution of the population by main age groups: 0-14 years, 15-64 years and 65 years and more. As well as the official projections, even the stochastic forecasts based on expert opinions confirm further aging of the Italian population. The young population would be affected by a decrease of 1.2 million, from 8.5 in 2011 to a median value of 7.3 million by 2065. On the contrary, population aged 65 years and more would increase quickly, from 12.3 million in the base year to a peak of 20.6 million by 2049, after which it would begin to decline with a median value of 18.8 million by 2065. Stochastic forecasts would give a rapid decrease of the working-age population in the medium-long run: from about 40 million of persons in 2025 to 30.6 million in 2065, according to the median forecast. About this latter age group, we also find a significant difference with respect to DET projections. The main scenario, in fact, provides a more optimistic evolution, giving a projection of 33.5 million people by 2065.

The uncertainty of our stochastic estimates, expressed in absolute numbers, varies from age group to age group: it is lower for the class 0-14 years and higher for the class 15-64 years, especially in the last two decades.

Differences between EBM and DET are very low if we take into account the old age dependency ratio (OADR), which is a key indicator of the level of population aging. Looking at our stochastic median value, we can see a fast increase of OADR in 2011-2045, from 30% to 60% , and an almost constant trend in the long term. The result is closely in line with the one provided by the main DET scenario but, EBM provides higher levels of uncertainty than the differences produced under the deterministic scenarios.

Figure 11 – Future stochastic trends for the population by main age groups and old age dependency ratios – 2011-2065

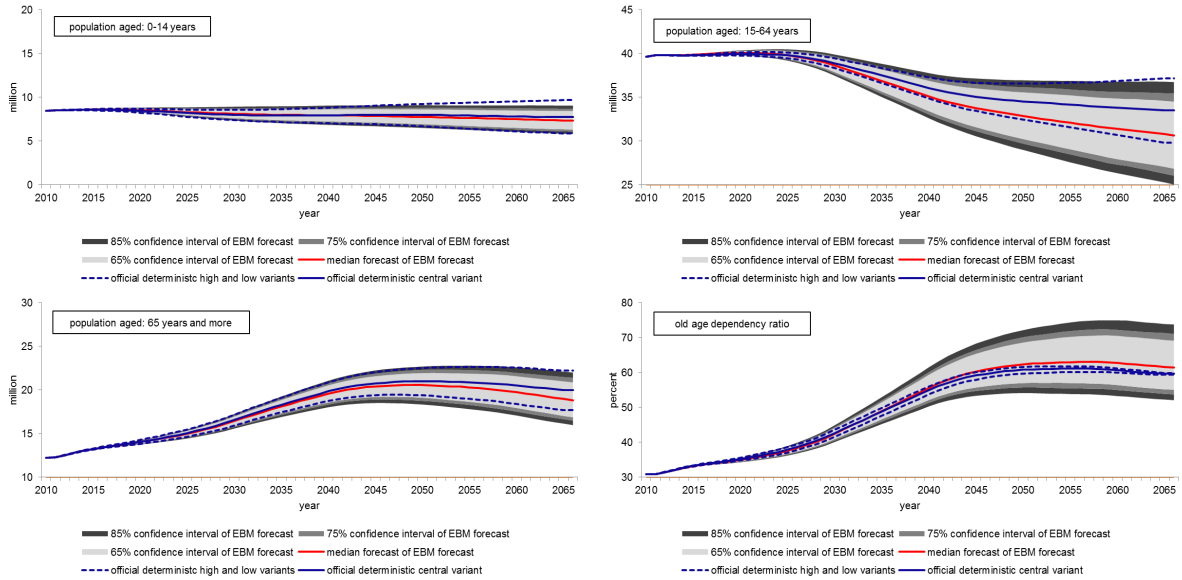


Figure 12 – Future stochastic trends for the components of the population change – 2011-2065

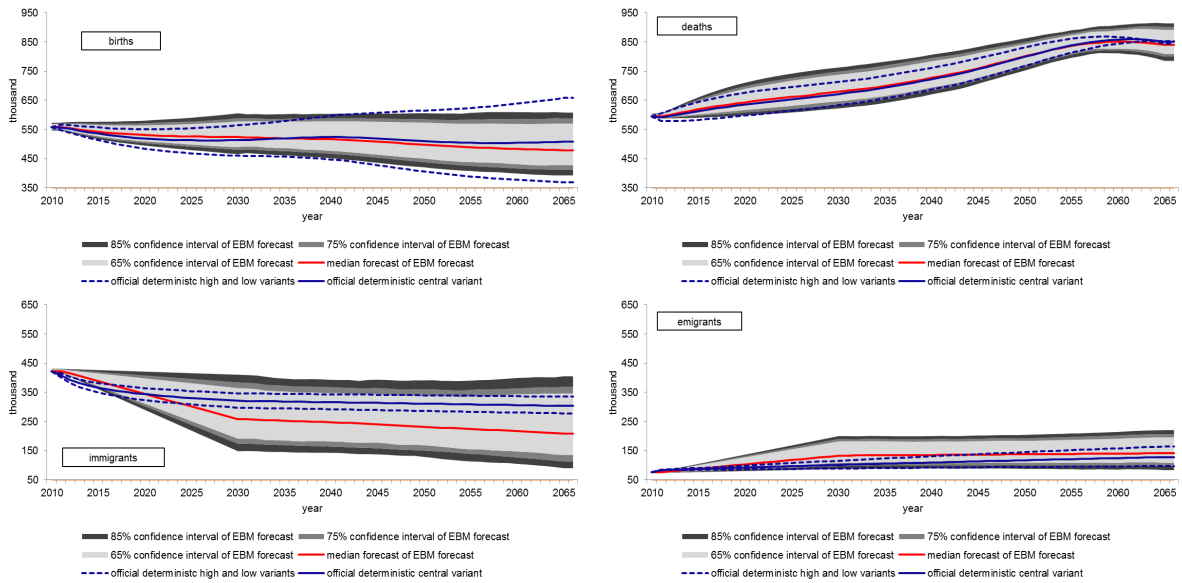


Figure 12 shows the forecast on the components of the demographic change. Births and Immigrations are the components affected by higher uncertainty. The comparison with DET projections is very interesting.

Except for births, low and high scenarios are well within the stochastic forecast intervals; it means that the deterministic range is very narrow and therefore there is a low probability of containing the true prediction. From this point view the deterministic trend shown by deaths is very explicative. The three official scenarios converge to the same value by 2065. Then we could say that the number of deaths for that year is a value determined without error. Forecasters know that this conclusion is far from being true and that, although mortality could be considered an easier component on which to speculate, there is always a degree of uncertainty to be considered, as it is pointed out by the stochastic confidence intervals.

7. Conclusions

As an outcome of a joint scientific group consisting of experts from Bocconi University and Istat, in this paper we described methodological assumptions and operational choices to derive expert-based stochastic population forecasts through a random-scenario approach. This activity is necessary to outline Istat's plans for future work in this area, with particular regard to the next round of official population projections for Italy and his territories, to be released next year.

For each component of the population change the opinions were collected thanks to a questionnaire submitted to seventeen Italian experts in late Spring 2012. In the paper we focus on the opinions provided by the experts and on the way we derived from them a synthesis for running our stochastic forecasts. Our questionnaire includes independent responses on expectations of an indicator and responses to be elicited in a conditional way with regard to one or two indicators. The former responses were used to obtain assumptions on the mean value of an indicator, the latter to derive assumptions on its variance or its correlation with another indicator across time.

Although this is still a preliminary study, the proposed method has produced satisfactory results in term of data collection and data processing, allowing us the definition of a good set of hypotheses for our forecasting exercise. In this context and for further development of our methodology it is appropriate to analyze the problems raised during the operations. The search for solutions to these issues would greatly improve the overall quality of the forecasts production process. These solutions concern primarily the overall assessment of the questionnaire and the possible ways to refine it, secondly the way we manage the synthesis of the responses.

For instance, we consider as appropriate to simplify the structure of the questionnaire and to plan actions to facilitate the expert's task. This objective can be achieved by better refining the parts of the questionnaire where the expert is asked to give "independent" opinions and those parts where he is requested to think in a conditional way.

Another possible simplification regards the choice of the correlations between indicators. In this study we selected two time points (2030, 2065) and two correlations to be investigated: TFR against IMM and male/female LE. A possible alternative to this choice could be the removal of correlation between indicators, holding only the correlation of an indicator with itself across time. This solution would allow us to increase the number of time-points, so reducing the length of the intervals to be interpolated.

In addition to issues about the nature of the questionnaire, we are also taking the opportunity to improve the treatment of the responses. In this exercise, expert opinions are combined by simply averaging them, the same weight being then assigned to each expert opinion. We intend to evaluate other combination methods, as weighted means, in which the weights might be defined on the basis of an evaluation of their expertise in the field, measured by means of their involvement in the literature or in the production of projections within official agencies.

Another source of error we intend to examine is the procedure we adopted for interpolating the assumptions between consecutive time-points. In this study we chose linear (IMM, EMI) or quadratic functions (TFR, MAC, LE), the choice between them depending on the best fit to observed past trends. In fact, we should better understand what kind of stochastic response we get, due to the fact that the optimal interpolating function can take different forms from those selected by us.

Last, except for fertility, where TFR and MAC are both subjected to expert opinion, we should better study the relationship between an indicator of overall intensity (LE, IMM, EMI) elicited by experts and its chosen age-pattern. Actually, this latter really impacting input is also taken for running the stochastic forecasts (output of Lee Carter model for mortality and output of Castro-Rogers model for migration), but with a level of uncertainty that, although existing, we do not take in consideration in this study.

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