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Item 7 – Forecasting demographic components: mortality

Mortality projections in Portugal[1]

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ABSTRACT

Population forecasts are used for important policy decisions both in the public and private sector. Forecasts produced using the cohort-component method requires, for each cohort, a projection of the fertility, migration and mortality components. In this paper we describe the methodology used in the projection of the component mortality within the 2008 Portuguese Population Projections exercise. The methodology is based on a combination of extrapolative and expert-opinion based methods. Specifically, we use the Lee and Carter (1992) log-bilinear model and its extension by Brouhns et al. (2002) based on heteroskedastic Poisson error structures, together with a new variant of the model proposed by Bravo (2007) in which the Poisson-Lee-Carter framework includes a limit life table to which future mortality improvements converge. This allows us to explicitly consider expert judgment together within a statistical extrapolative model, which is important to ensure that forecasted values based on past trends in mortality are within biologically reasonable boundaries. Additionally, we describe the methodology used to close life tables at older ages. Finally, we give an example on how to use this model considering mortality scenarios concerning the future development of mortality generated by the Heligman-Pollard mortality law for the Portuguese female population.

1. INTRODUCTION

Population forecasts are used for important policy decisions both in the public and private sector. The 2008 Portuguese Population Projections exercise provides projections of resident population through 2060, discriminated by age and sex. The projections originate with a base population from 1st January 2008 estimates of resident population and are produced using the cohort-component method. The three components of population change (fertility, mortality, and net migration) are projected separately for each birth cohort (persons born in a given year). The base population is advanced each year by using projected survival rates and net international migration by single year of age and sex. Each year, a new birth cohort is added to the population by applying the projected age specific fertility rates to the female population aged 15 to 45 years, and updating the new cohort for the effects of mortality and net international migration. The assumptions underlying the three components of population change are based on a blend of past trend analysis, expert judgment and stochastic models.

In this paper we describe the methodology used in the projection of the component mortality within the 2008 Portuguese Population Projections exercise. The methodology is based on a combination of extrapolative and expert-opinion based methods. Specifically, we use the Lee and Carter (1992) log-bilinear model and its extension by Brouhns et al. (2002) based on heteroskedastic Poisson error structures, together with a new variant of the model proposed by Bravo (2007) in which the Poisson-Lee-Carter framework includes a limit life table to which future mortality improvements converge. This allows us to explicitly consider expert judgment together within a statistical extrapolative model, which is important to ensure that forecasted values based on past trends in mortality are within biologically reasonable boundaries. The estimated parameters obtained from these models and forecasts for the time trend were used

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in the models' framework to create mortality schedules required for the cohort-component method. Expert's judgments on the future of human longevity in Portugal were accounted for as boundaries to the decline of death rates over the projection period.

The Lee-Carter model assumes that a single dynamic temporal process drives the changes in death rates at all ages over time. This method implicitly assumes that the errors are homoskedastic, which has proven to be unrealistic since the logarithm of the observed death rates is much more volatile at older ages. Brouhns *et al*. (2002) developed a maximum likelihood estimation solution of the Lee-Carter model based on the assumption that the number of deaths follows a Poisson distribution. The Poisson Lee-Carter method is used to model and projected the mortality in the 2008 Population Projections for Portugal.

The Lee-Carter method and its extensions are based on the assumption that the future mortality will continue to improve at the same rate as in the past. Being an extrapolative method, in contexts characterised by the decline of death rates, the model leads us invariably to asymptotic death rates approaching zero, an unlikely scenario according to the experts' judgment on the mortality phenomenon. In order to consider the experts' opinions we resort to an extension of the Lee-Carter model with a limit life table developed by Bravo (2007). This allows us to explicitly consider the expert judgment, which is an important factor in ensuring that the model is not forecasting mortality levels that are beyond biologically feasible levels. Two assumptions where established concerning the future development of mortality: one optimistic and another moderate. In the optimist assumption, the Poisson Lee-Carter was applied. In what concerns the moderate assumption, limits to the decline of age specific deaths rates were imposed through the use of the Heligman and Pollard (1980) mortality law, based on opinions of national experts.

