FAMILY OF INDICES FOR DIFFERENT PURPOSES*

Invited paper submitted by the ISTAT, Italy**

* Due to the late submission, this paper could neither be translated nor reproduced.

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1. Introduction

The problem of constructing a family of price index numbers – and especially of Consumer Price Indices (CPIs) – has noteworthy theoretical and applied importance. In fact, during the hold and recent debate on the relevance and reliability of CPIs constructed by the National Statistical Offices (NSIs), the discussions and controversies on the sources of possible bias in a specific CPI originated mostly from the fact that some discussants often sought a specific «true» or «unique» formulation for the index, to be utilised for all kinds of purposes and uses. In order to solve the controversies, many users are asking for different CPIs to be used for different purposes and for a family of indices to measure inflation also for sub-groups of population.

The paper first recalls, in Section 2, the strict relationships existing among purpose, definition, formula and actual computation of the price index numbers from both a theoretical and a practical point of view, remarking that different purposes will require different indices, to be chosen on a case-by-case basis. Then, in Section 3, the paper focuses on the need to construct a cost of living index (COLI) - as a measure of change in cost of living -, and different CPIs, for the other purposes, and emphasises how important it is to analyse and evaluate the factors that affect the difference between two different CPIs. Finally, in Section 4, the need for computation of different CPIs for sub-groups of population, to be used for price inflation adjustments or for assessing the impact of price changes on the disposal incomes of households, is analysed. The problems of their practical construction and the convenience of doing so, together with some suggestions on the analysis to be carried out are highlighted.

2. Need for different consumer price indices for different purposes

2.1. The process of consumer price index construction

In a paper presented at the 4th Meeting of the Ottawa Group on Price Indices (Biggeri, 1998), we showed that, frequently, the indices obtained from the different statistical and economic approaches are identical, and, above all, that there is no “ideal” consumer price index that is valid for all the purposes and that every procedure or formula satisfies particular needs. We concluded that most of the index number formulae, including the “superlative” indices, are in somehow good approximations of the so-called “true” index, but if this were so, the problem of the choice of the index is then not addressed. It is important to stress that the economic and statistical theory of index numbers can –and need – be integrated and, in any case, the theory needs to be used essentially as a guide for the desired characteristics of the indices to be constructed. However, it is evident that the validity and the choice of the different index numbers can be judged only on a case-by-case basis, with reference to the purposes for which they are used. It follows, in our opinion that we must rely on a general setting that will allow us to make the proper choices for the particular problem we are dealing with and for the data at hand.

In order to do it, in the construction of a specific index, we have to refer to a clearly defined production process (Biggeri, 1996) that starts from the user needs and should take into account the input process. The input process should consider the different steps and decisions – conceptual and operational - and the specific operations to be carried out for the construction of the price index P with specific quality characteristics. The process design will, therefore, depend on:
• the definition of the purpose for which the index is constructed;
• the definition of the operational concepts to be measured, with reference to the specified purpose;
• the choice of the reference population (types of: population or households; commodities and services acquired, used or paid for; outlets; prices);
• the choice of the formulae for calculating the indices at micro and at aggregate levels and the system of weights, including the reference;
• the specific manner in which the necessary information (outlets, prices and weights) has to be collected (sampling designs and data collection processes);
• the treatment of the specific items or issues as for non-market services, seasonal items, discount and sales prices; special markets, changes in the quality of commodities, etc..

These phases and decisions depend on the purpose of the index (Turvey, 1989; OECD, 2002; ILO, 2003); they often overlap and influence each other, and take on precise shapes for the related operations only with reference to a concrete case.

2.2. The choice of formulae for the different purposes of the indices and as a guide for calculating the synthetic index

Once the purpose of the index is defined, one important matter is the formulae to be chosen for calculating the index. Instead to choose “a priori” formulae, the following approach can be applied that can turn useful both from theoretical and empirical point of view.

