Bayesian functional models for population forecasting

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Joint work with Peter W. F. Smith, Jakub Bijak, Arkadiusz Wiśniowski and James Raymer
Motivation

1. Accurately forecast population via cohort component projection model
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2. Produce probabilistic forecasts
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1. Accurately forecast population via cohort component projection model
2. Produce probabilistic forecasts
3. Forecast age-specific male and female population together when possible
4. As a generalisation of Lee-Carter model, functional data analysis provides a flexible approach
5. Bayesian model averaging between the independent and coherent functional models
Population and mortality data (1975–2009) are from Human Mortality Database (http://www.mortality.org/)
Data: United Kingdom


2. Fertility data are from Human Fertility Database (http://www.humanfertility.org/cgi-bin/main.php)
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3. Emigration and immigration data are from Office for National Statistics (http://www.statistics.gov.uk/hub/index.html)
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Aim to forecast population by age and gender from 2010 to 2030
1. Mortality rates for single year of age (0–90+) between 1975 and 2009

2. Mortality rates dip in early childhood, climb in the teen years, stabilise in the early 20s, and then steadily increase with age
Graphical display of fertility rates

1. Age-specific fertility rates for ages (13–51) between 1975 and 2009
2. Increase in fertility rates at higher ages in more recent years caused by a tendency to postpone child-bearing
1. A slight increase over time in the emigration, but the patterns seem to be more volatile.

2. Emigration data come from the International Passenger Survey.
Immigration counts have been rapidly increasing from 1975 to 2009.
Cohort component projection model

\[
\begin{bmatrix}
P_{1,t+1}^F \\
P_{2,t+1}^F \\
\vdots \\
P_{z-1,t+1}^F \\
P_{z,t+1}^F
\end{bmatrix}
= \begin{bmatrix}
0 & \ldots & b_{13,t}^F & \ldots & b_{51,t}^F & \ldots & 0 \\
s_{0,t}^F & 0 & 0 & \ldots & \ldots & 0 \\
0 & s_{1,t}^F & 0 & \ldots & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \ldots & s_{z-2,t}^F & 0 & 0 \\
0 & 0 & \ldots & 0 & s_{z-1,t}^F & s_{z,t}^F
\end{bmatrix}
\times
\begin{bmatrix}
P_{1,t}^F \\
P_{2,t}^F \\
\vdots \\
P_{z-1,t}^F \\
P_{z,t}^F
\end{bmatrix}
+ I_t^F,
\]

- Growth matrix includes survival rates \( s_{i,t}, i = 0, \ldots, 90+ \) based on mortality and emigration, and fertility rates \( b_{j,t}, j = 13, \ldots, 51 \)
Cohort component projection model

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\begin{bmatrix}
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\vdots & \vdots & \vdots & \ddots & \vdots \\
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\end{bmatrix} \times \begin{bmatrix}
P_{1,t}^F \\
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\vdots \\
P_{z-1,t}^F \\
P_{z,t}^F
\end{bmatrix} + \mathbf{I}_t^F,
\]

- Growth matrix includes survival rates \( s_{i,t}, i = 0, \ldots, 90+ \) based on mortality and emigration, and fertility rates \( b_{j,t}, j = 13, \ldots, 51 \)

- \( \mathbf{I}_t^F = [I_{0,t}^F, \ldots, I_{+90,t}^F]^\top \) denotes female immigration counts
Functional data analysis

1 Received increasing amount of attention (Hyndman and Ullah, 2007; Lazar and Denuit, 2009; Yang et al., 2010; Dowd et al., 2010; Cairns et al., 2011; D’Amato et al., 2011)
Functional data analysis

1. Received increasing amount of attention (Hyndman and Ullah, 2007; Lazar and Denuit, 2009; Yang et al., 2010; Dowd et al., 2010; Cairns et al., 2011; D’Amato et al., 2011)

2. Let $m_{t,i}$ represent the original data for ages $i = 0, \ldots, 90+$ in year $t$. The Box-Cox transformation is applied to each component of population to alleviate heteroscedasticity and non-normality

$$g_{t,i} = \begin{cases} \frac{1}{\lambda}(m_{t,i}^\lambda - 1) & \text{if } 0 < \lambda \leq 1 \\ \ln(m_{t,i}) & \text{if } \lambda = 0 \end{cases}$$

where $\lambda$ is the Box-Cox transformation parameter
We apply a smoothing technique to transform the set of discrete data points, $g_{t,i}$, to continuous function

$$g_t(x_i) = \tau_t(x_i) + \sigma(x_i)\epsilon_t, \quad t = 1, \ldots, n,$$

where $\sigma(x_i)$ models the variability for each age $x_i$, and $\epsilon_t \overset{iid}{\sim} N(0, 1)$.
Independent decomposition step