The projection of mortality at advanced ages is particularly important due to the increasingly concentration of deaths at ages more and more advanced, with reductions of mortality beyond these ages having a growing contribution to future gains in life expectancy. Observed death rates at oldest ages, however, show a rather erratic pattern justified by the fact that these figures may be heavily contaminated by random fluctuations, due to the small number of those surviving up to very old ages as well as to the probable misreporting of ages occurred at the censuses to at very old ages. Following the study of mortality in old age in the calculation of complete life tables for Portugal (Coelho, Magalhães and Bravo, 2007), the probability of dying at ages above 85 years for men and women are estimated using the method of Denuit and Goderniaux (2005). The paper is organized as follows. In Section 2 we describe the major trends observed in the Portuguese mortality and the major expected improvements for the next 50 years. In Section 3, we briefly describe the classical Lee-Carter method, the extension proposed by Brounhs *et al.* (2002) and the limit table extension proposed by Bravo (2007). In Section 4, we describe the methodology used to close life tables at older ages. In Section 5 we briefly describe the results of implementing the approaches described above in projecting mortality for the Portuguese female population during the 2008 population projection exercise. Section 6 concludes.

2. PAST AND CURRENT TRENDS IN MORTALITY AND LONGEVITY IN PORTUGAL

Life expectancy at birth in Portugal has nearly doubled in less than a century, attaining 75.18 years for men and 81.57 years for women in the period 2005- 2007. Similar to other developed countries, two major trends dominated the mortality decline in Portugal during the last century: a huge decrease in infant mortality, more evident during the first half of the century, and a decrease in mortality at older ages, more pronounced during the second half. This pattern is somehow expected since the ongoing increase in life expectancy is driven by the mortality decline among older persons. This has important consequences in many areas, ranging from population projections to the provision of health care or the management of social security systems.

In the opinion of experts, further improvements in mortality among the elderly might still be expected in the future. A common prediction among researchers on the limits of human longevity is that the decline in premature deaths will continue to occur in the future. The modal age at death, which can be interpreted as an indicator of the mean longevity at a given time, increased steadily in the last decades. Among adults, the variability of age at death decreased during the last decades, signalling a compression of mortality (rectangularization of the survival curve). As a consequence, the proportion of those surviving up to older ages increased and the age of maximum mortality gradually shifted towards older ages.

In what concerns infant mortality, there is not great scope for further gains as the current levels are already very low. In 2007, infant mortality rate was 3.4 deaths per thousand live births. This figure is not far from the estimated endogenous mortality (mortality occurring in the first year of life resulting from congenital, hereditary defects or injuries caused during delivery) that, by definition, is not preventable. Experience from other developed countries regarding the effects of improvements in legislation on occupational safety and traffic accidents supports expectations of further declines in mortality at young and medium adult ages.

Concerning differences in longevity between men and women, the pace of growth of average life expectancy of women has been historically higher, contributing to the increase of the longevity gap between men and women. However, in recent years we observe a reduction in the gender life expectancy at birth gap. This convergence phenomenon has been already observed for some time in countries like the Netherlands, Sweden and Denmark but is relatively recent in Portugal. Between 1997 and 2007, the difference in life expectancy at birth declined from 7.3 years to 6.4 years. In the opinion of experts, the evolution of gender differences in mortality is consistent with lifestyle (e.g., smoking) patterns. Recent changes will probably reduce sex differences in longevity in the coming decades.

To sum up, recent trends in mortality indicate that further declines in mortality can be expected, particularly at advanced ages. Longevity improvements are also expected from the decline in "avoidable" mortality at adults ages, particularly associated with the reduction of risk of death from external causes, particularly among males (flattening of the accident hump). Increases in average life expectancy of the population will continue to occur in the future, however at a slower pace than in the past.

The model of mortality in Portugal has changed profoundly in the last century. The probability of occurrence of similar reductions in mortality levels, namely the dramatic reductions in infant mortality, is virtually zero. Future improvements in mortality will tend to be similar to the most recent trends in mortality. So, more recent data on mortality is the most relevant to the establishment of assumptions about future behaviour of mortality. Therefore, for modelling and projection of mortality we have considered estimates of the resident population on 1st January 2008, by sex and single age, the number of deaths by sex, age and year of birth and the number of live births by sex, for the period 1980 - 2007.

3. LOG-BILINEAR MODEL FOR MORTALITY FORECASTING: LEE-CARTER MODEL AND EXTENSIONS

Mortality forecasting methods currently in use can be classified in many different ways. Roughly speaking, they can be clustered into extrapolative methods, explanatory methods and expert-opinion based methods (Booth and Tickle, 2008). Extrapolative methods assume that future mortality patterns can be estimated by projecting into the future trends observed in the recent to medium-term past. This approach includes the classical and relatively simple extrapolation of aggregate measures such as life expectancy, as well as stochastic and more complex methods such as the Lee-Carter method. Explanatory methods of mortality forecasting are based on structural or causal epidemiological models and analyse the relationship between age-specific risk factors (e.g., smoking, obesity, socio-economic status, marital status) and their effects on mortality. Expert-opinion methods involve the use of informed expectations about the future, often accompanied by some alternative low and high scenarios, or a targeting approach. These methods have the advantage of incorporating, in a qualitative way, demographic, epidemiological, medical and other relevant knowledge, but its relative subjectivity and potential for bias should be taken into attention. The mortality component of the 2008 Portuguese Population Projections exercise is based on a combination of extrapolative and expert-opinion based methods. Specifically, we use the log-bilinear Lee and Carter (1992) model and its extension by Brouhns et al. (2002) and a new variant proposed by Bravo (2007) in which the Poisson-Lee-Carter framework includes a limit life table to which future mortality improvements converge.

