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Item 7 – Forecasting demographic components: mortality

Lee-Carter mortality projection with "Limit Life Table"

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ABSTRACT

The Lee-Carter Model and extensions have been used for decades by official Statistic Bureaus has the standard framework for projecting future mortality rates in population projections. Intrinsically, the model assumes that the dynamics of death rates over time are driven by a single time-varying parameter and that mortality forecasts rely on the extrapolation of this index using appropriate statistical time-series methods. Despite its simplicity and appealing features, the asymptotic behaviour of mortality rates projected by LC model cannot be considered satisfactory. Empirical studies conducted using LC model show a decreasing pattern for the time index parameter k_t, combined with positive finite parameters α_x and β_x. In this scenario, it can easily be shown that the extrapolation of past time index trends into the future will invariably lead to zero mortality rates at all ages. In this paper we develop a new variant of the so called Poisson Lee-Carter model in which mortality projections are bounded by a limit life table to which future mortality improvements converge over time. This model explicitly assumes that over a fixed time range there are lower bounds to mortality rates. We assume that these limit rates are exogenously determined, either by expert subjective judgements on the limits to human longevity, or by considering that the limit table resembles that of a more advanced population in terms of socio-economic conditions (target life table), or by admitting that the limit table can be expressed by a parametric mortality law that conveys information on the main trends in population mortality.

1. INTRODUCTION

The Lee-Carter Model and extensions have been used for decades by official Statistic Bureaus has the standard framework for projecting future mortality rates in population projections. Intrinsically, the model assumes that the dynamics of death rates over time are driven by a single time-varying parameter and that mortality forecasts rely on the extrapolation of this index using appropriate statistical time-series methods.

The Lee-Carter method and its extensions belong to a class of extrapolative methods which assume that future mortality patterns can be estimated by projecting into the future trends observed in the recent to medium-term past. Despite its simplicity and appealing features, the asymptotic behaviour of mortality rates projected by LC model cannot be considered satisfactory. In fact, empirical studies conducted using LC model show a decreasing pattern for the time index parameter k_t, combined with positive finite parameters α_x and β_x. In this scenario, it can easily be shown that the extrapolation of past time index trends into the future will invariably lead to zero mortality rates at all ages, an unlikely scenario according to the experts' judgment on the mortality phenomenon.

As an alternative to extrapolative methods, expert-opinion methods involve the use of informed expectations about the future, often accompanied by some alternative low and high scenarios, or a targeting approach. These methods have the advantage of incorporating, in a qualitative way, demographic, epidemiological, medical and other relevant knowledge, but its relative subjectivity and potential for bias should be taken into attention.

In this paper, we develop a new variant of the LC model, formulated within a Generalized Linear Model framework with a generalised error distribution, in which mortality projections are bounded by a limit life table to which future

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mortality improvements converge over time. This model explicitly assumes that over a fixed time range there are lower bounds to mortality rates. We assume that these limit rates are exogenously determined, either by expert subjective judgements on the limits to human longevity, or by considering that the limit life table resembles that of a more advanced population in terms of socio-economic conditions (target life table), or by admitting that the limit life table can be mathematically represented by a parametric mortality law that conveys information on the main trends in population mortality. The methodology is based on a combination of extrapolative and expert-opinion based methods and has been used in the projection of the component mortality within the 2008 Portuguese Population Projections exercise. This methodology allows us to explicitly consider expert judgment together within a statistical extrapolative model, which is important to ensure that forecasted values based on past trends in mortality are within biologically reasonable boundaries. The paper is organized as follows. In Section 2, we briefly describe the classical age-period Lee-Carter mortality forecasting method, focusing on the asymptotic properties of projected mortality rates. In Section 3, we develop an extension of the LC considering the existence of a limit life table. We discuss the critical aspects in the application of this LC variant, namely the alternatives approaches that can be followed to select an appropriate limit life table. Section 4 concludes.

2. CLASSICAL LEE-CARTER MORTALITY MODELLING

The classical LC modelling approach has dominated the recent literature in the field of mortality forecasting (see Brouhns et al. 2002, Booth 2006, Booth and Tickle 2008 and further references therein). According to Booth and Tickle (2008), the LC-based approach is broadly considered in the current literature to be among the most efficient and transparent methods used to generate plausible life expectancy forecasts for use within population projection exercises, prospective life table construction or life insurance actuarial calculations.