To model female and male data independently,

\[ \tau_t^F(x) = \mu^F(x) + \sum_{k=1}^{K} \beta_{t,k}^F \phi_k^F(x) + e_t^F(x), \quad x \in [0, 90+] \quad (1) \]

where
- \( \mu^F(x) \) represents the mean function for females
Independent decomposition step

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where

- \(\mu^F(x)\) represents the mean function for females
- \(\{\phi_1^F(x), \ldots, \phi_K^F(x)\}\) is a set of first \(K\) functional principal components
Independent decomposition step

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\[
\tau_t^F(x) = \mu^F(x) + \sum_{k=1}^{K} \beta^F_{t,k} \phi^F_k(x) + e_t^F(x), \quad x \in [0, 90+] \tag{1}
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where

- \( \mu^F(x) \) represents the mean function for females
- \( \{\phi^F_1(x), \ldots, \phi^F_K(x)\} \) is a set of first \( K \) functional principal components
- \( \{\beta^F_{t,1}, \ldots, \beta^F_{t,K}\} \) is a set of corresponding principal component scores

\( K < n \) is the number of retained components, which explain at least 99% of the total variation in data.
Independent decomposition step

To model female and male data independently,

\[ \tau_t^F(x) = \mu^F(x) + \sum_{k=1}^{K} \beta_{t,k}^F \phi_k^F(x) + e_t^F(x), \quad x \in [0, 90+] \] (1)

where

- \( \mu^F(x) \) represents the mean function for females
- \( \{\phi_1^F(x), \ldots, \phi_K^F(x)\} \) is a set of first \( K \) functional principal components
- \( \{\beta_{t,1}^F, \ldots, \beta_{t,K}^F\} \) is a set of corresponding principal component scores
- \( e_t^F(x) \sim N(0, v(x)) \) is the residual function
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- \( \{\phi_1^F(x), \ldots, \phi_K^F(x)\} \) is a set of first \( K \) functional principal components
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- \( e_t^F(x) \sim N(0, \nu(x)) \) is the residual function
- \( K < n \) is the number of retained components, which explain at least 99% of the total variation in data
The terms in equation (1) can be estimated by

\[ \hat{\mu}^F(x) = \frac{1}{n} \sum_{t=1}^{n} \tau_t^F(x) \]
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- $\hat{\mu}^F(x) = \frac{1}{n} \sum_{t=1}^{n} \tau^F_t(x)$
- $\{\hat{\phi}_1^F(x), \ldots, \hat{\phi}_K^F(x)\}$ are the sample principal components, obtained by singular value decomposition
Independent decomposition step

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\[ \hat{\mu}_F(x) = \frac{1}{n} \sum_{t=1}^{n} \tau_t^F(x) \]

\[ \{\hat{\phi}_1^F(x), \ldots, \hat{\phi}_K^F(x)\} \] are the sample principal components, obtained by singular value decomposition

\[ \hat{\beta}_{t,k}^F \] and \[ \hat{v}(x) \] are drawn from their respective posterior distribution, where the likelihood is the Box-Cox normal and the prior densities of the variance parameters are inverse gamma distributions.
We model female and male data jointly via multilevel functional principal component regression

\[
\tau_t^F(x) = \mu(x) + w^F(x) + R_t(x) + U_t^F(x) + \varepsilon_t^F(x),
\]

(2)

where

- \( \mu(x) \) represents the overall mean function
We model female and male data jointly via multilevel functional principal component regression

\[ \tau^F_t(x) = \mu(x) + w^F(x) + R_t(x) + U^F_t(x) + \varepsilon^F_t(x), \]  

where

- \( \mu(x) \) represents the overall mean function
- \( w^F(x) \) is the female deviation from the overall mean function
We model female and male data jointly via multilevel functional principal component regression

\[ \tau_t^F(x) = \underbrace{\mu(x) + w^F(x) + R_t(x) + U_t^F(x) + \varepsilon_t^F(x)}_{\mu^F(x)}, \]  

where

- \( \mu(x) \) represents the overall mean function
- \( w^F(x) \) is the female deviation from the overall mean function
- \( R_t(x) \) models the common trend for female and male data
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- \( U^F_t(x) \) models the gender-specific trend in the female data
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- \(R_t(x)\) models the common trend for female and male data
- \(U_t^F(x)\) models the gender-specific trend in the female data
- \(\varepsilon_t^F(x)\) represents the error term with finite variance \(\delta(x)\)
Motivation