3.1. The Lee-Carter Model

The Lee-Carter method (Lee and Carter, 1992) combines a demographic model, describing the historical change in mortality, a method for fitting the model and a time series model for the time component which is used for forecasting. The classical two-factor Lee-Carter model is

$$\ln\left(m_{x,t}\right) = \alpha_x + \beta_x k_t + \varepsilon_{x,t} \qquad (1)$$

where $m_{x,t}$ denotes the central mortality rate at age x in year t, k_t represents a time-index level of mortality, α_x and β_x are vectors of age-specific constants denoting, respectively, the general (average over time) pattern of mortality by age and the relative rate of response at age x to changes in the overall level of mortality over time, and $\varepsilon_{x,t}$ is the residual. The residual term, $\varepsilon_{x,t}$, with mean 0 and variance σ_{ε}^2, reflect particular age-specific historical influences not captured by the model. The equation underpinning the Lee-Carter model is known to be over parameterized. To ensure model identification, Lee and Carter (1992) add the following constraints to the parameters

$$\sum_{x=x_{\min}}^{x_{\max}} \beta_x = 1, \qquad \sum_{t=t_{\min}}^{t_{\max}} k_t = 0 \qquad (2)$$

to obtain unique parameter estimates. As a result of these constraints, the parameter α_x is calculated simply by averaging the $\ln(m_{x,t})$ over time.

The main statistical tool of Lee and Carter (1992) is least-squares estimation via singular value decomposition of the matrix of $\ln(m_{x,t})$. The authors incorporated an adjustment to the estimated k_t so that fitted deaths match observed total deaths in each year. To forecast, Lee and Carter assume that α_x and β_x remain constant over time and forecast future values of k_t using a standard ARIMA univariate time series model.

One of the main criticisms to the Lee-Carter method (see, e.g., Lee and Miller, 2001, Booth and Tickle, 2008) refers to the hypothesis that residuals $\varepsilon_{x,t}$ are normally distributed with constant variance. This homoskedasticity hypothesis is quite unrealistic since the logarithm of the observed mortality rates is much more variable at older ages than at younger ages because of the much smaller absolute number of deaths at older ages. Between the many improvements to the Lee-Carter estimation basis Brouhns et al. (2002) developed an alternative approach to mortality forecasting based on heteroskedastic Poisson error structures. In this extension of the LC method, ordinary least-squares are replaced with Poisson regression for the death counts and model parameters are estimated by maximizing a Poisson log-likelihood.

3.2. The Poisson Lee-Carter Model

Assume that the age-specific forces of mortality are constant within bands of age and time, i.e., within each rectangle of the Lexis diagram, but allowed to vary from one band to the next. More formally, given any integer age x and calendar year t, we assume that

$$\mu_{x+\xi,t+\tau} = \mu_{x,t} \text{ for } 0 \le \xi,\ \tau < 1. \tag{3}$$

Under this constant force of mortality assumption, $\mu_{x,t}$ may be estimated as the quotient between the number of deaths and the number of exposed to the risk of dying or $m_{x,t}$. Brouhns *et al*. (2002) developed a maximum likelihood estimation solution of the Lee-Carter model based on the assumption that $D_{x,t}$, the number of deaths recorded at age x during calendar year t, follows a Poisson distribution, i.e.,

$$D_{x,t} \sim Poisson(\mu_{x,t} E_{x,t}) \tag{4}$$

with

$$\mu_{x,t} = \exp(\alpha_x + \beta_x k_t) \tag{5}$$

These deaths originate from an exposure-to-risk $E_{x,t}$. The model preserves the log-bilinear structure for $\mu_{x,t}$ but replaces the classical assumptions on the error term $\varepsilon_{x,t}$ by a Poisson law for $d_{x,t}$. In spite of this, parameters α_x, β_x and k_t maintain, in essence, their original interpretation. Instead of resorting to SVD procedures, parameter estimates maximize the following log-likelihood function