The classical age-period AP LC model was first introduced by Lee and Carter (1992), combining a demographic model for the mortality rate, dependent only on factors related to age and period, describing the historical change in mortality, a method for fitting the model and a time series model for the time component which is used for forecasting. The classical AP LC model is expressed as

$$\ln\left(m_{x,t}\right) = \alpha_x + \beta_x k_t + \varepsilon_{x,t} \qquad (1)$$

where $m_{x,t}$ denotes the central mortality rate at age x in year t, k_t represents a time-index level of mortality, α_x and β_x are vectors of age-specific constants denoting, respectively, the general (average over time) pattern of mortality by age and the relative rate of response at age x to changes in the overall level of mortality over time, and $\varepsilon_{x,t}$ are Gaussian distributed $N(0,\sigma_\varepsilon^2)$ random effects by age and time. The equation underpinning the Lee-Carter model is known to be over parameterized due to the log-bilinear multiplicative term $\beta_x k_t$. Because of this, the identifiability problem is traditionally resolved by ensuring that parameters β_x and k_t satisfy the following constraints

$$\sum_{x=x_{\min}}^{x_{\max}} \beta_x = 1, \qquad \sum_{t=t_{\min}}^{t_{\max}} k_t = 0 \qquad (2)$$

As a result of these constraints, the parameter α_x is calculated simply by averaging the $\ln(m_{x,t})$ over time. The main statistical tool of Lee and Carter (1992) is least-squares estimation via SVD decomposition of the matrix of $\ln(m_{x,t})$. The authors incorporated an adjustment to the estimated k_t so that fitted deaths match observed total deaths in each year. In the classical AP LC type modelling approach, the age effects (α_x and β_x) are assumed to be constant in time and the time-variant period effects are projected forward using autoregressive time series models. Specifically, the period factors k_t are extrapolated in time by standard univariate stochastic ARIMA processes in order to make forecasts of the future force of mortality and, implicitly, future (period- and cohort-based) life expectancy.

In terms of forecasting, the LC family of models are part of the extrapolative stochastic methods that assume that future mortality patterns can be estimated by projecting into the future the historical trends of human mortality observed in the recent to medium-term past. Although the validity of these assumptions is frequently debated (see, e.g., Gutterman and Vanderhoof 2000), the inherent complexity of the factors affecting human mortality and the current lack of understanding of the intricate mechanisms governing the aging process, together with the relative stability of the past trends, is used by many authors as a justification for the use of past trends as a reliable basis for future projections.

The way the demographic model is defined in the LC family of models ensures that death rates exhibit a pattern of exponential decrease, without imposing any arbitrary asymptotic limit to future gains in life expectancy. Although this behaviour is consistent with the pattern of mortality decline observed in developed countries, the asymptotic behaviour of deaths rates (or life expectancy) projected by the LC family of models should somehow be considered unsatisfactory. In fact, most empirical studies conducted using the LC models (see, e.g., Lee and Carter 1992, Brouhns et al. 2002), including some on the Portuguese population (Bravo 2007, INE 2008), show a clear downward trend for the estimated time index $\hat{k}_t$ and positive estimates of β_x, a result anticipated in a context characterised by a mortality decline over time. Assuming that the age effects are constant in time, the use of time-series methods to extrapolate $\hat{k}_t$ over long-term horizons leads us invariably to asymptotic deaths rates approaching zero. Formally, given positive β_x's and finite α_x's it is clear that

$$\lim_{k\to-\infty} \mu_{x,t} = \lim_{k\to-\infty} \exp\left(\hat{\alpha}_x + \hat{\beta}_x \hat{k}_t\right) = 0 \qquad (3)$$

This is an unlikely scenario based on our current understanding of the mortality phenomena. The linear projections of log of mortality rates implicit in the LC method may produce implausible age patterns in the long run if any assumed rate trends differ, since these differences will be augmented in the projections. For instance, linear extrapolation of past LE trends for Portugal and Japan will lead to huge future differences between these two countries because of their different past trends. The same argument is valid for extrapolation of male and female LE. In effect, since in most countries male and female LE at birth are converging, simple extrapolation of these trends will inevitably lead to male LE exceeding female LE. Moreover, forecasts based on the LC model are likely to imply increasing divergence in life expectancy in the long run, contradicting the observation made by Wilson (2001) who documented a global convergence in mortality. This motivated modifications in the LC method to ensure non-divergence (see, e.g., Lee 2000).

