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Item 5 (a) of the provisional agenda
Proposals of amendments to ATP:
Pending proposals

Clarification in ATP regarding the overall coefficient of heat transfer of the bodies of special equipment (the K coefficient), replacing the margin of error with uncertainty, and in the ATP Handbook, regarding the recommended methods to find the uncertainty of measurement of the K coefficient

Transmitted by the Russian Federation

Summary

Executive summary: In annex 1, appendix 2, to ATP, paragraph 2.3.2 indicates maximum margins of error for the measurement of the overall heat transfer coefficient (the K coefficient) of bodies of special equipment.

In the scientific community, it is now accepted practice to refer not to margins of error, i.e. not to the maximum error in determining the true value of a physical quantity, (a value that can never reliably be known), but instead to refer to uncertainty, which establishes the limits of the interval within which the value of the quantity being measured can be expected to fall, with a specified likelihood.

The reference to margins of error in ATP dates back to a time when the distinction between the concepts of error and uncertainty of measurement had not yet been sufficiently established. References to margin of error should be replaced with uncertainty whenever possible so that ATP will fully conform with current scientific practice.

Action to be taken: Change the wording of paragraph 2.3.2 in annex 1, appendix 2, to ATP with provisions relating to the accuracy of measurement of the K coefficient on the basis of uncertainty and not the margin of error of the measurement result. Introduce the corresponding changes to model test reports Nos. 2 A and 2 B in ATP.

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	Clarify the commentary to annex 1, appendix 2, of the ATP Handbook.
Related documents:	GOST R 54500.3-2011/ISO/IEC Guide 98-3:2008 “Uncertainty of measurement”. Part 3: – Guide to the expression of uncertainty in measurement (GUM: 1995) (IDT).

Introduction

1. At the seventy-first session of WP.11, experts from the Russian Federation noted that the test method set out in ATP did not contain a specific indication of how to calculate the margin of error when determining the K coefficient.

2. At the seventy-second session of WP.11 the Russian Federation prepared proposals to amend ATP and the ATP Handbook with the relevant provisions concerning the type of margin of error in measuring the K coefficient and the methodology for calculating this margin of error on the basis of a given mathematical model for the tests (see document ECE/TRANS/WP.11/2016/4).

3. During the discussion at the seventy-second session of WP.11 of document ECE/TRANS/WP.11/2016/4, the expert from France noted that the concept of uncertainty of measurement, and not margin of error, was currently used. In the light of the similarity of the mathematical methods for defining the margin of error and the uncertainty of measurement, and consequently of the quantitative evaluation of the inaccuracy of measurement of the K coefficient, the proposal of the Russian Federation was accepted.

4. In preparation for the seventy-third session of WP.11, the experts from the Russian Federation carefully studied the comments made by France concerning document ECE/TRANS/WP.11/2016/4, which the French delegation had provided to them. ISO/IEC Guide 98-3:2008, which had been translated in the Russian Federation into Russian and now serves as national standard GOST R 54500.3-2011, was also examined. As a result of this work, a joint opinion was forged with the French experts, according to which it is advisable in ATP to use the concept of uncertainty of measurement of the K coefficient instead of the margin of error. The Russian Federation has thus prepared the corresponding amendments to ATP and the ATP Handbook.

5. Despite the fact that the previous proposal of the Russian Federation on this question (contained in document ECE/TRANS/WP.11/2016/4), adopted at the seventy-second session of WP.11 for the comments in the ATP Handbook relating to ATP annex 1, appendix 2, sub-section 2.3.2, for the calculation method for the margin of error of measurement of the K coefficient, it is proposed at the seventy-third session of WP.11 to adopt new wording for these comments. The changes here relate to the use of uncertainty of measurement of the K coefficient, instead of margin of error. It is also proposed to make the appropriate corrections in ATP annex 1, appendix 2, sub-section 2.3.2, and also to test models Nos. 2 A and 2 B.

6. For convenience, the new amendments are to be introduced in the current version of ATP (as amended on 19 December 2016) and the current version of the ATP Handbook (from the ECE website, as it appeared on 5 June 2017).

Proposals

7. Reword ATP annex 1, appendix 2, sub-section 2.3.2, as follows:[1]

"2.3.2 Accuracy of measurements of the K coefficient

Testing stations shall be provided with the equipment and instruments necessary to ensure that the K coefficient is determined with ~~a maximum margin of error~~ an expanded uncertainty ± 10% when using the method of internal cooling and ± 5% when using the method of internal heating. In calculating the expanded uncertainty of measurement of the K coefficient, the confidence level should be at least 95%."