$$L(\alpha_x, \beta_x, k_t) = \sum_{x=x_{\min}}^{x_{\max}} \sum_{t=t_{\min}}^{t_{\max}} \left\{ d_{x,t}(\alpha_x + \beta_x k_t) - E_{x,t} \exp(\alpha_x + \beta_x k_t) \right\} + c \tag{6}$$

where c is a constant. The presence of the log-bilinear term $\beta_x k_t$ in (5) prevents the estimation of model parameters using standard statistical packages that include Poisson regression. Because of this, we resort to an iterative algorithm for estimating log-bilinear models developed by Goodman (1979) based on a Newton-Raphson algorithm.[5] Finally, a reparametrization of the model is necessary in order guarantee that the parameter estimates α_x, β_x and k_t generated by the ML procedure verify the model identification constraints. To forecast, as in the Lee–Carter method we use the above time series methods to make long-run forecasts of age–sex-specific mortality rates.

The Poisson-Lee-Carter model has some advantages over the classical version of the model that make it especially attractive. First, the model explicitly recognizes the integer nature of $D_{x,t}$ unlike the Lee-Carter method. Second, the

[5] Details of the fitting procedure can be found in Brouhns et al. (2002).

model drops the assumption of homoscedasticity of the error term and recognizes the greater variability of $\mu_{x,t}$ at older ages. Third, the possibility of using maximum likelihood methods to estimate the parameters instead of using the least squares method implemented by singular value decomposition makes the estimation more efficient. Finally, contrary to the classical LC approach there is thus no need of a second-stage estimation of k_t since the error applies directly on the number of deaths in the Poisson regression approach.

One of the virtues of both the classical and the Poisson versions of the Lee-Carter method concerns the way the demographic model is defined, which ensures that death rates exhibit a pattern of exponential decrease, without imposing any arbitrary asymptotic limit or restriction that limit the gains in life expectancy. Although this behaviour is consistent with a pattern of mortality decline observed in developed countries, when the model parameters are estimated from empirical mortality data we usually find positive β_x's and a decreasing trend for k_t. Positive β_x's imply that the death rates are decreasing in the k_t's or, put in another way, if the k_t's are projected to decline in the future life lengths will continue to increase indefinitely. Given the current knowledge on human longevity, this result is unacceptable and should somehow be incorporated in the projection exercise.

3.3. Poisson Lee-Carter Model with Limit Life Table

The asymptotic behaviour of deaths rates projected by the Lee-Carter model and the Poisson Lee-Carter might, however, in some cases, prove to be unsatisfactory. Empirical studies conducted on this models (including some on the Portuguese population) show a clear downward trend for the estimated time index $\hat{k}_t$, and positive estimates of β_x, a result anticipated in a context characterised by a mortality decline over time. In this sense, the use of time-series methods to extrapolate $\hat{k}_t$ over long-term horizons leads us invariably to asymptotic deaths rates approaching zero. Formally, given positive β_x's and finite α_x's it is clear that

$$\lim_{\hat{k}\to-\infty} \exp\left(\hat{\alpha}_x + \hat{\beta}_x \hat{k}_t\right) = 0 \quad (7)$$

This is an unlikely scenario based on our current understanding of the mortality phenomena. Empirical studies showed that extrapolation without constraints usually produces implausible results in the long run. Thus, choosing an upper limit to life span is a reasonable approach in projecting mortality. Concerns over the asymptotic behaviour of projection models motivated the development of solutions that require, in principle, an arbitrary positive number for deaths rates in the long run. Among the arguments for a limited duration of life, the most critical is the one that states that there is a decline in the physiological parameters associated with ageing in humans, but other arguments include stylized facts such as the slowdown in life expectancy at birth increases observed in Portugal and in other several developed countries.

The approach used in these studies lies in the field of so-called projection models with limit table (or objective table). Initially developed by Bourgeois-Pichat (1952), these models admit the existence of an "optimal" life table to which longevity improvements over time converge. In other words, these models explicitly admit that there are natural limits to human longevity, i.e., mortality levels below which it is considered impossible to descend in the future (or at least in a given time horizon).

In this line of research, Bravo (2007) developed an extension of the Poisson Lee-Carter model in which future mortality developments are guided by a particular limit life table to which future longevity improvements tend to converge. Let $\mu_x^{\lim}$ and $q_x^{\lim}$ denote the instantaneous death rates and probabilities of death corresponding to this target life table. The incorporation of a limit life table on the Poisson model requires the replacement of the hypothesis (4) by

$$D_{x,t} \sim Poisson\left(E_{x,t}(\mu_{x,t}^{\lim} + \mu_{x,t}^{ad}\right) \quad (8)$$

with

$$\mu_{x,t}^{ad} = \exp\left(\alpha_x + \beta_x k_t\right) \quad (9)$$

and the usual identification restrictions.