Data

Model and methods

Results

Discussion

Coherent decomposition step

4. We model female and male data jointly via multilevel functional principal component regression

\[ \tau_t^F(x) = \mu(x) + w^F(x) + R_t(x) + U_t^F(x) + \varepsilon_t^F(x), \quad (2) \]

where

- \( \mu(x) \) represents the overall mean function
- \( w^F(x) \) is the female deviation from the overall mean function
- \( R_t(x) \) models the common trend for female and male data
- \( U_t^F(x) \) models the gender-specific trend in the female data
- \( \varepsilon_t^F(x) \) represents the error term with finite variance \( \delta(x) \)
- Similarly, equation (2) also holds for males
The terms in equation (2) can be estimated by

\[
\hat{\mu}(x) = \frac{1}{2n} \sum_{j=1}^{2n} \tau_t^{(j)}(x), \quad \hat{w}^F(x) = \hat{\mu}^F(x) - \hat{\mu}(x),
\]

\[
\hat{R}_t(x) \approx \sum_{k=1}^{K} \hat{\beta}_{t,k} \hat{\phi}_k(x), \quad \hat{U}_t^F(x) \approx \sum_{l=1}^{L} \hat{\gamma}_{t,l}^F \hat{\psi}_l^F(x),
\]

where \(\{\hat{\beta}_{t,1}, \ldots, \hat{\beta}_{t,K}\}\) and \(\{\hat{\gamma}_{t,1}^F, \ldots, \hat{\gamma}_{t,L}^F\}\) represent sample principal component scores of \(\hat{R}_t(x)\) and \(\hat{U}_t^F(x)\)
The terms in equation (2) can be estimated by

\[ \hat{\mu}(x) = \frac{1}{2n} \sum_{j=1}^{2} \sum_{t=1}^{n} \tau^{(j)}_t(x), \quad \hat{w}^F(x) = \hat{\mu}^F(x) - \hat{\mu}(x), \]

\[ \hat{R}_t(x) \approx \sum_{k=1}^{K} \hat{\beta}_{t,k} \hat{\phi}_k(x), \quad \hat{U}^F_t(x) \approx \sum_{l=1}^{L} \hat{\gamma}_{t,l}^F \hat{\psi}_l^F(x), \]

where

1. \( \{\hat{\beta}_{t,1}, \ldots, \hat{\beta}_{t,K}\} \) and \( \{\hat{\gamma}_{t,1}^F, \ldots, \hat{\gamma}_{t,L}^F\} \) represent sample principal component scores of \( \hat{R}_t(x) \) and \( \hat{U}_t^F(x) \)

2. \( \Phi = \left\{ \hat{\phi}_1(x), \ldots, \hat{\phi}_K(x) \right\} \) and \( \Psi = \left\{ \hat{\psi}_1^F(x), \ldots, \hat{\psi}_L^F(x) \right\} \) represent the sample principal components of \( \hat{R}_t(x) \) and \( \hat{U}_t^F(x) \)
The terms in equation (2) can be estimated by

\[ \hat{\mu}(x) = \frac{1}{2n} \sum_{j=1}^{n} \sum_{t=1}^{\tau}(j) \tau_t(x), \quad \hat{w}^F(x) = \hat{\mu}^F(x) - \hat{\mu}(x), \]

\[ \hat{R}_t(x) \approx \sum_{k=1}^{K} \hat{\beta}_{t,k} \phi_k(x), \quad \hat{U}_t^F(x) \approx \sum_{l=1}^{L} \hat{\gamma}_{t,l}^F \psi_{l}^F(x), \]

where

1. \( \{\hat{\beta}_{t,1}, \ldots, \hat{\beta}_{t,K}\} \) and \( \{\hat{\gamma}_{t,1}^F, \ldots, \hat{\gamma}_{t,L}^F\} \) represent sample principal component scores of \( \hat{R}_t(x) \) and \( \hat{U}_t^F(x) \)
2. \( \Phi = \{\hat{\phi}_1(x), \ldots, \hat{\phi}_K(x)\} \) and \( \Psi = \{\hat{\psi}_1^F(x), \ldots, \hat{\psi}_L^F(x)\} \) represent the sample principal components of \( \hat{R}_t(x) \) and \( \hat{U}_t^F(x) \)
3. \( K \) and \( L \) are the number of retained components, which explain at least 95% and 90% of the total variations in data
Forecasting step of coherent decomposition