Let us assume we want to measure, by means of a synthetic index \( P_t \), the variation of a vector of prices between time \( r \) and time \( t \) (binary comparison). Furthermore, let us assume that, for the purpose for which we want to make such a measurement, there are two correctly defined price vectors, \((p_{1r}, ..., p_{kr}, ..., p_{nr})\) and \((p_{1t}, ..., p_{kt}, ..., p_{nt})\), of \( n \) goods and services at time \( r \) and at time \( t \) and that the two vectors are technically comparable. This means the goods and services whose prices are collected in period \( r \) and period \( t \) have the same characteristics with regard to the type and the quality of the goods, the place of exchange, the market type, the distribution channels, etc.. Finally, let us assume that we have correctly observed the two price vectors or we hold the relatives or the elementary price indices \((r_{P_{1r}}, ..., r_{P_{kt}}, ..., r_{P_{nt}})\), where \( r_{P_{kt}} = \frac{p_{kt}}{p_{kr}} \).

The problem of determining a synthetic index \( P_t \) can be solved, in a general way, using the «equivalence (invariance) conditions», such as:

\[
F (r_{P_{1r}}, ..., r_{P_{kt}}, ..., r_{P_{nt}}) = F (P_t),
\]

by which the average synthetic index is computed in such a way as not to alter the measurement of the relative variation in the function \( F \) (invariant) against which the equivalence is judged.

If, for example, the function \( F \) is specified as \( \Sigma g (r_{P_{k,t}}) a_k \), where the \( a_k \) are generic non negative weights and, if \( g^{-1}(\cdot) \) represents the inverse function of \( g(\cdot) \), the equivalence condition becomes

\[
\Sigma g (r_{P_{k,t}}) a_k = \Sigma g (P_t) a_k
\]
from which we get

\[ r, \mathbf{\bar{P}}_t = g^{-1} \left( \sum g \left( \mathbf{r} \mathbf{P}_{k,t} \right) a_k / \sum a_k \right) \]  \hspace{1cm} (3)

and, by appropriately specifying \( g \), we get most of the index formulae proposed by different authors.

This general approach to the problem is very useful especially from an empirical point of view since, by defining the equivalence condition with reference to the purpose for which the index is being constructed, it has the advantage of guiding us in the choice of the type of mean we need to compute. In order to give an economic meaning to an index constructed in this fashion, the invariant should necessarily express a concept that is concretely and economically relevant.

Regarding price variations, it is easily to demonstrate that very often the most suitable invariant function is the expenditure or value necessary to acquire, use or paid for a fixed basket of goods and services whose structure depends on the «position» (in terms of type of expenditure and behaviour) of the economic agent (real or hypothetical) who has an interest in the computation of the price index. Imposing this type of invariance condition leads to synthetic index of elementary indices, with a weight structure that depends precisely on the «position» of the economic agent. For example, if the economic agent, either consumer or producer, has a conservative behaviour, - that is, if between period 0 and period \( t \) he does not change the structure of his consumption or inputs, and maintains the same structure as in period 0 - , the invariant quantity will be \( \sum p_{k,t} q_{k,0} = \sum 0 \mathbf{P}_{k,t} (p_{k,0} q_{k,0}) \), thus producing a Laspeyres type index. By contrast, if the economic agent has a speculative behaviour and, anticipating the price variations, changes the structure of his consumption or inputs, then his expenditure - immediately after time 0 - will already refer to the quantity \( q_{k,t} \) and the invariant quantity will be \( \sum p_{k,t} q_{k,t} = \sum 0 \mathbf{P}_{k,t} (p_{k,0} q_{k,t}) \) thus producing a Paasche type index. It is easy to observe that the economic indices can also be obtained following this approach by imposing an equivalence condition in which the invariant quantity is the expenditure necessary to obtain a specified level of utility.

Following this line of thought, we should then have, at least theoretically, as many indices as there are intermediate consumer (or producer) positions. Then, an index constructed to satisfy the needs of a hypothetical economic agent, with his specific position, will not necessarily satisfy the needs of another agent in a different position.

Actually this approach based on equivalent condition allows, case by case and in a coherent way, the choice of the most suitable formulas that justify - with reference to the purpose and character of the research - the substitution of the price variations of the single goods and services with the «global variation».

In our opinion, the main advantage of this approach is the fact that the reference to an equivalence condition (implicit in any synthetic index) forces us to consider the purpose the index was computed and used for and the meaning of the «position» we refer to. It also compels us both to evaluate, with an acceptable degree of approximation, whether different conditions can satisfy our needs and, anyway, to be coherent with such condition throughout the various phases of the construction of the index. Sometimes, the general characteristics of the formulae are the same but with different specifications of the variables depending on the purpose of the index.

As consequence, from a practical point of view, the validity and the choice of the different index numbers can obviously be assessed only with reference to the purposes for which they
are used. Therefore, the choice must be made on a case-by-case basis, and we must be able to evaluate the accuracy and the validity of the computed indices.

3. The main purposes of the consumer price indices and the related family of indices

3.1. The different purposes of the consumer price indices: COLI and CPIs

There is a wide variety of purposes for constructing consumer price indices (Turvey, 1989; Hill, 1996; Australian Bureau of Statistics, 1997a; Cook and Lewington, 1997; OECD, 2002 and ILO, 2003), that we can summarise as follows:

- as a measure of change in the cost of living;
- as a measure of inflation (including the analysis of the inflation process);
- as a measure for income adjustment process;
- for general indexation of public and private sectors’ contracts (in particular for indexation by government, prices, wage and salary adjustment in contracts);
- as a measure for the deflation of consumption aggregates in the national accounts.

There are further different specifications within the five main groups of purposes.

From a practical point of view, we can agree that the so-called Cost of Living Index (COLI) is to be used for the first purpose, whereas the so-called CPIs, computed with different specifications of Laspeyres formula, are used for the other purposes.

COLI is used for the first purpose because its objective is to measure the effects of price changes on the cost necessary to maintain the same level of standard of living (i.e.: utility or welfare). As it is well known, a general COLI derived from this approach consists of the ratio between two expenditures or values born in different situations (of prices) but referred to the same level of utility. Hence, the index is obtained in the following manner:

\[
0P_t^E = \frac{C (p_{1t}, ..., p_{kt}, ..., p_{nt}; U)}{C (p_{10}, ..., p_{k0}, ..., p_{n0}; U)}
\] (4)

where \(C\) is a cost function and \(U\) the level of utility. However, the computation of this index is not easy, because COLI cannot be directly calculated. Many approximations have been proposed (Diewert, 2001a), but very few NSIs actually produce this kind of index. The US Bureau of Labor Statistics has been trying to do it, and has recently started calculating an index on an experimental basis, introducing for this purpose the so-called Chained Consumer Price Index (C-CPI-U), that employs a superlative Tornquist formula (Cage et Al., 2003).

Due to ease of computation, timeliness and clear meaning, the most widely used formulae for calculating the CPIs are of the Laspeyres type and have the following structure:

\[
rP_t = \sum_k r w_k rP_{k,t}
\] (5)

where \(k\) denotes the generic modality of any classification (for example of commodities), \(r (r = 1, ..., T)\) and \(t (t = 1, ..., T)\) indicate, respectively, the base and the current period. These indices are then obtained by weighted arithmetical averages of elementary or aggregate indices with weights \(w_k\) (usually the expenditure share), such that \(\Sigma_k w_k = 1\). Obviously, this kind of indices have different meanings depending on the choices and decisions mentioned above.
Taking into account the approach suggested in par. 2.2, to construct specific CPIs for different purposes it is necessary to specify the equivalent condition, that is the most suitable invariant function in term of expenditure or value necessary to acquire, use or paid for a fixed basket of goods and services whose structure depends on the «position» (in terms of type of expenditure and behaviour) of the economic agent (real or hypothetical) who has an interest in the computation of the price index. Therefore, a more detailed invariant function needs to be used in order to take into account all the choices necessary to satisfy the purpose of a specific index.

In general terms, to locate this problem, let us assume that all the necessary information is available, namely the complete matrices of price and weights at various time period in which we are interested. The index formula coming from an adequate detailed invariant function as expenditure or value necessary to acquire, use or paid for a fixed basket of commodities and services could be, for example, specified as follows:

\[ rP_t = \sum_{j, scgdil} r w_j, scgdil \ r P_{j, scgdil t} \]  

where we use

- s to indicate the s-th ‘specific object’, i.e. a pre-established quality of commodity (s=1,…,S);
- c to indicate the c-th commodity or service acquire, used or paid for (c=1,…,C);
- g to indicate the g-th group of commodities or services (g=1,…,G);
- d to indicate the d-th distribution channel or specific type of outlets (d=1,…,D);
- i to indicate the i-th municipalities or lower level (i=1,…,I);
- l to indicate the l-th territorial area as province or region (l=1,…,L);
- j to indicate the j-th household (j=1,…,J);

and where in (6) the relatives or elementary indices for the household j-th should be computed jointly with reference to specific quality of commodities, commodity groups, distribution channels, municipalities and territorial areas. Obviously, a different order in the variables could be chosen, but in any case the invariant function should be based on the different purposes specifying adequately the contents of the variables and subscripts (in terms of type of expenditure and behaviour of the consumers).

It is clear that sub-indices can be computed for each subscript as well as for various combination of subscripts and obviously there are many ways of obtaining the general index \( rP_t \) through direct calculation from the elementary indices or by means of successive aggregations of sub-indices following different “paths”. The meaning of the \( rP_t \) depends on the choices made on variables and subscripts and the resulting index will be adequate for the specific purpose for which it has been computed (as measure of inflation or for the indexation).

An important consequence of the consideration outlined above is that when collecting elementary data - which obviously can be based only on samples - and in the procedure for estimating indices, it is necessary to make reference to a stratified sample. Moreover, in the practical application of what has been stated above, the main problem to be faced, is the lack of adequate and detailed statistical information which prevents the exact computation of the desired index (suggested by the theory).

In practice, a CPI is constructed in two stages. The first stage is represented by the calculation of the price relatives by a pre-established quality of commodities. In practice, in the construction of the CPI, the first stage is represented by the calculation of the price relatives...
by 'specific object' (i.e. a pre-established quality of the commodity) and, subsequently, the
calculation of elementary indices for each commodity or commodity group (using an adequate
arithmetical average, weighted or not, formula or a geometric average formula). The second
stage consists of aggregating the relatives and elementary indices into a higher-level indices.

3.2. The divergence between two different CPIs

When two or more CPIs are calculated, as weighted averages, to satisfy different purposes it
could be useful to analyse and decompose the differences in the numerical results obtained.
In fact, as is well known, starting from the Bortkiewicz’s theorem, it is possible to decompose
the divergence between prices indices associated with different systems of weights. This can
be done by different factors and elements (see for example Schultz, 1997).

Along the same lines, it is also possible (Biggeri, 1998a) to consider some slightly different
decomposition of the divergence between two indices , and, referring to the same time
interval, when the divergence depends only on the differences in the weighting system.
With reference to formula (5), if by \( d_k = \bar{w}_k^a - \bar{w}_k^b \) we denote the difference between the
standardised weights used to calculate the two indices; by \( s_p \) and \( s_d \) respectively the standard
deviations of elementary (or partial) indices and of differences between weights \( d_k \); by \( R_{pd} \)
the linear correlation coefficient between the elementary (or partial) price indices \( P_k \) and the
difference in the corresponding weights (that is \( R_{pd} = \frac{\Sigma_k (P_{k,t} - \bar{P}_k^m) (d_k - d^m)}{n s_p s_d} \),
where \( n \) is the number of commodities and services (or commodity groups), \( \bar{P}_k^m \) and \( d^m \) are
the arithmetic means of \( P_{k,t} \) and \( d_k \)), then the difference between the two indices can be
decomposed as follows:

\[
\rho_t^a - \rho_t^b = \sum_k \rho_{k,t} d_k = n s_p s_d R_{pd} . \tag{7}
\]

It is important to emphasise that the difference between the two indices vanishes when there is
no relationship between the price variations of commodities and the differences between the
weights attributed to them, and when one of the standard deviations of elementary indices or
of the differences between weights is equal to zero. The same decomposition can be applied
to the different indices and sub-indices obtained using (6).

Sometimes, both the elementary indices and the system of weights used for calculating two
indices \( \rho_t^a \) and \( \rho_t^b \) are different. Therefore, formula (7) has to be accordingly modified by
introducing the differences between \( \rho_{k,t}^a \) and \( \rho_{k,t}^b \).

From an empirical point of view, this kind of decomposition is also interesting because it
makes possible, by making suitable hypotheses, to estimate a conjectural weighting system in
order to judge whether the computed index is reasonable compared to the desired one,
particularly when not enough data is available.

Furthermore, sometimes, the difference between two CPIs may be a result of the differences
in their coverage. If an index \( \rho_t^a \) has a smaller coverage than a more complete \( \rho_t^b \), it is
possible to measure the factors that influence the difference between them.

If by \( \rho_{n-s,t}^a \) we denote the index for the commodities excluded from \( \rho_t^a \) in comparison with
\( \rho_{n,t}^b \) and by \( W_{k,t}^a \) and \( W_{k,t}^{n-s} \), the normalised weights for the commodity groups of the two
indices (or sub-sets of elementary indices), the more complete index will be equal to:

\[
\rho_{n,t}^b = r_{w_k^{n-s},k,t} P_{n-s,t}^b + \sum_{k=1}^{n} r_{w_k^{n-s},k,t} P_{n-s,t}^b = (1 - r_{w_k^{n-s},k,t}) \rho_{n,t}^b + r_{w_k^{n-s},k,t} P_{n-s,t}^b =
(1 - r_{w_k^{n-s},t}) \left[ \sum_{k=1}^{n} r_{w_k^{n-s},k,t} P_{n,s,t}^b \right] + r_{w_k^{n-s},k,t} P_{n-s,t}^b \tag{8}
\]
and then

\[ \Delta P^n_t - \Delta P^s_t = (\Delta P^{n-s}_t - \Delta P^{s}_t) \cdot w^{n-s}_t \]  

Therefore, the difference will depend on the weights of the excluded commodities and on the difference between the two indices; that is on the different evolution of the set of elementary indices included in the computed index and of the set of elementary indices excluded from the computation. Moreover, taking into account what have been shown above, (9) may be decomposed as in (7) thus providing interesting information on its characteristics.

The importance of studies on both elementary indices and weights variability should be evident. It should also be clear how important is to study the relationships between indices and weights for all commodities or for groups of commodities in order to estimate the magnitude of each single component of the differences between different CPIs. Obviously, this kind of study requires more detailed information than that normally used for the computation of the index. Yet, National Statistical Offices could undoubtedly obtain such information through the new technology used for data capture (for example using scanner data), at least on the occasion of the so-called 'benchmark' surveys. The proposed analyses would facilitate the comparison between the desired and the surrogate index and would surely contribute to better choose CPIs adequate for the different purposes.

4. Family of Consumer Price Indices for sub-groups of population

NSIs are continuously faced with requests from different users asking to compute different price indices to measure the inflation process and to use them for wage and salary adjustment or for assessing the impact of price changes on the disposal incomes of sub-groups of population.

In Biggeri (1998b), we have already tackled the issues concerning the inflation measures and the need to built up a system of price indices (consumer, producer, export, import, etc. price indices and national accounts implicit price deflators) and satellite or sub-consumer price indices (by groups of goods and services, by geographical area, etc.), including a measure of “core inflation”. A good analytical framework for price indices can be found in an information paper of the Australian Bureau of Statistics (1997a).

As far as the computation of different CPIs for sub-groups of population is concerned there are issues both related to their usefulness and to the difficulties in computing them. Most NSIs and International Statistical Organisations suggest to clearly state the principal (primary) purpose to compute the CPI and to produce an index strictly devoted to it. This would make the meaning and the methods of the index construction more understandable and would avoid confusion and lack of confidence from the users (see for example Australian Bureau of Statistics, 1997b; Cook and Lewington, 1997; OECD, 2002; ILO, 2003).

In any case, if we would like to compute adequate CPIs for sub-groups of population, we need to start from the following base formula, coming from (6):
\[ r_{P_{ht}} = \sum_{j,scgdil} r_{w_{j,scgdil}} r_{P_{j,scgdil};t} \]  

(10)

where \( h \) indicates the \( h \)-th type or sub-group of households \((h=1, \ldots, H)\) and the \( h \)-th sub-group includes a certain number of households (therefore in (10) the \( i \)-th household belong only to one group of households, that is \( j=1,\ldots,J_h \)).

Therefore, the CPIs for different sub-groups must assure that the system of weights and elementary indices fulfil the condition of characteristicity (Biggeri and Giommi, 1987); that is to say, they must be as typical or characteristic as possible for the groups of households the indices refer to.

Because of the difficulties and costs in collecting all the requested information, no one NSI computes adequate CPIs for sub-group of households or population.

Few NSIs are computing and publishing specific CPIs for given socio-economic groups (OECD, 2002), or have been conducting experiments to compute CPIs for elderly consumers and for the poor (Amble and Stewart, 1994; Garner and Al., 1996) or for selected types of households (Australian Bureau of Statistics, 2001). Also Istat (2003), has conducted experiments to assess the impact of price changes on the disposable incomes of different types of households. However all the resulting sub-indices are not adequate to the purpose because they do not satisfy the condition of characteristicity.

Actually, NSIs - or the authors of experiments- usually compute indices for sub-groups of households applying the following formula:

\[ r_{P_{ht}} = \sum_{c,\text{eg}} r_{w_{h,c,\text{eg}}} r_{P_{c,\text{eg};t}} \]  

(11)

where \( c \) and \( g \) now indicate the \( c \)-th commodity and \( g \)-th group of commodities acquired, used or paid for by a specific sub-group of households \((c=1,\ldots,C_h; \ g=1,\ldots,G_h)\). In other words, indices for different sub-groups of households are computed by using the same vector of elementary price indices (computed for the general CPI at national level) aggregated through system of weights differentiated for each sub-group of households; that is assuming that every sub-group of households faces the same price for each commodity (Australian Bureau of Statistics, 2001, constructed the CPIs for different selected households types using also lower level price data for few elementary price indices).

This is obviously not true, because usually the different sub-groups of households face different prices when purchasing commodities of different quality with different level of price, in different kind of outlets or distribution channels, in specific municipalities and territorial areas. Moreover, frequently, households belonging to a sub-group (of households) considered for the computation of \( r_{Pht} \) are heterogeneous from the point of view of their consumer behaviour; for example the sub-groups of pensioner households or poor households include households with different number of members, with different level of disposable income, etc.

It is important to point out that the results obtained in the mentioned experimental indices showed minimal differences (of few decimals) among the different consumer price indices for the sub-groups of households and the overall CPI. In our view, these indices are not so useful because they do not provide an accurate measure of price inflation as experienced by different population groups. In fact, due to the above-mentioned reasons, the \( r_{P_{ht}} \) indices do not take into account the actual consumer behaviour of the different households, the variability of the
prices and of the elementary price indices by quality of commodities, types of outlets and geographical areas.

Looking at the formulas (7) and (9), it is clear that the differences between the standardised weights used to calculate two different indices \( r_{P_1} \) and \( r_{P_2} \) affect the results only partially. The latter depend also on:

- the different evolution (computed by the standard deviation) of the set of elementary (or partial) indices (if they are the same in the two indices);
- the standard deviation of differences between elementary indices;
- the standard deviation of differences between weights;
- the linear correlation coefficient between the elementary (or partial) price indices (or the differences between the elementary indices) and the differences in the corresponding weights;
- the number of commodities and services (or commodity groups) included in the indices.

Using formula (11), most of the above mentioned components of the differences between different indices for sub-groups of households are equal to zero, because of the assumption made in the mentioned experiments. Therefore it is obvious that the resulting indices for different population groups are quite similar. Moreover, sometimes also the mentioned linear correlation coefficient is very small.

Therefore, when CPIs for sub-groups of households are calculated also on an experimental basis, it is important to make some of the mentioned analyses and evaluations, in order to get information on the potential magnitude of the components of the differences between different CPIs.

However, if we want to compute adequate CPIs for sub-groups of households, it is much more important to try to carry out specific researches and pilot surveys aiming to: (i) verify the real differences in the consumer behaviour of the various sub-groups of households, in terms of purchase of commodities of different quality with different level of price, in different kind of outlets or distribution channels, in specific municipalities and territorial areas; (ii) collect the corresponding prices, and (iii) verify the cost of the collection of so detailed data. As the cost of this kind of surveys is surely too high to be extended to the overall computation of the general CPI and sub-groups of CPIs, the results of the pilot surveys could be used to make adequate assumptions to compute indices that can be good estimates of the desired CPIs, as specified in (10).
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