As can be observed, the model stipulates that the number of deaths expected at age x in year t is determined by the exposure $E_{x,t}$ and by a force of mortality that results from the sum of the limit value $\mu_x^{\lim}$ with the additional value (contemporary) μ_x^{ad}, defined by the classical Lee-Carter equation. Given the above assumptions, parameter estimates are again obtained by maximizing the log-likelihood function, resorting to an iterative algorithm adapted from Goodman (1979). Finally, the initial parameter estimates generated by the algorithm are adjusted so as to comply with the identification constraints.[6]

One of the critical aspects in the application of projection models with limit table refers to the selection of the limit table, i.e. the definition of what are considered the plausible limits to human longevity. To determine this limit, a number of subjective or informed assumptions about the future development of a set of important biological, economic and social variables have to be made. In practical applications, this prospective exercise may however reveal very difficult or even impossible. In this case, a different way of interpreting the model is to consider the limit table as a life table that reflects the pattern of mortality in a more advanced population in terms of economic and social development (target population) to which current experience will converge in a give time horizon. An alternative solution is to take some parametric function (mortality law) on which to examine different scenarios on the main trends in human longevity (e.g., rectangularization of the survival function, evolution of life expectancy, mode of the survival function, entropy,...) by incorporating recent statistical information and expert-opinion judgements.

Among the many mortality laws considered, the 2008 Portuguese population projection exercise finally used the so-called second Heligman and Pollard (1980) mortality law, defined by

$$q_x = A^{(x+B)^C} + D\exp\left[-E(\ln x - \ln F)^2\right] + \frac{GH^x}{1+KGH^x} \tag{10}$$

where q_x denotes the death probability at age x and A, B, C, D, E, F, G, H and K are parameters to be estimated by non-linear weighted least squares methods.

Equation (10) includes three distinct terms, each reflecting a separate component of mortality. The first term, an exponential function rapidly decreasing, represents the decrease in mortality during the first years of life. The second term, a sort of lognormal function, represents the mortality at intermediate adult ages and is referenced in demographic and actuarial literature as describing the incidental mortality for both sexes, and maternal mortality for the female population. The third term reflects the traditional Gompertz mortality law, which reflects the exponential growth of mortality at older ages.

4. CLOSING THE LIFE TABLE AT OLDER AGES

The estimate of the gross death rates is in general possible only up to an age limit relatively far away from the maximum survival age. The calculation of crude age specific mortality rates at advanced ages suffers from several problems. The main issue concerns the quality and the availability of data on population estimates for the oldest-old. Effectively, although data on deaths are in general of good quality, mortality rates may be contaminated by random fluctuations due to either the small number of those surviving up to very old ages, to age misreporting problems or to the lack of coherence between deaths and the number of those exposed to risk.

In order to construct complete life tables, it was decided to remove fluctuations by smoothing crude estimates via a projection (closing) method. Various methodologies have been proposed for estimating mortality rates at oldest ages[7]. From these, we adopted the method proposed by Denuit and Goderniaux (2005), a method that is applied directly to crude death probabilities and establishes a limiting age for the life table. Formally, the following log-quadratic model is fitted by weighted-least squares:

$$\ln \hat{q}_x = a + bx + cx^2 + \varepsilon_x \quad , \quad \varepsilon_x \sim N\left(0, \sigma^2\right) \tag{11}$$

to age-specific death probabilities observed at advanced ages. Two restrictions are imposed to equation (11), as to assure a concave configuration to the mortality curve at older ages and restrict a horizontal tangent at the maximum age point considered. The imposed restrictions are:

$$q_{x_{\max}} = 1 \tag{12}$$

$$q'_{x_{\max}} = 0 \tag{13}$$

The inclusion of (12) and (13) into (11) will lead to a new expression of the model equation, given by

[6] Details of the fitting procedure can be found in Bravo (2007).
[7] For a review see, *e.g.*, Buettner (2002) and Pitacco (2004).

$$\ln \hat{q}_x = \left(x_{\max}^2 - 2x(x_{\max}) + x^2\right)c + \varepsilon_x \quad , \quad \varepsilon_x \sim N\left(0, \sigma^2\right) \qquad (14)$$

To understand the influence of the limit age on the performance of the model, we tested three different versions of (11) considering $x_{\max} \in \{110, 115, 120\}$. The final value for $x_{\max}$ is chosen to be the one that better describes the data.