3. LEE-CARTER MODEL WITH "LIMIT LIFE TABLE"

Concerns over the asymptotic behaviour of projection models motivated the development of solutions that require, in principle, an arbitrary positive number for deaths rates in the long run. The approach used in these studies lies in the field of so-called projection models with limit table (or objective table). Initially developed by Bourgeois-Pichat (1952), these models admit the existence of an "optimal" life table to which longevity improvements over time converge. In other words, these models explicitly admit that there are (at least in a limited time horizon) natural limits to human longevity, i.e., mortality levels below which it is considered impossible to descend in the projection interval. There are many arguments used as a justification for a limited duration of life, the most critical is the one that states that there is a decline in the physiological parameters associated with ageing in humans, but other arguments include stylized facts such as the slowdown in life expectancy at birth increases observed in many developed countries.[2]

In this paper we present and upgrade an extension of the Lee-Carter model developed by Bravo (2007) in which future mortality developments are guided by a particular limit life table to which future longevity improvements tend to converge. Let $\mu_x^{\lim}$ and $q_x^{\lim}$ denote, respectively, the instantaneous death rate and the probability of death corresponding to this target life table. Assume that, given any integer age x and calendar year t, the age-specific forces of mortality are constant within each rectangle of the Lexis diagram but allowed to vary from one to the next, i.e.,

$$\mu_{x+\xi,t+\tau} = \mu_{x,t} \text{ for } 0 \le \xi,\ \tau < 1. \qquad (4)$$

We follow Brouhns *et al*. (2002) and assume that the LC model can be formulated within a Generalized Linear Model (GLM) framework with a generalised error distribution. Specifically, we assume that the age- and period-specific numbers of deaths $D_{x,t}$ are independent realizations from a Poisson distribution with parameters

$$E\left[D_{x,t}\right] = E_{x,t}\mu_{x,t} \text{ and } Var\left[D_{x,t}\right] = \phi E\left[D_{x,t}\right] \qquad (5)$$

where $E_{x,t}$ denotes the number of individuals exposed-to-risk at age x during calendar year t, and ϕ is a measure of over-dispersion to allow for heterogeneity. To incorporate an upper limit to life span in the LC family of models, we replace parameterization (1) by

[2] We note that this slowdown in LE increases is paralleled by an accelerated decline in mortality at older ages. Moreover, the fastest decline of mortality has been observed in countries with the lowest levels of old-age mortality, i.e., the opposite of what is expected if mortality were pushing against an upper limit.

$$\mu_{x,t} = \mu_x^{\lim} + \mu_{x,t}^{ad} \tag{6}$$

with

$$\mu_{x,t}^{ad} = \exp\left(\alpha_x + \beta_x k_t\right) \tag{7}$$

As can be observed, the model stipulates that the number of deaths expected at age x in year t is determined by the corresponding exposure $E_{x,t}$ and by a force of mortality that results from the sum of the limit value $\mu_x^{\lim}$ with an additional (contemporary) value $\mu_{x,t}^{ad}$, defined by the classical Lee-Carter parameterization. In spite of this, parameters α_x, β_x and k_t maintain, in essence, their original interpretation. Equation (6) provides a simple (additive) solution to incorporate the existence of a limit life table within the LC framework. Alternative formulations could be considered, e.g., by assuming different rates of converge to the target value over time.

Equations (5), (6) and (7) correspond to a GLM model of the response variable $D_{x,t}$ with logarithmic link and non-linear parameterized predictor $\eta_{x,t}$:

$$\begin{aligned} \eta_{x,t} &= \log\left(\hat{d}_{x,t}\right) = \log\left(E_{x,t}\mu_{x,t}\right) \\ &= \log\left(E_{x,t}\right) + \log\left(\mu_x^{\lim}\right) + \alpha_x + \beta_x k_t \end{aligned} \tag{8}$$

In order to obtain unique parameter values, the above model is formulated in line with the together with usual identification restrictions (2), while $\log\left(E_{x,t}\right) + \log\left(\mu_x^{\lim}\right)$ is treated as an offset value during fitting.

Model (8) is conceptually different from the original LC framework (1), because the modelling errors have a generalised class of distribution (member of the exponential family) that are determined by the direct fitting of the number of deaths instead of the logarithmic transform of the mortality rates. That is, the GLM regression is based on ML methods with theory-based distributional assumptions in contrast to the SVD fitting, which relies on empirical measures (i.e. least squares). Moreover, the parameter estimates under the original framework (1) can be considered a particular case of the GLM regression since they can derived within the GLM approach by adjusting the target variable to $D_{x,t} = \log\left(m_{x,t}\right)$ and applying the identity link function with a Normal error structure.