6 Conditional on the estimated principal components $\Phi$ and $\Psi$ and smooth functions $\Gamma$, point forecasts of $g_{n+h}(x)$ are

$$\hat{g}_{n+h|n}(x) = E \left[ g_{n+h}(x) \mid \Phi, \Psi, \Gamma \right]$$

$$= \hat{\mu}(x) + \hat{w}^F(x) + \sum_{k=1}^{K} \hat{\beta}_{n+h|n,k} \phi_k(x) + \sum_{l=1}^{L} \hat{\gamma}_{n+h|n,l}^F \psi_l^F(x)$$

where $\hat{\beta}_{n+h|n,k}$ and $\hat{\gamma}_{n+h|n,l}^F$ are the forecasted principal component scores, obtained from a univariate time-series method, such as the ARIMA
Prediction interval

Prediction interval of $g_{n+h|n}^F(x)$ can be obtained using parametric bootstrap samples, given by

$$
\hat{g}_{n+h|n}^{b,F}(x) = \hat{\mu}(x) + \hat{w}^F(x) + \sum_{k=1}^{K} \hat{\beta}_{n+h|n,k}^b \hat{\phi}_k(x) + \sum_{l=1}^{L} \hat{\gamma}_{n+h|n,l}^b \hat{\psi}_l^F(x) + \hat{\epsilon}_{n+h}^b(x) + \hat{\sigma}(x)\hat{\epsilon}_{n+h}^b, \quad b = 1, \ldots, B,
$$

where

1. $\hat{\beta}_{n+h|n,k}^b$ is the forecast of bootstrapped principal component scores $\{\hat{\beta}_{1,k}^b, \ldots, \hat{\beta}_{n,k}^b\}$
Prediction interval

Prediction interval of $g_{n+h|n}^F(x)$ can be obtained using parametric bootstrap samples, given by

$$
\tilde{g}_{n+h|n}^{b,F}(x) = \hat{\mu}(x) + \hat{w}^F(x) + \sum_{k=1}^{K} \hat{\beta}_{n+h|n,k}^b \hat{\Phi}_k(x) + \sum_{l=1}^{L} \hat{\gamma}_{n+h|n,l}^{b,F} \hat{\Psi}_l^F(x) + \hat{\varepsilon}_{n+h}(x) + \hat{\sigma}(x) \hat{\varepsilon}_{n+h}^b, \quad b = 1, \ldots, B,
$$

where

1. $\hat{\beta}_{n+h|n,k}^b$ is the forecast of bootstrapped principal component scores $\{\hat{\beta}_{1,k}^b, \ldots, \hat{\beta}_{n,k}^b\}$

2. $\hat{\gamma}_{n+h|n,l}^{b,F}$ is the forecast of bootstrapped principal component scores $\{\hat{\gamma}_{1,l}^{b,F}, \ldots, \hat{\gamma}_{n,l}^{b,F}\}$
Prediction interval

Prediction interval of $g_{n+h|n}^F(x)$ can be obtained using parametric bootstrap samples, given by

$$
\hat{g}^{b,F}_{n+h|n}(x) = \hat{\mu}(x) + \hat{\nu}^F(x) + \sum_{k=1}^{K} \hat{\beta}^{b}_{n+h|n,k} \hat{\phi}_k(x) + \sum_{l=1}^{L} \hat{\gamma}^{b,F}_{n+h|n,l} \hat{\psi}_l^F(x) + \hat{\varepsilon}^b_{n+h}(x) + \hat{\sigma}(x)\hat{\varepsilon}^b_{n+h}, \quad b = 1, \ldots, B,
$$

where

1. $\hat{\beta}^{b}_{n+h|n,k}$ is the forecast of bootstrapped principal component scores $\{\hat{\beta}_{1,k}^b, \ldots, \hat{\beta}_{n,k}^b\}$
2. $\hat{\gamma}^{b,F}_{n+h|n,l}$ is the forecast of bootstrapped principal component scores $\{\hat{\gamma}_{1,l}^b, \ldots, \hat{\gamma}_{n,l}^b\}$
3. $\hat{\varepsilon}^b_{n+h}(x)$ is drawn from $N(0, \hat{\delta}(x))$
Prediction interval