One of the critical aspects related to this model is determining the adequate age from which to replace the gross mortality probabilities by its correspondent adjusted estimates. The ad hoc method suggested in Denuit and Goderniaux (2005) and Bravo (2007), among others, recommends choosing the age so that the regression coefficient R^2 is maximized, by ranging the initial age of the calibration procedure in the $[50, 85]$ interval. Attention is also drawn to the possible need for smoothing the mortality series around the cut age point, so as to avoid abrupt discontinuities between the two series. The suggestion is to replace initial estimates $\hat{q}_x$ by a five-year geometric average of the death probabilities around ages $x = x_0 - 5, ..., x_0 + 5$.

5. SOME ILLUSTRATIVE RESULTS OF THE 2008 MORTALITY PROJECTION EXERCISE

In this section we briefly describe the results, published by Statistics Portugal, of implementing the approaches described above in projecting mortality and estimating life expectancy for the Portuguese female population during the 2008 population projection exercise.[8] The database used in this exercise comprises two elements: the observed number of deaths and the population size at December 31 of each year. These two data sets were published by Statistics Portugal and are classified by age (ranging from 0 to 100 or older) and sex and were considered in the estimation window 1980-2007. Figure 1 gives a first indication of the evolution of the female mortality in the estimation period. Three trends dominate the mortality decline: a reduction in mortality at younger ages; a decline in the importance of the mortality hump in the age interval 15-30; a decline in the mortality at older ages.

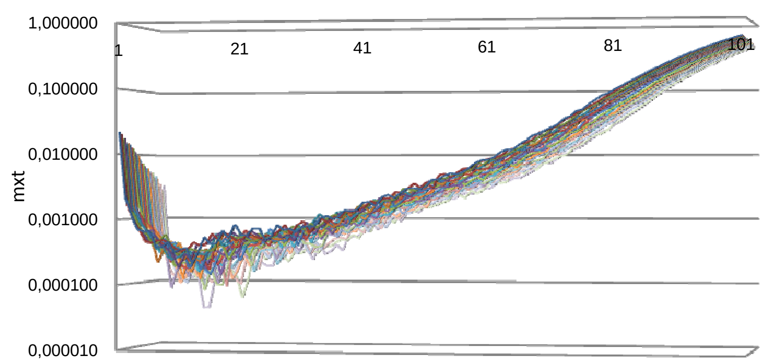

Figure 1 – Crude mortality curves for the period 1980-2007, female

We apply the Poisson-Lee-Carter with and without limit table models to the Portuguese female mortality data. First, we calibrate the second Heligman and Pollard (1980) mortality law to data by estimating the model parameters using non-linear weighted least squares methods. Next, we define a set of high and low mortality scenarios by changing the model parameters in order to reflect alternative expert-opinion judgements about the evolution of the rectangularization of the survival function, about the evolution of life expectancy and so on. Each scenario for the model parameters defines a scenario for the limit life table in the horizon 2060. Figure 2 gives an example of a mortality scenario generated by the Heligman-Pollard (HP) mortality law based on the following parameters

[8] For a detailed analysis of the results see INE (2008).

Parameter	A	B	C	D	E	F	G	H	K
Coefficient	0,00023	0,05	0,10	0,00150	2,200	85,0	0,00001	1,10350	0,060

Table 1: Parameter values of a specific mortality scenario generated by the HP mortality law

This is a relatively conservative scenario that stipulates little margin for mortality improvements at younger ages but leaves significant room for longevity increases at middle-aged and older ages.

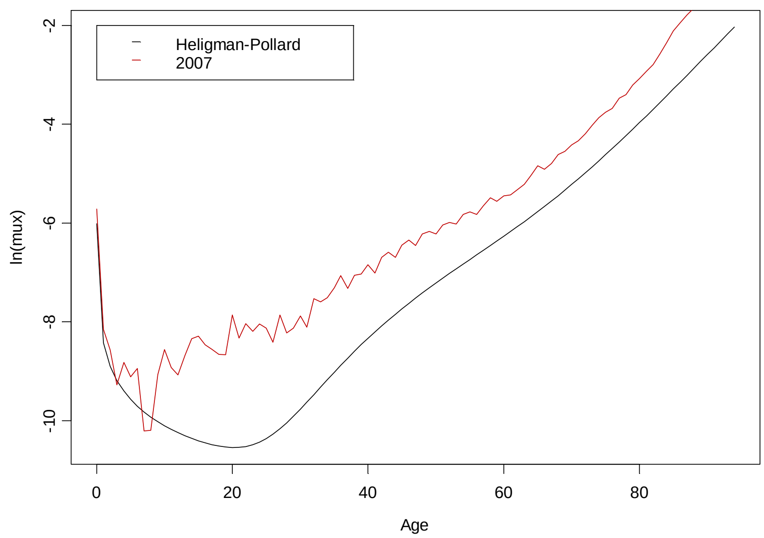

Figure 2: Crude (2007) mortality rates and mortality scenario generated by the Heligman-Pollard mortality law for the horizon 2060

Next, for each scenario we estimate the parameters of the Poisson-Lee-Carter model with (restricted) and without (unrestricted) limit table using the maximum-likelihood estimation procedure presented above. In the Poisson-Lee-Carter with limit table case, the target life table is given by the corresponding HP mortality law scenario. The iterative algorithm is started considering the following initial values: $\hat{\alpha}_x^{(0)} = 0$, $\hat{\beta}_x^{(0)} = 1$ and $\hat{k}_t^{(0)} = 0.1$. The criterion used to stop the iterative algorithm is a very small increase in the log-likelihood function (in our case 10^{-6}). The routine was implemented within the SAS statistical package.

Figures 3 and 4 plot the estimated α_x, β_x and k_t (for the female population) generated by the standard Poisson-Lee-Carter (PLC) model and by the PLC with limit table for the above HP scenario. As can be seen, the parameter estimates of PLC model with limit table exhibit, roughly speaking, the same patterns generated by the classical model.

The fitted values of α_x, i.e., the general shape of the mortality schedule is, has expected, slightly lower in the limit table case, particularly for those ages for which the limit table scenario considered stipulates some room for mortality improvements.

The fitted values of β_x, that represent the age-specific patterns of mortality change, are very similar in both models. However, we can observe that the shape of the β_x profile tells us that, roughly speaking, mortality rates for ages below (above) 50 are relatively more (less) sensitive to changes in the time trend under the traditional PLC model when compared with its limit table version. Finally, estimates of k_t show that the (negative) slope of the shape of the time trend is slightly more pronounced in PLC with limit table case.

To have a more comprehensible perception on the influence of the inclusion of a limit life table within the PLC model we represent in Figure 5 the crude estimates of $\hat{\mu}_{80,t}$ for the period 1980-2007, together with its projected values generated by the two version of the PLC mortality model for the period 2008-2060.

As can be seen, the convergence of projected mortality rates towards a zero mortality level implicit in the traditional PLC model is replaced by a steady convergence of projected mortality rates towards a positive lower limit $\mu_{80,t}^{\lim} = 0{,}01864$ defined by the target limit life table.

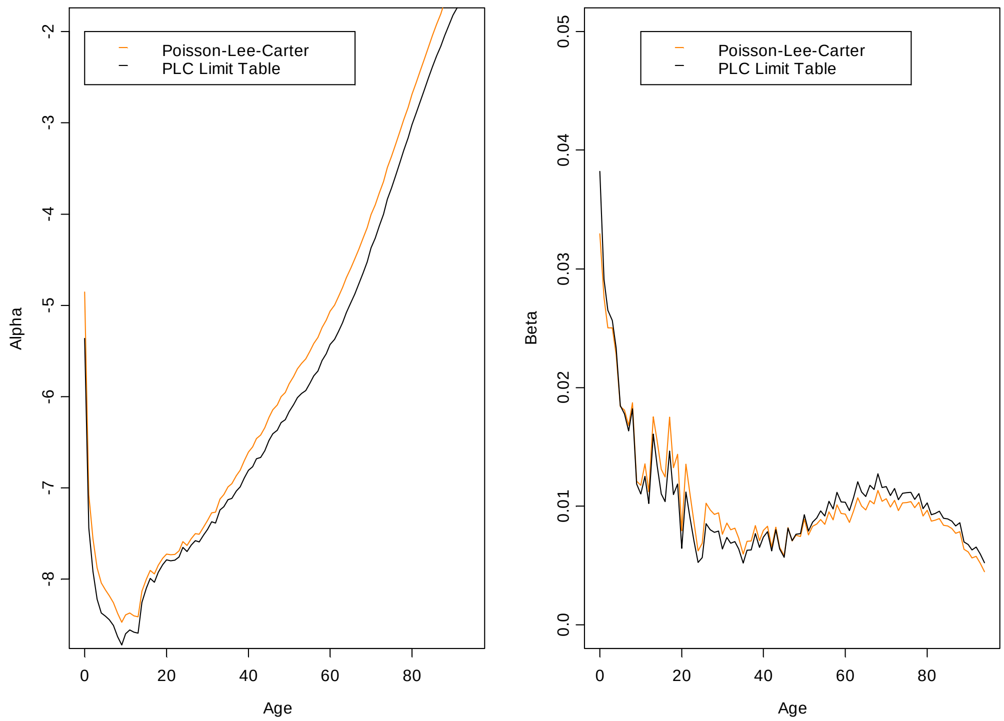

Figure 3: Estimates of α_x and β_x generated by the Poisson-Lee-Carter with/without limit table

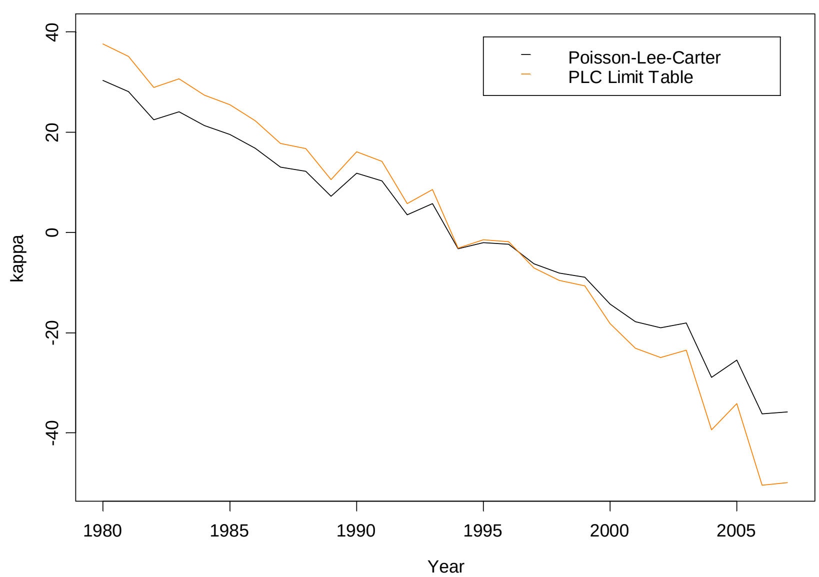

Figure 4: Estimates of k_t generated by the Poisson-Lee-Carter (PLC) with/without limit table

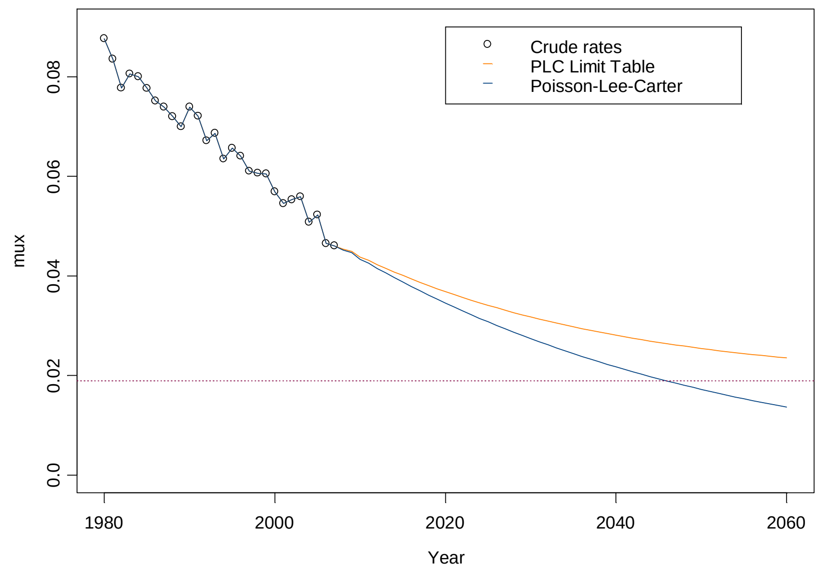

Figure 5: Crude and projected mortality rates for the period 1980-2060, age 80

6. CONCLUSION

Mortality projections are one the most important tasks in determining the level of the population's natural growth, population's growth rate, in evaluating the population health and social levels, in calculating mortality prospects and creating life tables. In this paper we describe the methodology used in the projection of the component mortality within the 2008 Portuguese Population Projections exercise. The methodology is based on a combination of extrapolative and expert-opinion based methods.

The paper briefly summarizes and compares three different models that have been developed in the literature for modelling and forecasting human mortality rates over the age range. The first two models are the classical Lee-Carter log-bilinear model and its extension considering heteroskedastic Poisson error structures. The third model is a new variant of the LC model in which the Poisson-Lee-Carter framework includes a limit life table. The model is implemented by setting up scenarios for future mortality rates using the second Heligman and Pollard mortality law.

We give an overview of the methods, discuss their applications and evaluate their performance.

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