Given equations (5), (6) and (7) and model assumptions, it can be shown that the parameter estimates are obtained by maximizing the following log-likelihood function

$$\begin{aligned} L(\alpha_x, \beta_x, k_t) &= \ln\left\{ \prod_{x=x_{\min}}^{x_{\max}} \prod_{t=t_{\min}}^{t_{\max}} \left(\frac{\lambda_{x,t}^{d_{x,t}} \exp\left(-\lambda_{x,t}\right)}{\left(d_{x,t}\right)!} \right) \right\} \\ &= \sum_{x=x_{\min}}^{x_{\max}} \sum_{t=t_{\min}}^{t_{\max}} \left\{ d_{x,t} \ln(\mu_x^{\lim} + \exp\left(\alpha_x + \beta_x k_t\right)) - E_{x,t} \exp(\alpha_x + \beta_x k_t) + c \right\} \end{aligned} \tag{9}$$

where $\lambda_{x,t} = E\left[D_{x,t}\right] = E_{x,t}\left(\mu_x^{\lim} + \exp\left(\alpha_x + \beta_x k_t\right)\right)$ and c denotes a constant term. The presence of the log-bilinear term $\beta_x k_t$ in (9) prevents the estimation of model parameters using standard statistical packages that include GLM Poisson regression. Because of this, we resort to an iterative algorithm for estimating log-bilinear models developed by Goodman (1979) based on a Newton-Raphson algorithm. The algorithm proceeds as follows: in iteration step $v+1$, a single set of parameters is updated fixing the other parameters at their current estimates using the following updating scheme

$$\hat{\theta}_j^{(v+1)} = \hat{\theta}_j^{(v+1)} - \frac{\dfrac{\partial L(\alpha_x, \beta_x, k_t)^{(v)}}{\partial \theta_j}}{\dfrac{\partial^2 L(\alpha_x, \beta_x, k_t)^{(v)}}{\partial \theta_j^2}} \tag{10}$$

Similar to the original LC model, we have in our application three sets of parameters, namely the α_x, β_x and k_t terms. For example, the updating scheme for parameter α_x can be represented, for a given starting value $\hat{\alpha}_x^{(0)}$, as follows

$$\begin{aligned}
\hat{\alpha}_x^{(v+1,1)} &= \sum_{t=t_{\min}}^{t_{\max}} \left(d_{x,t} \frac{\hat{\mu}_{x,t}^{ad(v,v,v)}}{\mu_x^{\lim} + \hat{\mu}_{x,t}^{ad(v,v,v)}} - E_{x,t} \hat{\mu}_{x,t}^{ad(v,v,v)} \right) \\
\hat{\alpha}_x^{(v+1,2)} &= \sum_{t=t_{\min}}^{t_{\max}} \left(d_{x,t} \frac{\mu_x^{\lim} \hat{\mu}_{x,t}^{ad(v,v,v)}}{\left(\mu_x^{\lim} + \hat{\mu}_{x,t}^{ad(v,v,v)}\right)^2} - E_{x,t} \hat{\mu}_{x,t}^{ad(v,v,v)} \right) \\
\hat{\alpha}_x^{(v+1)} &= \hat{\alpha}_x^{(v)} - \frac{\hat{\alpha}_x^{(v+1,1)}}{\hat{\alpha}_x^{(v+1,2)}}
\end{aligned} \tag{11}$$

where $\hat{\mu}_{x,t}^{ad(v,v,v)} = \exp\left(\alpha_x^{(v_\alpha)} + \beta_x^{(v_\beta)} k_t^{(v_k)}\right)$. Similar schemes are derived for parameters β_x and k_t. Finally, the initial parameter estimates generated by the algorithm are adjusted in order to fulfil the identification constraints, i.e.,

$$\begin{aligned}
&\hat{\alpha}_x \leftarrow \hat{\alpha}_x + \hat{\beta}_x \bar{k}, \qquad \bar{k} = \frac{1}{\left(t_{\max} - t_{\min} + 1\right)} \sum_{t=t_{\min}}^{t_{\max}} \hat{k}_t \\
&\hat{k}_t \leftarrow \left(\hat{k}_t - \bar{k}\right) \hat{\beta}_\xi, \qquad \hat{\beta}_\xi = \sum_{x=x_{\min}}^{x_{\max}} \hat{\beta}_x \\
&\hat{\beta}_x \leftarrow \frac{\hat{\beta}_x}{\hat{\beta}_\xi}
\end{aligned} \tag{12}$$

One of the critical aspects in the application of the LC mortality projection model with limit life table refers to the selection of the limit table, i.e., to the definition of what are considered the plausible limits to human longevity or to what are the extreme levels of mortality which will be reasonably reached over a given limited time horizon. Although there are biological arguments in favour of an upper age limit, there is an ongoing debate about the value of this limit with many proposals for the maximum of $\dot{e}_x$ or life span found in the literature (see, e.g., Harman, 2001).

To determine this limit, a number of subjective or informed assumptions about the future development of a set of important biological, economic and social variables have to be made. Using a biodemographic approach, Olshansky (1990) refers to the schedule of age-specific death rates as an "intrinsic mortality signature", which might change only when the forces of selection acting to maintain the genetic composition of a population are disrupted (either through environmental challenges, interventions, or diseases).

In practical applications, this prospective exercise may however reveal very difficult or even impossible. In this case, a different way of interpreting the model is to consider the limit table as a life table that reflects the pattern of mortality in a more advanced population in terms of economic and social development (target population) to which current experience will converge in a given time horizon. For instance, Oeppen and Vaupel (2002) recommended using the observed gaps between countries and regions. In this case, the life expectancy in the countries with the highest life expectancy can set as the achievable limit for all.

Alternative approaches include a combination of the lowest mortality rates observed by sex-age groups, or gaps between countries (i.e., considering the time needed by a specific country to catch up the most advanced countries), or estimates of the lowest achievable cause-specific death rates. Eradication of one or more causes of deaths and the resulting change in mortality rates have been used to predict achievable gains in life expectancy (see, e.g., Nusselder et al. 1996). One of the problems here is the lack of independency between causes of deaths. Alternatively, attainable (target) life expectancy can be estimated by combining the lowest mortality rates observed worldwide. Recently, Vallin and Meslé (2008) used a similar approach by combining the lowest age- and cause-specific mortality rates worldwide from 1950 to 2000. In 2000, the authors conclude that the resulting $\dot{e}_0$ would reach 84.4 years in men and 88.9 years in women. By comparing the highest observed $\dot{e}_0$ and the potential $\dot{e}_0$ resulting from the model the authors argue that observed $\dot{e}_0$ reached potential $\dot{e}_0$ about 25-30 calendar years later.

An alternative solution is to take some parametric function - mortality law (Gompertz-Makeham, Weibull, Heligman-Pollard,...) - on which to examine different scenarios on the main trends in human longevity (e.g., rectangularization of the survival function, evolution of life expectancy, mode of the survival function, entropy,...) by incorporating recent statistical information and expert-opinion judgements. In this line of research, Duchêne and Wunsch (1988) proposed a hypothetical limit life table based on the Weibull mortality law

$$\mu_x^{\lim} = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} \tag{13}$$

with $\beta = 95$ and $\alpha = 14.40198275$. The authors assume also that the highest attainable age is 115 years old, that the modal age is 95 years old, that $q_0 = 0.002$ and that of a total of one million of newborns only one will survive until age 114. From (13), the corresponding survival function is given by

$$l_x = l_x \exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right] \tag{14}$$

and is represented in Figure 1.

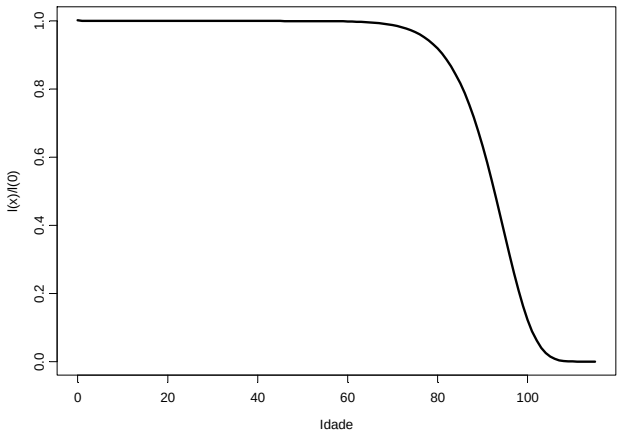

Figure 1: Duchêne-Wunsch limit life table, survival function $x \to l_x / l_0$

We note that this limit life table establishes a significant rectangularization of the survival curve. As one of the many possible alternatives, the 2008 Portuguese population projection exercise used the so-called second Heligman and Pollard (1980) mortality law, defined by

$$q_x = A^{(x+B)^C} + D\exp\left(-E(\ln x - \ln F)^2\right) + \frac{GH^x}{1+KGH^x} \tag{15}$$

where q_x denotes the death probability at age x and A, B, C, D, E, F, G, H and K are parameters to be estimated by non-linear weighted least squares methods. Equation (13) includes three distinct terms, each reflecting a separate component of mortality. The first term, an exponential function rapidly decreasing, represents the decrease in mortality during the first years of life. The second term, a sort of lognormal function, represents the mortality at intermediate adult ages and is referenced in demographic and actuarial literature as describing the incidental mortality for both sexes, and maternal mortality for the female population. The third term reflects the traditional Gompertz mortality law, which reflects the exponential growth of mortality at older ages. Figure 1 gives an example of a mortality scenario generated by the Heligman-Pollard (HP) mortality law based on the following parameters:

Parameter	A	B	C	D	E	F	G	H	K
Coefficient	0,00023	0,05	0,10	0,00150	2,200	85,0	0,00001	1,10350	0,060

Table 1: Parameter values of a specific mortality scenario generated by the HP mortality law

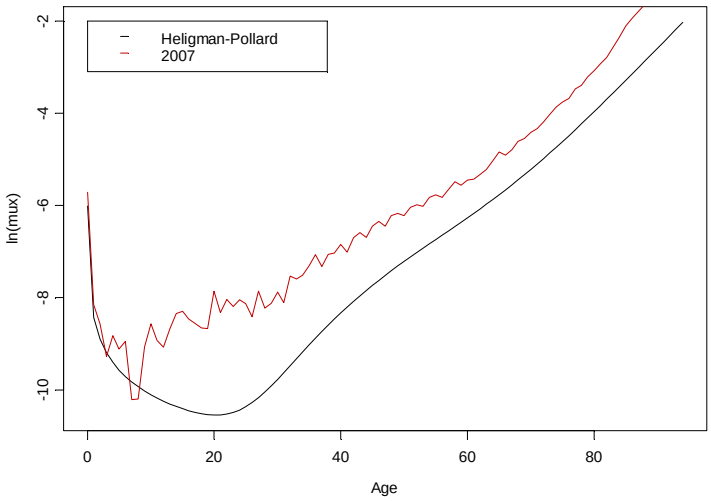

Figure 2: Crude (2007) Portuguese female mortality rates and mortality scenario generated by the Heligman-Pollard mortality law for the horizon 2060

INE (2008) and Bravo *et al*. (2010) briefly describe the results, published by Statistics Portugal, of implementing the LC with limit life table approach described above in projecting mortality and estimating life expectancy for the Portuguese female population during the 2008 population projection exercise.

Models based on the "rectangularization" of the survival curve belong to these "analytical" approaches. The theory states that at a given period, there is strong compression of mortality around a given age (and a stiffer resistance to further increase is expected), without stating what will be the value of this limit. Based on the inverse relationship between the modal age at death and the standard deviation of the age at death above the mode, Kännisto (2001) developed his hypothesis of an “invisible wall” to the extension of human longevity: as the modal age at death increases, the right-hand slope of the distribution of age at death becomes more and more vertical. Up to now, there is little empirical evidence backing up this hypothesis. Moreover, it is unlikely that the survival curve will become totally rectangular, due to the heterogeneity of the human population, so the question will be up to what degree of rectangularization the survival curve will evolve. Finally, apart from biological determinants of mortality, man-made determinants should be considered.

4. CONCLUSION

The LC family of models are part of the extrapolative stochastic methods that assume that future mortality patterns can be estimated by projecting into the future the historical trends of human mortality observed in the recent to medium-term past. Although this is a reasonable approach for mortality forecasting, the asymptotic behaviour of deaths rates projected by the LC family of models should be considered unsatisfactory. In this paper we argue that a combination of expert-opinion and extrapolative methods can be used to forecast mortality rates within the LC framework. Specifically, we develop a new extension of the age-period LC method in which forecasted mortality rates are bounded below by a limit life table to which future mortality improvements converge over a given time horizon. We discuss alternative approaches in implementing this model, namely those regarding the definition of the limit life table.

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