Prediction interval of \( g_{n+h|n}^F(x) \) can be obtained using parametric bootstrap samples, given by

\[
\tilde{g}_{n+h|n}^{b,F}(x) = \hat{\mu}(x) + \hat{w}^F(x) + \sum_{k=1}^{K} \hat{\beta}_{n+h|n,k}^{b} \hat{\phi}_k(x) + \sum_{l=1}^{L} \hat{\gamma}_{n+h|n,l}^{b,F} \hat{\psi}_l^F(x) + \hat{\varepsilon}_{n+h}(x) + \hat{\sigma}(x)\hat{\varepsilon}_{n+h}, \quad b = 1, \ldots, B,
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where

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2. \( \hat{\gamma}_{n+h|n,l}^{b,F} \) is the forecast of bootstrapped principal component scores \( \{\hat{\gamma}_{1,l}^{b,F}, \ldots, \hat{\gamma}_{n,l}^{b,F}\} \)
3. \( \hat{\varepsilon}_{n+h}(x) \) is drawn from \( \text{N}(0, \hat{\delta}(x)) \)
4. \( \hat{\varepsilon}_{n+h}^{b} \) is simulated from \( \text{N}(0, 1) \)
Let \( M_1 \) and \( M_2 \) be two functional models considered.
Bayesian model averaging

1. Let $M_1$ and $M_2$ be two functional models considered
2. $\theta_1$ and $\theta_2$ be vector of parameters associated with each model
Bayesian model averaging

1. Let $M_1$ and $M_2$ be two functional models considered.
2. $\theta_1$ and $\theta_2$ be vector of parameters associated with each model.
3. Denote $\Delta$ as the quantity of interest, such as a combined forecast of age-specific mortality, then its posterior distribution given data $D$ is

$$
\Pr(\Delta|D) = \sum_{r=1}^{2} \Pr(\Delta|M_r, D)\Pr(M_r|D)
$$

$$
= \sum_{r=1}^{2} \Pr(\Delta|M_r, D) \frac{\Pr(D|M_r)\Pr(M_r)}{\sum_{l=1}^{2} \Pr(D|M_l)\Pr(M_l)}
$$
Given equal prior probability for the two models, the posterior odds can be obtained by the ratios of two marginal likelihoods averaged over all Markov chain Monte Carlo iterations.
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The marginal likelihood introduced by Newton & Raftery (1994) can be written as

$$\Pr(D|M_r) \approx \left( \frac{1}{T} \sum_{t=1}^{T} \Pr(D|\theta_{M_r}^{(t)}, M_r)^{-1} \right)^{-1}$$
Harmonic estimator of marginal likelihood

1. Given equal prior probability for the two models, the posterior odds can be obtained by the ratios of two marginal likelihoods averaged over all Markov chain Monte Carlo iterations.

2. The marginal likelihood introduced by Newton & Raftery (1994) can be written as

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3. For mortality and emigration rates, coherent functional model has weight one. For immigration counts, independent functional model has weight one.
Forecasts of mortality

Log mortality rates are likely to drop in general, except for men aged around 30.
1) Total fertility rate will decrease until 2015, then increase thereafter.
2) The slight declining, yet uncertain, fertility rates signal a possibility of another period of postponement.
Forecasts of migration

A continuing increase in emigration rates and immigration counts for ages between 20 and 40
Forecasts of out-of-sample population in 2030

<table>
<thead>
<tr>
<th>Year</th>
<th>Total female population (million)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>26</td>
</tr>
<tr>
<td>2000</td>
<td>30</td>
</tr>
<tr>
<td>2020</td>
<td>34</td>
</tr>
<tr>
<td>2030</td>
<td>38</td>
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</tbody>
</table>

<table>
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</tr>
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</tr>
<tr>
<td>2000</td>
<td>60</td>
</tr>
<tr>
<td>2020</td>
<td>70</td>
</tr>
</tbody>
</table>

Baseline 2009 Forecasts
1. Present an independent and a coherent functional model to forecast population via cohort component projection model.
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2. The coherent one models age-specific female and male data jointly, both functional models estimate uncertainty via parametric bootstrapping.
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3. We aim to propose a fully Bayesian functional data approach.
Discussion

1. Present an independent and a coherent functional model to forecast population via cohort component projection model.
2. The coherent one models age-specific female and male data jointly, both functional models estimate uncertainty via parametric bootstrapping.
3. We aim to propose a fully Bayesian functional data approach.
4. It remains a future research to include cohort effect, at least in some of the four demographic components.
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4. It remains a future research to include cohort effect, at least in some of the four demographic components.

5. Thank you.
References:


