SUPPORTING PAPER No 6(17)*
24 October 2001

ENGLISH ONLY

STATISTICAL COMMISSION and ECONOMIC COMMISSION FOR EUROPE

INTERNATIONAL LABOUR ORGANIZATION (ILO)

CONFERENCE OF EUROPEAN STATISTICIANS

Joint ECE/ILO Meeting on Consumer Price Indices
(Geneva, 1-2 November 2001)

STUDY OF VARIATIONS OF MIDPOINT-YEAR BASKET INDICES

Contributed Paper submitted by Statistics Bureau of Japan**

* Paper posted on internet as submitted by the country.

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Summary

Hill [1999] proposed a kind of fixed basket index which uses a basket in the third year intermediate between the base year and the observation year as an alternative to superlative indices and/or pure price indices which uses some average of baskets in the base year and the observation year. Following his proposal, Okamoto [2001a] studied midpoint-year basket indices using the 1995-base Japanese CPI dataset and showed these types of indices are close to superlative indices and/or chained superlative indices in addition to their good features for practical use including the possibility of real-time compilation. Shultz [1998] also applied the identical formula named'single year, mid-term basket index' to price and volume indices for final domestic demand and price index series of industrial production.

Recently, an additional study was made in order to estimate an impact of inclusion of PCs on issues in aggregation formulas. The results are surprising. Fisher index falls far below chained Paashe index, and 'midpoint-year basket index - arithmetic-mean type' falls significantly below chained superlative indices while Tornqvist index and 'midpoint-year basket index - geometric mean type' are still sufficiently close to chained superlative indices. After this finding, search for variations of midpoint-year basket indices in additive form, sufficiently close to chained superlative indices as well as suitable for practical use, is being carried out. At present, an approximation in additive form derived from a variation of midpoint year basket index (a combination of arithmetic-mean type and geometric-mean type) is considered to have a possibility of success.

1. Hill [1999] proposed a kind of fixed basket index which uses a basket in the third year intermediate between the base year and the observation year as a practical alternative to superlative indices or pure price indices which uses some average of baskets in the base year and the observation year. Following his proposal, Okamoto [2001a] inquired into 'midpoint-year basket index - arithmetic-mean-type' defined as (1) on the next page. Okamoto [2001a] also introduced several variations including geometric-mean-type defined as (5) on the next page, taking price elasticity of demand into consideration. All these midpoint-year basket indices are proved to be second-order approximations to superlative indices and Divisia index at the base period with respect to time supposing prices and quantities change smoothly.

2. In fact, a test calculation using the 1995-base Japanese CPI dataset showed midpoint-year basket indices are sufficiently close to superlative indices and chained superlative indices as shown in Chart 1 and Table 1.

Chart 1. Comparison of different index formulas (Overall index)

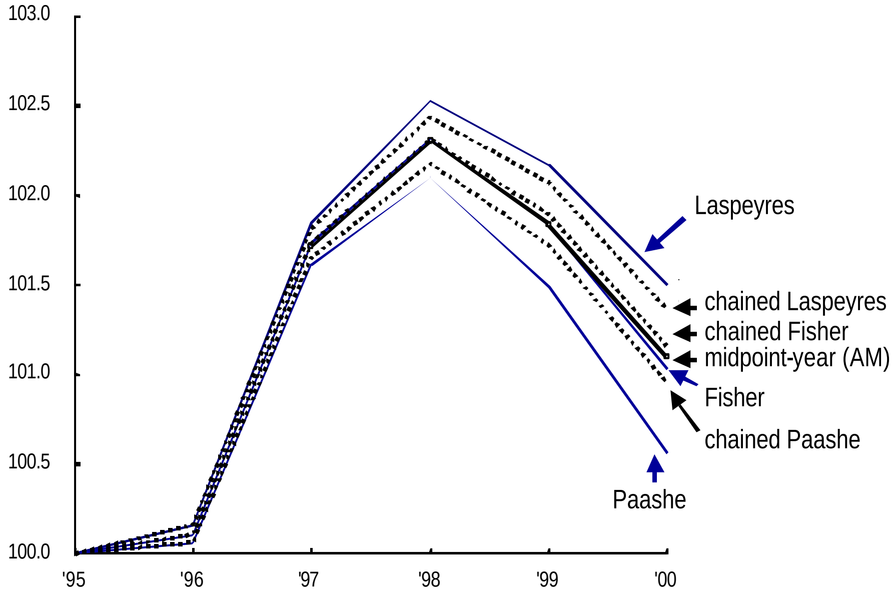

Table 1. Comparison of different index formulas (Overall index)

	Fixed-base index				Chained index				Midpoint-year basket index		
	Laspeyres	Paasche	Fisher	Tornqvist	Laspeyres	Paasche	Fisher	Tornqvist	arithmetic-mean-type (3)	arithmetic-mean-type (4)	geometric-mean-type (5)
1995	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000		100.000
1996	100.162	100.061	100.111	100.112	100.162	100.061	100.111	100.112			
1997	101.844	101.618	101.731	101.738	101.817	101.652	101.734	101.736	101.715		101.729
1998	102.523	102.095	102.309	102.318	102.441	102.175	102.308	102.308	102.306	102.308	102.272
1999	102.169	101.492	101.830	101.857	102.072	101.721	101.896	101.897	101.844		101.898
2000	101.503	100.560	101.030	101.074	101.362	100.957	101.159	101.160	101.094	101.094	101.181

Arithmetic-mean-type (AM)	Geometric-mean-type (GM)
Principle formula : $AM = \frac{\sum q_{t/2} p_t}{\sum q_{t/2} p_0} = \frac{\sum \frac{w_{t/2}}{I_{t/2}} I_t}{\sum \frac{w_{t/2}}{I_{t/2}}}$ (1) where p_t : price at the observation - year t p_0 : price at the base - year 0 $p_{t/2}$: price at the midpoint - year $t/2$ $q_{t/2}$: quantity at the midpoint - year $t/2$ $w_{t/2} = p_{t/2} q_{t/2}$ $I_t = \frac{p_t}{p_0}, I_{t/2} = \frac{p_{t/2}}{p_0}$	Principle formula : $GM = \exp\left(\sum s_{t/2} I_t\right)$ (5) where $s_{t/2} = \frac{w_{t/2}}{\sum w_{t/2}}$: share at the midpoint - year $t/2$ $I_t = \frac{p_t}{p_0}$
Actual formula (in the case of the base year is 1995) : the observation year : 1997, 1999 ⇒ 'single midpoint - year case' $AM = \frac{\sum \frac{w_{96}}{I_{96}} I_{97}}{\sum \frac{w_{96}}{I_{96}}}, \frac{\sum \frac{w_{97}}{I_{97}} I_{99}}{\sum \frac{w_{97}}{I_{97}}}$ (2) the observation year : 1998, 2000 ⇒ 'plural midpoint - year case' $AM = \frac{\sum \left(\frac{w_{96}}{I_{96}} + \frac{w_{97}}{I_{97}}\right) I_{98}}{\sum \left(\frac{w_{96}}{I_{96}} + \frac{w_{97}}{I_{97}}\right)}, \frac{\sum \left(\frac{w_{97}}{I_{97}} + \frac{w_{98}}{I_{98}}\right) I_{00}}{\sum \left(\frac{w_{97}}{I_{97}} + \frac{w_{98}}{I_{98}}\right)}$ (3) ⇒ arithmetic mean of baskets or $= \frac{\sum \sqrt{\frac{w_{96}}{I_{96}} \frac{w_{97}}{I_{97}}} I_{98}}{\sum \sqrt{\frac{w_{96}}{I_{96}} \frac{w_{97}}{I_{97}}}}, \frac{\sum \sqrt{\frac{w_{97}}{I_{97}} \frac{w_{98}}{I_{98}}} I_{00}}{\sum \sqrt{\frac{w_{97}}{I_{97}} \frac{w_{98}}{I_{98}}}}$ (4) ⇒ geometric mean of baskets	Actual formula (in the case of the base year is 1995) : the observation year : 1997, 1999 ⇒ 'single midpoint - year case' $GM = \exp\left(\sum s_{96} I_{97}\right), \exp\left(\sum s_{97} I_{99}\right)$ (6) the observation year : 1998, 2000 ⇒ 'plural midpoint - year case' $GM = \exp\left(\sum \left(\frac{s_{96} + s_{97}}{2}\right) I_{98}\right), \exp\left(\sum \left(\frac{s_{97} + s_{98}}{2}\right) I_{00}\right)$ (7)

3. Following the good results, it was decided to add some variation of midpoint-year basket index to the family of annual supplementary indices[1] in the 2000-base CPI, and continue to study possibilities of practical use for the compilation of the monthly official index. In order to choose a specific formula for the compilation of the supplementary index, impact of inclusion of PCs on the overall index was examined lately, making use of hedonic index for PCs estimated by Okamoto and Sato [2001] because price index for PCs, which is to be added at the

[1] Chained Laspeyres index has been compiled as a supplementary index since the 1980 base revision.

2000 base revision, tends to fall at an annual rate of 30 to 40 percent - extremely faster relative to other goods, and there is a fear that thes e extreme products has an impact on issues in aggregation formulas.

4. In the 1995-base CPI, expenditure on PCs was assigned to weights for Japanese word processors supposing Japanese word processors represent changes of PC prices also. Thus, weights for Japanese word processors are split into two for the additional test. The results are strikingly different from the previous results as shown in Chart 2 and Table 2. Fisher index falls substantially below chained Paashe index although Tornqvist index is still close to both chained Fisher index and chained Tornqvist index. 'Midpoint-year basket index - arithmetic-mean type' also falls significantly below chained Fisher index and chained Tornqvist index while 'midpoint-year basket index - geometric-mean type' is close to those chained superlative indices.

Chart 2. Comparison of different index formulas (Overall index, in the case of inclusion of PCs)

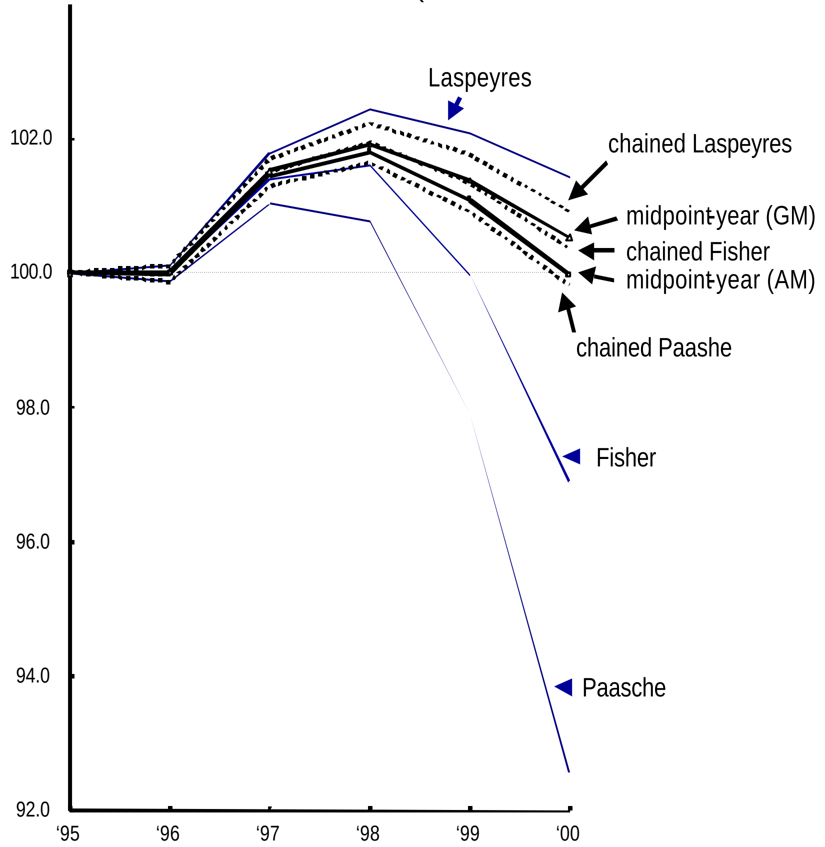

Table 2. Comparison of different index formulas (Overall index, in the case of inclusion of PCs)

	Fixed-base index				Chained index				Midpoint-year basket index				
	Laspeyres	Paashe	Fisher	Tornqvist	Laspeyres	Paashe	Fisher	Tornqvist	arithmetic-mean-type (3)	geometric-mean-type (5)	combination type (8)	approximate I (10)	approximate II (11)
1995	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
1996	100.113	99.866	99.990	100.013	100.113	99.866	99.990	100.013					
1997	101.781	101.014	101.397	101.537	101.700	101.295	101.497	101.532	101.449	101.512	101.499	101.503	101.508
1998	102.452	100.772	101.608	102.013	102.233	101.638	101.936	101.977	101.805	101.916	101.955	101.960	101.971
1999	102.092	97.899	99.974	101.313	101.757	100.914	101.335	101.393	101.093	101.376	101.331	101.336	101.348
2000	101.426	92.568	96.896	100.246	100.905	99.814	100.358	100.437	99.979	100.524	100.450	100.453	100.463

5. Obviously, the results indicate the choice of aggregation formula becomes more critical in the case that PCs are added. Supposing chained superlative indices are the target indices or near the target index, 'midpoint-year basket index – geometric-mean type' is still a good approximation. However, for practical reasons such as definability of monthly index and sub-index, arithmetic mean type is desirable. In this context, practical approximations in additive form derived from some combination of arithmetic-mean-type and geometric-mean-type are being studied at present.

6. Formula (8) is a kind of midpoint-year basket index. This formula can be regarded as a specific form[2] of (9), defined by Okamoto [2001a] (formula (18) in Annex 2) as variations of midpoint-year basket indices supposing different elasticity among elementary indices.

Combination type (CM)

Priciple formula :

$$CM_t = \exp\left\{(1 - s_{t/2}) \ln AM_{t,\text{non-PC}} + s_{t/2} \ln I_{t,PC}\right\}$$

$$= \exp\left\{(1 - s_{t/2}) \ln\left(\frac{\sum_{\text{non-PC}} \frac{w_{t/2}}{I_{t/2}} I_t}{\sum_{\text{non-PC}} \frac{w_{t/2}}{I_{t/2}}}\right) + s_{t/2} \ln I_{t,\text{PC}}\right\} \quad (8)$$

where $s_{t/2}$: PCs' share in consumption at the midpoint - year $t/2$

$$CM_t = \left\{\frac{\sum_k a_{t/2,k}\left(\sum \frac{s_{t/2}}{I_{t/2}^{1-\eta_k}} I_t^{1-\eta_k}\right)^{\frac{1-\eta}{1-\eta_k}}}{\sum_k a_{t/2,k}\left(\sum \frac{s_{t/2}}{I_{t/2}^{1-\eta_k}}\right)^{\frac{1-\eta}{1-\eta_k}}}\right\}^{\frac{1}{1-\eta}} \quad (9)$$

where η_k : price elasticity of demand for items in subgroup k,

η : price elasticity of demand for subgroups

$s_{t/2}$: share in expenditure on subgroup k at period $t/2$

$a_{t/2,k}$: share of subgroup k in the total consumption at period $t/2$

[2] Set η_1 and η_2 - price elasticity of demand for all items except PCs and that of PCs respectively - for 0, and let η - price elasticity of demand for the two subgroups - approach 1 in formula (9).

7. As PCs' share in the total consumption $s_{t/2}$ in formula (8) is very small - less than 0.6 percent, formula (8) defined above can be approximated by the following formula (10). Furthermore, formula (10) can be replaced with formula (11) supposing index for goods and services other than PC does not change substantially relative to the contribution of PC prices. The test calculation indicates that a combination type index (8), its approximations (10) and (11) are good approximations to chained superlative indices as shown in Table2. Differences among formula (8), (10), (11) and chained Tornqvist index remain less than 0.1 percent point from 1997 - 2000.

Approximate I

Priciple formula :

$$AM_{t,\text{non-PC}} \exp ? s_{t/2} \ln I_{t,\text{PC}} ? \quad ? \; AM_{t,\text{non-PC}} ? AM_{t,\text{non-PC}} ? \exp ? s_{t/2} \ln I_{t,\text{PC}} ? ? 1 ? \qquad (10)$$

Approximate II

Priciple formula :

$$AM_{t,\text{non-PC}} ? ? \exp ? s_{t/2} \ln I_{t,\text{PC}} ? ? 1 ? ? AM_{t,\text{non-PC}} ? (I_{t,\text{PC}}^{s_{t/2}} ? 1) \qquad (11)$$

8. Approximation defined by (11) is in near additive form. Sub-index for each item group can be defined naturally. As for definability of monthly index, it seems unable to define monthly indices averaged to (11) because of existence of the second term while the first term of formula (11) has no problem (see Okamoto [2001a]). However, yearly average of the second term calculated using monthly price index for PCs dose not so differ from the second term calculated using annual price index for PCs, within 0.02 percent point as shown in Table 3 on the next page. Thus, monthly index also seems definable practically.

9. The formula to be used for the compilation of a new supplementary index has not been determined yet. 'Midpoint-year basket index – geometric-mean type' (5) is surely one of the good choices. A combination type index (8) also seems a good choice unless there is the emergence of new products of which quality continuously improves as fast as PCs, while approximate II defined by (11) needs to be studied furthermore although it looks hopeful.

10. Lastly, a related matter should be mentioned. Paasche check has been carried out every five years in Japan in order to see a gap between the official Laspeyres index and lower limit of the most appropriate index. However, after inclusion of PCs, there is a fear that Paasche check becomes inappropriate because Paasche index is likely to fall extremely far from the most appropriate index. Thus, another check was added supposing chained Paasche index as a lower limit of the most appropriate index.

Table 3. Monthly and annual figures of the 2nd term in formula (11)

	1995		1996		1997		1998	
	Price index for PCs	2nd term	Price index for PCs	2nd term	Price index for PCs	2nd term	Price index for PCs	2nd term
Jan.	140.8	0.195	59.5	-0.295	34.5	-0.604	21.4	-0.874
Feb.	127.9	0.140	55.6	-0.334	33.2	-0.626	20.5	-0.900
Mar.	123.3	0.119	53.9	-0.352	32.0	-0.648	19.8	-0.918
Apr.	121.0	0.109	53.0	-0.361	31.2	-0.661	19.5	-0.927
May	117.4	0.092	52.1	-0.371	30.7	-0.671	18.6	-0.953
Jun.	103.6	0.020	48.9	-0.407	29.5	-0.693	16.6	-1.020
Jul.	95.4	-0.027	47.2	-0.427	28.4	-0.715	15.5	-1.056
Aug.	89.5	-0.063	45.9	-0.443	27.4	-0.736	15.1	-1.072
Sep.	81.7	-0.115	43.7	-0.470	26.1	-0.762	14.8	-1.082
Oct.	72.9	-0.180	41.6	-0.499	24.5	-0.799	14.1	-1.110
Nov.	65.3	-0.242	38.6	-0.541	23.4	-0.825	13.2	-1.149
Dec.	61.1	-0.280	36.4	-0.574	22.3	-0.851	12.9	-1.160
Average of monthly figures		-0.019		-0.423		-0.716		-1.018
Annual figures	100.0	0.000	48.0	-0.417	28.6	-0.711	16.8	-1.010

Note. Calculation is performed on the assumption that PCs' share in the total consumption is constant to its share 0.57 % in 2000 from 1995 to 1999.

	Paasche check $\frac{P ? L}{L}$	Alternative check# $\frac{CP ? L}{L}$
Official CPI	-0.9%	-0.5%
Addition of PCs	-8.7%	-1.6%

Note. The base year is 1995, and the observation year is 2000.

$\text{Alternative check} ? \frac{\text{chained Paashe} ? \text{Laspeyres}}{\text{Laspeyres}} ? \frac{CP ? L}{L}$

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