[1] Here and throughout, new text has been underlined and text to be deleted has been struck-out. The original formatting is to be maintained throughout.

8. Reword the comments in the ATP Handbook relating to ATP annex 1, appendix 2, sub-section 2.3.2, as follows:[2]

"*Comments to 2.3.2:*

1. Examples for the ~~errors~~ uncertainty which are normally taken into account by the test stations are temperature, ~~power~~ heat output (or cold production) and the surface area of the body.

~~The method of calculating the error, which is usually applied, is the total admissible error ε:~~

$$\varepsilon = \sqrt{\left(\frac{\Delta S}{S}\right)^2 + \left(\frac{\Delta W}{W}\right)^2 + \left(2 \cdot \frac{\Delta T}{T_e - T_i}\right)}$$

~~or the absolute error: ε_m~~

$$\varepsilon_m = \frac{\Delta S}{S} + \frac{\Delta W}{W} + 2 \cdot \frac{\Delta T}{T_e - T_i}$$

~~where:~~

~~S is the mean surface area of the vehicle body (geometric mean of the internal and external surfaces);~~

~~W is the power dissipated inside the vehicle body in the steady state;~~

~~T_e and T_i are the respective external and internal temperatures of the vehicle body under test.~~ T_i

The expanded uncertainty of the measurement of the K coefficient, U(K), can be obtained using the recommendations in paragraph 6.3.3 of ISO/IEC Guide 98-3:2008. In this case:

$$U(K) = k \cdot u_c(K)$$

where:

k – is the coverage factor for the selected confidence level (for a confidence level or 95% this may be taken as 2; for 99%, 3);

$u_c(K)$ – is the combined standard uncertainty of the measurement of the K coefficient.

The combined standard uncertainty of the measurement of the K coefficient is an approximation of the standard deviation of the K coefficient and characterizes the range of values which may reasonably be assigned to the K coefficient.

Since the K coefficient is determined by a functional dependence that includes such physical values as heat output (or cold production) of heat exchangers, external and internal temperatures of the body and the mean surface area of the body, which are in turn measured with some standard uncertainty, the combined uncertainty of the measurement of the K coefficient can be calculated on the basis of the law of the propagation of uncertainty described in section 5 of ISO/IEC Guide 98-3:2008, taking into consideration the correlation (over time) of the internal and external temperatures of the body, the heat output (or cold production) and the inside temperature of the body:

[2] To ensure clarity, the added formulas, including the descriptions of their arguments, are not underlined.

$$u_c(K) = \sqrt{\begin{matrix}\left(\frac{u_c(W)}{\bar{S}\cdot(\bar{T}_e-\bar{T}_i)}\right)^2+\left(\frac{\bar{W}\cdot u_c(S)}{\bar{S}^2\cdot(\bar{T}_e-\bar{T}_i)}\right)^2+\cdots \\ \frac{\bar{W}^2\cdot\left(u_c(T_i)^2+u_c(T_e)^2+2\cdot r(T_e,T_i)\cdot u_c(T_e)\cdot u_c(T_i)\right)}{\bar{S}^2\cdot(\bar{T}_e-\bar{T}_i)^4}+\cdots \\ \frac{2\cdot\bar{W}\cdot r(W,T_i)\cdot u_c(W)\cdot u_c(T_i)}{\bar{S}^2\cdot(|\bar{T}_e-\bar{T}_i|)^3}\end{matrix}}$$

where:

$\bar{W}, \bar{T}_e, \bar{T}_i, \bar{S}$ – *are mean values respectively for the heat output (or cold production), in W; the external and internal temperatures of the body, in °C; and the area of the average surface of the body, in m²;*

$u_c(W)$, $u_c(T_i)$, $u_c(T_e)$, $u_c(S)$ – *are the combined standard uncertainties of measurement, respectively of the heat output (or cold production), in W; the external and internal temperatures of the body, in °C; and the area of the average surface of the body, in m²;*

$r(T_e,T_i)$, $r(W,T_i)$ – *are the correlation coefficients, respectively, of the value vectors of the external and internal temperatures of the body, and of the heat output (or cold production) and the internal temperature of the body.*

The correlation coefficient may be calculated as a linear correlation coefficient (Pearson correlation coefficient). However, it should be borne in mind that changes in the values of the vectors for heat output (or cold production), and particularly for the external temperature of the body, produce corresponding changes in the vector of the internal temperature of the body, with some shift (or lag) over time. This time lag is due to heat exchange processes in the "air inside the special equipment-insulation-environment" system. If there is a change in the external temperature of the body, this may take several hours. The actual time lag can be established either visually (by looking at graphs of the changing values) or by selecting the maximum linear correlation coefficient, with consistent selection of the shift variants for the internal temperature vector.

The combined standard uncertainty of measurement of the heat output (or cold production), and that of the external and internal temperatures of the body, can be determined using the recommendations in sections 4 and 5 of ISO/IEC Guide 98-3:2008, according to the following formulae:

$$u_C(W) = \sqrt{u_A(\bar{W})^2+u_B(W)^2} = \sqrt{\frac{\sum_{k=1}^{n}(W_k-\bar{W})^2}{n\cdot(n-1)}+u_B(W)^2}$$

$$u_C(T_i) = \sqrt{u_A(\bar{T}_i)^2+u_A\left(\bar{\bar{T}}_i\right)^2+u_B(T_i)^2} = \sqrt{\begin{matrix}\max\limits_{1\le k\le n}\left(\frac{\sum_{i=1}^{l}\left(T_{i_{i,k}}-\overline{T_{i_k}}\right)^2}{l\cdot(l-1)}\right)+\cdots \\ \frac{\sum_{k=1}^{n}\left(\overline{T_{i_k}}-\bar{T}_i\right)^2}{n\cdot(n-1)}+\cdots \\ u_B(T_i)^2\end{matrix}}$$

$$u_C(T_e) = \sqrt{u_A(\bar{T}_e)^2+u_A\left(\bar{\bar{T}}_e\right)^2+u_B(T_e)^2} = \sqrt{\begin{matrix}\max\limits_{1\le k\le n}\left(\frac{\sum_{j=1}^{m}\left(T_{e_{j,k}}-\overline{T_{e_k}}\right)^2}{m\cdot(m-1)}\right)+\cdots \\ \frac{\sum_{k=1}^{n}\left(\overline{T_{e_k}}-\bar{T}_e\right)^2}{n\cdot(n-1)}+\cdots \\ u_B(T_e)^2\end{matrix}}$$

$$u_C(S) = \sqrt{\frac{\left(\overline{S_i} \cdot u_c(S_e)\right)^2 + \left(\overline{S_e} \cdot u_c(S_i)\right)^2}{4 \cdot \overline{S_e} \cdot \overline{S_i}}}$$

where:

$u_A(\overline{W})$, $u_A(\overline{T_\iota})$, $u_A(\overline{T_e})$, $u_A(\overline{\overline{T_\iota}})$, $u_A(\overline{\overline{T_e}})$ – *are the standard uncertainties of measurement of the average values, respectively for: the heat output (or cold production), in W; and the internal and external temperatures of the body (within the limits of a single measurement on the basis of simultaneous readings of 12 thermometers), in K; and the internal and external temperatures of the body (steady state), in K, using type A evaluation;*

$u_B(W)$, $u_B(T_i)$, $u_B(T_e)$ – *are the standard uncertainties of measurement respectively of the heat output (or cold production), in W; and of the internal and external temperatures of the body, in K, using type B evaluation;*

$u_c(S_e)$, $u_c(S_i)$ – *are the combined standard uncertainties of the values of the areas respectively of the internal and external surfaces of the body of the vehicle being tested (disregarding corrugation), in m²;*

W_k – *is the value of the heat output (or cold production) obtained at the kth measurement (in all, when n measurements are taken at the end of the steady state, for the period of measurement), in W;*

$T_{i_{i,k}}$, $T_{e_{j,k}}$ – *are the temperatures measured at the kth measurement, respectively using instrument i on the interior of the body of the vehicle under test (in all, with one measurement, simultaneously taken by l uniformly precise thermometers) and by instrument j on the exterior of the body of the vehicle under test (in all, with one measurement, simultaneously taken by m uniformly precise thermometers), in °C;*

$\overline{W}$, $\overline{T_\iota}$, $\overline{T_e}$ – *are the calculated average values (steady state), respectively, of the heat output (or cold production), in W; and the internal and external temperatures of the body, in °C;*

$\overline{T_{\iota_k}}$, $\overline{T_{e_k}}$ – *are the calculated average values (within the limits of the -th measurement), respectively, of the internal and external temperatures of the body, in °C;*

$\overline{S_i}$, $\overline{S_e}$ – *are the calculated average values of the areas, respectively of the internal and external surfaces of the body of the vehicle being tested (disregarding corrugation), in m².*

$$\overline{W} = \frac{\sum_{k=1}^{n} W_k}{n}$$

$$\overline{T_\iota} = \frac{\sum_{k=1}^{n} \sum_{i=1}^{l} T_{i_{i,k}}}{n \cdot l}$$

$$\overline{T_e} = \frac{\sum_{k=1}^{n} \sum_{j=1}^{m} T_{e_{j,k}}}{n \cdot m}$$

$$\overline{T_{\iota_k}} = \frac{\sum_{i=1}^{l} T_{i_{i,k}}}{l}$$

$$\overline{T_{e_k}} = \frac{\sum_{j=1}^{m} T_{e_{j,k}}}{m}$$

If the heat output (or cold production) of the heat exchangers has been determined on the basis of the values of electric energy consumption consumed by the heat exchangers, then the mathematical dependence on the basis of which the required calculations are carried out must be factored into the final result of the uncertainty as well.

Section 4.3 of ISO/IEC Guide 98-3:2008 addresses the evaluation of standard uncertainties for type B evaluation. In this commentary we provide a design formula to obtain the standard uncertainty on the basis of known boundaries (upper and lower limits) for the evaluation of the measured physical values. Such situations often occur in practice and correspond with concepts such as the accuracy class of the instrumentation and its margin of error. If the interval of the evaluations of measured physical values, x, is denoted as 2a (corresponding to the common notation for the margin of error of the instrumentation as ±a), then:

$$u_B(x) = \frac{a}{\sqrt{3}}$$

2. *Under normal ~~test~~ conditions, ~~S~~ $\overline{S_i}$ and $\overline{S_e}$ can be measured with a high degree of accuracy. The combined standard uncertainty for such conditions may be accepted as equal to ± 1%. However, there are cases where it is impossible to measure with this precision.*

Generally, the following method may be used to determine the combined standard uncertainty of S_i and S_e,which are used to determine the heat transfer surface area of the body, S.

If S_i and $S_e$are presented as functions of a series of repeated measurements, p_i and p_e (for example, of the length, width and height measured at various places in the body of the vehicle):

$$S_i = f_1\left(p_{i_1}, p_{i_2}, \dots, p_{i_y}, \dots, p_{i_Y}\right)$$

$$S_e = f_2\left(p_{e_1}, p_{e_2}, \dots, p_{e_z}, \dots, p_{e_Z}\right)$$

then their combined standard uncertainties can be calculated according to the formulae:

$$u_c(S_i) = \sqrt{\sum_{y=1}^{Y}\left(u_c\left(p_{i_y}\right)\cdot\frac{\partial f_1}{\partial p_{i_y}}\right)^2}$$

$$u_c(S_e) = \sqrt{\sum_{z=1}^{Z}\left(u_c\left(p_{e_z}\right)\cdot\frac{\partial f_2}{\partial p_{e_z}}\right)^2}$$

where:

$\frac{\partial f_1}{\partial p_{i_y}}, \frac{\partial f_2}{\partial p_{e_z}}$ *– are respectively the partial derivatives for the functions for calculating S_i and S_e;*

$u_c\left(p_{i_y}\right), u_c\left(p_{e_z}\right)$ *– are the combined standard uncertainties for the parameters p_{i_y} and p_{e_z}*

$$u_c\left(p_{i_y}\right) = \sqrt{\frac{\sum_{v=1}^{V}\left(p_{i_{y_v}} - \overline{p_{i_y}}\right)^2}{V\cdot(V-1)} + u_B\left(p_{i_y}\right)^2}$$

$$\overline{p_{i_y}} = \frac{\sum_{v=1}^{V} p_{i_{y_v}}}{V}$$

where:

V – is the quantity of measurements carried out to determine the average value of parameter p_{i_y};

$p_{i_{y_v}}$ – *is the measured value of parameter* p_{i_y} *at the -th measurement;*

$u_B\left(p_{i_y}\right)$ – *is the standard uncertainty of parameter* p_{i_y} *assessed by type B evaluation (for further details on uncertainty evaluation methods and techniques using type B evaluation see section 4.3 of ISO/IEC Guide 98-3:2008).*

$\overline{p_{e_z}}$ *and* $u_c(p_{e_z})$ *are calculated in a fashion similar to* $\overline{p_{i_y}}$ *and* $u_c\left(p_{i_y}\right)$.

~~*The error of W does not exceed 1 %, although certain test stations use equipment giving a greater error.*~~

~~*Temperature is measured with an absolute accuracy of 0.1 K. The measurement of a temperature difference* $(T_e - T_i)$ *of the order of 20 K therefore gives an error of twice 0.5 %, i.e. ±1%.*~~

~~*Taking this into account, the total admissible error is* $\varepsilon = \pm \sqrt{0{,}0003} = \pm 0{,}017$*, i.e., ±1,7%. The maximum admissible error is* $\varepsilon_m = \pm 3\%$.~~

3. *Other ~~errors~~ uncertainties which have not been taken into consideration can have an effect on accuracy in determining the K coefficient.*

   (a) Latent errors due to admissible variations in the internal and external temperatures, which are a function of the thermal inertia of the walls of the equipment, the temperature and time;

   (b) ~~Errors~~ Uncertainties due to the variation of air velocity at the boundary layer and its effect on the thermal resistance.

   If the internal and external air velocities are of equal value, the possible ~~error~~ expanded uncertainty will be about 2.5% as between 1 and 2 m/s for a mean K coefficient of 0.40 W/m²K. For a K coefficient of 0.70 W/m²K, this ~~error~~ expanded uncertainty will be nearly 5%. If there are significant thermal bridges, the influence of the speed and direction of the air will be greater.

~~*4. Finally, because of the error in the estimation of the surface area of the body, an error which in practice is difficult to calculate when dealing with non standard equipment, (this estimation involving factors of a subjective nature), one could envisage the determination of the error in the measurement of the overall heat transfer per degree temperature difference:*~~

$$\frac{W}{T_e - T_i} = K \cdot S$$

9. In model test reports Nos. 2 A and 2 B, recast the line on the margin of error for the definition of the K coefficient, as follows:

"~~Maximum error~~ Expanded uncertainty of measurement with test used … per cent (coverage factor k = ... for a confidence level of … %."

Sample calculations

10. A sample calculation for the uncertainty of the measurement of the K coefficient carried out using Mathcad is given in annex A.

Justification

11. This document calls for the use of uncertainty instead of error, primarily for the following reasons:

- It is widespread practice throughout the world to use uncertainty in describing measurement results (error is used more often for measurement instruments).
- ISO/IEC Guide 98-3:2008 has been translated into Russian and has become the national standard of the Russian Federation.

- There is the possibility of greater practical use, as uncertainty is related to an actually obtained (measured) result and expresses a level of doubt in its veracity, while error relates to an abstract and unknowable "true value".
- ISO/IEC Guide 98-3:2008 establishes understandable, uniform rules for determining uncertainty, including through the exclusion of the main differences between the components of uncertainty arising from random effects and those associated with the correction for systematic effects, and also by taking into account the effects of possible correlations of measured values.

12. A commentary to paragraph 2.3.2 of ATP annex 1, appendix 2 is required because ISO/IEC Guide 98-3:2008 contains only a general classification and methods for determining uncertainties caused by various factors. The broad freedom in the choice of the mathematical models used for the measurements, the possibility of using an essentially infinite number of components of uncertainty and the taking into account of the effects of correlation of measured values create a great variety of specific methodologies that may be used to establish the uncertainty of measurement of the K coefficient. The experts from the Russian Federation, without in any way limiting that freedom, would like to see the ATP Handbook provide certain recommendations on the most important points that arise when finding the uncertainty of measurement of the K coefficient. With such information, it will become possible, inter alia, to carry out a justified assessment of the required classes of accuracy of measurement equipment for tests to measure the value of the K coefficient.

13. In paragraph 2.3.2 of appendix 2 to annex 1 of ATP, the term "maximum error" has been replaced with "expanded uncertainty", as it is the concept that gives the closest numerical equivalent to an expression of quantity.

At the same time, if expanded uncertainty is used, then it follows that the confidence level for the defined value of the coverage factor must be indicated. Paragraph 2.3.2 of appendix 2 to annex 1 of ATP indicates the minimum confidence level for solving most technical tasks. The coverage factor, which in turn may be defined in various ways, is indicated in the models for tests No. 2 A and 2 B, with the aim of allowing further reverse calculations of the combined standard uncertainty for the measurement of the K coefficient.

In comment 3 to 2.3.2 of appendix 2 to annex 1 of ATP, the term "error" has been replaced with "uncertainty" (without specification of the type) for cases where reference is made to the concept, without a specific form. In all other cases, the term "expanded uncertainty" has been used, for the reasons given above.

14. The use of a simplified coverage factor equal to 2 for a confidence level of 95% (and to 3 for a confidence level of 99%) is justified owing to the large, hard to establish number of effective degrees of freedom (inter alia, as a result of correlation) during the evaluation of $u_c(K)$. The values of the selected coverage factors approximately correspond with the condition of proximity to normal probability distribution with estimates for the values of the K coefficient and $u_c(K)$, which is justified in meeting the conditions for the central limit theorem in probability theory. Taking into account the number of repeated measurements of physical values that are in a relation of functional dependency with the K coefficient, and the fact that their mean values are being used with the corresponding estimates of uncertainties, it may be considered that the probabilities for the estimated values of the K coefficient and $u_c(K)$ are normally distributed.

Despite the overall similarity of the mathematical methods used to calculate errors and uncertainties, there are a number of important divergences from the document from last year. Specifically, there is the introduction of the correlation between the various arguments of the functional dependence used to calculate the value of the K coefficient. As can be seen in the sample calculation in annex A to this document, the calculation of the correlation between the parameters of the external and internal temperatures of the body and of the heat output (or cold production) and the internal temperature of the body introduces a significant component that influences the final value of the combined standard uncertainty of measurement of the K coefficient.

Costs

16. There are no additional costs.

Feasibility

17. The proposed amendments to ATP will remove ambiguity about the instrument's requirements for the accuracy of definition of the K coefficient in the testing of special equipment. The recommendations on methods for identifying the various components of the uncertainties in the measurement of the K coefficient help to build confidence between the Contracting Parties to ATP.

Enforceability

18. No problems are foreseen in the use of the proposed clarifications regarding expanded uncertainty of measurement of the K coefficient in ATP.

Sample calculation of uncertainty of measurements of the K coefficient of an insulated wagon

1 Input data

Power consumed by electrical heating appliances [QD], in W; internal [TiD] and external [TeD] temperature of the body, °C:

$$QD := \begin{pmatrix} 1852.7 \\ 1829.7 \\ 1850.6 \\ 1835.9 \\ 1856.9 \\ 1840.0 \\ 1854.8 \\ 1829.7 \\ 1838.0 \\ 1856.9 \\ 1833.8 \\ 1850.6 \\ 1821.3 \\ 1836.0 \\ 1817.2 \\ 1842.2 \\ 1823.4 \\ 1817.2 \\ 1842.2 \\ 1810.9 \\ 1831.8 \\ 1798.4 \\ 1821.3 \\ 1802.5 \\ 1821.3 \\ 1794.2 \\ 1810.9 \\ 1785.8 \\ 1779.7 \\ 1798.3 \\ 1771.3 \\ 1802.4 \\ 1783.7 \\ 1813.0 \\ 1777.5 \\ 1785.8 \\ 1806.7 \\ 1777.5 \\ 1798.4 \\ 1771.2 \\ 1794.2 \\ 1781.6 \\ 1792.1 \\ 1813.0 \\ 1790.1 \\ 1810.9 \\ 1779.6 \\ 1796.2 \\ 1763.0 \end{pmatrix} \quad TeD := \begin{pmatrix} 7.1 & 7.2 & 7.0 & 6.9 & 7.2 & 7.2 & 6.7 & 6.8 & 7.5 & 7.5 & 6.8 & 7.7 \\ 7.1 & 7.2 & 6.9 & 6.8 & 7.2 & 7.2 & 6.8 & 6.8 & 7.5 & 7.5 & 6.9 & 7.7 \\ 7.1 & 7.2 & 7.0 & 6.8 & 7.3 & 7.2 & 6.7 & 6.8 & 7.5 & 7.5 & 6.9 & 7.8 \\ 7.1 & 7.1 & 7.0 & 6.9 & 7.2 & 7.2 & 6.7 & 6.7 & 7.5 & 7.4 & 6.9 & 7.7 \\ 7.0 & 7.1 & 6.9 & 6.9 & 7.2 & 7.1 & 6.7 & 6.7 & 7.4 & 7.4 & 6.8 & 7.7 \\ 7.1 & 7.1 & 6.9 & 6.9 & 7.2 & 7.1 & 6.7 & 6.8 & 7.4 & 7.5 & 6.8 & 7.8 \\ 7.0 & 7.1 & 6.9 & 6.8 & 7.2 & 7.1 & 6.7 & 6.8 & 7.4 & 7.5 & 6.8 & 7.7 \\ 7.0 & 7.1 & 6.9 & 6.8 & 7.2 & 7.1 & 6.7 & 6.7 & 7.5 & 7.4 & 6.9 & 7.7 \\ 7.0 & 7.1 & 6.9 & 6.8 & 7.2 & 7.1 & 6.7 & 6.7 & 7.4 & 7.4 & 6.8 & 7.7 \\ 7.1 & 7.1 & 6.9 & 6.8 & 7.2 & 7.2 & 6.7 & 6.7 & 7.5 & 7.5 & 6.8 & 7.7 \\ 7.0 & 7.1 & 6.9 & 6.8 & 7.2 & 7.1 & 6.7 & 6.7 & 7.4 & 7.5 & 6.8 & 7.7 \\ 7.1 & 7.1 & 6.7 & 6.6 & 7.2 & 7.1 & 6.7 & 6.8 & 7.4 & 7.5 & 6.6 & 7.8 \\ 7.0 & 7.1 & 6.9 & 6.8 & 7.2 & 7.1 & 6.7 & 6.7 & 7.4 & 7.4 & 6.7 & 7.7 \\ 7.0 & 7.1 & 6.7 & 6.6 & 7.2 & 7.0 & 6.5 & 6.6 & 7.3 & 7.4 & 6.6 & 7.7 \\ 7.0 & 7.1 & 6.7 & 6.6 & 7.1 & 7.0 & 6.6 & 6.6 & 7.3 & 7.4 & 6.7 & 7.7 \\ 6.9 & 7.0 & 6.7 & 6.6 & 7.1 & 7.0 & 6.6 & 6.7 & 7.4 & 7.4 & 6.7 & 7.7 \\ 6.9 & 7.0 & 6.5 & 6.6 & 7.1 & 6.9 & 6.6 & 6.6 & 7.3 & 7.3 & 6.6 & 7.6 \\ 6.9 & 7.0 & 6.6 & 6.6 & 7.0 & 6.9 & 6.5 & 6.5 & 7.2 & 7.3 & 6.6 & 7.6 \\ 6.8 & 6.9 & 6.6 & 6.6 & 6.9 & 6.8 & 6.4 & 6.5 & 7.2 & 7.2 & 6.6 & 7.6 \\ 6.8 & 6.9 & 6.5 & 6.6 & 6.9 & 6.8 & 6.4 & 6.5 & 7.2 & 7.3 & 6.5 & 7.5 \\ 6.7 & 6.9 & 6.5 & 6.5 & 6.9 & 6.8 & 6.5 & 6.5 & 7.1 & 7.2 & 6.6 & 7.5 \\ 6.7 & 6.8 & 6.4 & 6.5 & 6.8 & 6.8 & 6.5 & 6.5 & 7.1 & 7.2 & 6.4 & 7.5 \\ 6.7 & 6.8 & 6.3 & 6.4 & 6.9 & 6.7 & 6.4 & 6.5 & 7.1 & 7.2 & 6.4 & 7.5 \\ 6.7 & 6.8 & 6.3 & 6.4 & 6.8 & 6.7 & 6.4 & 6.4 & 7.1 & 7.2 & 6.4 & 7.4 \\ 6.7 & 6.8 & 6.4 & 6.4 & 6.8 & 6.8 & 6.3 & 6.4 & 7.1 & 7.1 & 6.5 & 7.4 \\ 6.7 & 6.8 & 6.3 & 6.4 & 6.8 & 6.7 & 6.4 & 6.4 & 7.0 & 7.1 & 6.4 & 7.4 \\ 6.7 & 6.8 & 6.4 & 6.5 & 6.8 & 6.7 & 6.4 & 6.4 & 7.1 & 7.2 & 6.5 & 7.4 \\ 6.7 & 6.8 & 6.3 & 6.4 & 6.8 & 6.7 & 6.4 & 6.4 & 7.1 & 7.1 & 6.4 & 7.5 \\ 6.7 & 6.8 & 6.3 & 6.4 & 6.8 & 6.7 & 6.3 & 6.4 & 7.1 & 7.1 & 6.3 & 7.3 \\ 6.6 & 6.7 & 6.3 & 6.4 & 6.7 & 6.7 & 6.4 & 6.4 & 7.0 & 7.1 & 6.4 & 7.4 \\ 6.7 & 6.8 & 6.2 & 6.3 & 6.8 & 6.7 & 6.3 & 6.3 & 6.9 & 7.0 & 6.3 & 7.4 \\ 6.6 & 6.7 & 6.2 & 6.3 & 6.7 & 6.6 & 6.3 & 6.4 & 7.0 & 7.1 & 6.3 & 7.3 \\ 6.6 & 6.7 & 6.3 & 6.4 & 6.7 & 6.6 & 6.2 & 6.3 & 7.0 & 7.0 & 6.4 & 7.4 \\ 6.8 & 6.8 & 6.3 & 6.4 & 6.9 & 6.8 & 6.4 & 6.4 & 7.1 & 7.1 & 6.4 & 7.4 \\ 6.7 & 6.8 & 6.3 & 6.4 & 6.7 & 6.7 & 6.4 & 6.4 & 7.0 & 7.1 & 6.4 & 7.3 \\ 6.8 & 6.8 & 6.3 & 6.3 & 6.8 & 6.8 & 6.4 & 6.4 & 7.0 & 7.1 & 6.4 & 7.4 \\ 6.7 & 6.8 & 6.2 & 6.4 & 6.8 & 6.7 & 6.4 & 6.5 & 7.1 & 7.1 & 6.3 & 7.4 \\ 6.8 & 6.8 & 6.3 & 6.4 & 6.9 & 6.7 & 6.4 & 6.4 & 7.1 & 7.1 & 6.4 & 7.5 \\ 6.8 & 6.8 & 6.4 & 6.5 & 6.9 & 6.8 & 6.4 & 6.5 & 7.2 & 7.2 & 6.5 & 7.5 \\ 6.9 & 6.9 & 6.3 & 6.4 & 6.9 & 6.9 & 6.4 & 6.5 & 7.2 & 7.2 & 6.4 & 7.5 \\ 6.9 & 6.9 & 6.4 & 6.5 & 7.0 & 6.9 & 6.5 & 6.5 & 7.3 & 7.2 & 6.4 & 7.5 \\ 6.9 & 7.0 & 6.5 & 6.5 & 7.0 & 6.9 & 6.5 & 6.5 & 7.2 & 7.2 & 6.5 & 7.6 \\ 6.9 & 7.0 & 6.4 & 6.5 & 7.1 & 7.0 & 6.5 & 6.6 & 7.2 & 7.3 & 6.5 & 7.6 \\ 6.9 & 7.0 & 6.4 & 6.5 & 7.0 & 7.0 & 6.5 & 6.5 & 7.3 & 7.3 & 6.5 & 7.6 \\ 7.0 & 7.0 & 6.5 & 6.5 & 7.1 & 7.0 & 6.5 & 6.6 & 7.2 & 7.3 & 6.5 & 7.6 \\ 7.0 & 7.0 & 6.4 & 6.5 & 7.1 & 7.0 & 6.5 & 6.5 & 7.2 & 7.3 & 6.5 & 7.7 \\ 6.9 & 7.0 & 6.4 & 6.4 & 7.0 & 6.9 & 6.5 & 6.5 & 7.2 & 7.2 & 6.5 & 7.6 \\ 7.0 & 7.0 & 6.4 & 6.5 & 7.0 & 7.0 & 6.5 & 6.5 & 7.2 & 7.3 & 6.5 & 7.6 \\ 7.0 & 7.0 & 6.4 & 6.5 & 7.1 & 7.0 & 6.5 & 6.5 & 7.3 & 7.3 & 6.5 & 7.7 \end{pmatrix}$$

$$
TiD := \begin{pmatrix}
33.8 & 33.7 & 33.6 & 33.2 & 34.0 & 34.1 & 32.8 & 33.0 & 33.2 & 33.2 & 32.4 & 33.6 \\
33.8 & 33.7 & 33.8 & 33.2 & 34.1 & 34.1 & 32.8 & 33.0 & 33.2 & 33.2 & 32.3 & 33.6 \\
33.8 & 33.7 & 33.6 & 33.1 & 34.1 & 34.1 & 32.8 & 33.1 & 33.2 & 33.3 & 32.3 & 33.7 \\
33.8 & 33.7 & 33.8 & 33.2 & 34.1 & 34.1 & 32.8 & 33.1 & 33.3 & 33.2 & 32.4 & 33.7 \\
33.9 & 33.7 & 33.8 & 33.2 & 34.1 & 34.1 & 32.8 & 33.1 & 33.3 & 33.3 & 32.4 & 33.6 \\
33.9 & 33.7 & 33.6 & 33.2 & 34.1 & 34.1 & 32.8 & 33.1 & 33.4 & 33.3 & 32.4 & 33.6 \\
33.9 & 33.7 & 33.6 & 33.2 & 34.1 & 34.2 & 32.9 & 33.1 & 33.4 & 33.3 & 32.4 & 33.7 \\
33.9 & 33.8 & 33.6 & 33.2 & 34.1 & 34.2 & 32.8 & 33.1 & 33.3 & 33.3 & 32.4 & 33.7 \\
33.9 & 33.8 & 33.8 & 33.3 & 34.1 & 34.2 & 32.9 & 33.2 & 33.3 & 33.3 & 32.4 & 33.7 \\
33.9 & 33.8 & 33.8 & 33.2 & 34.1 & 34.2 & 32.9 & 33.2 & 33.4 & 33.4 & 32.4 & 33.7 \\
33.9 & 33.8 & 33.8 & 33.2 & 34.1 & 34.2 & 32.9 & 33.1 & 33.3 & 33.3 & 32.4 & 33.8 \\
33.9 & 33.8 & 33.8 & 33.3 & 34.1 & 34.2 & 32.9 & 33.1 & 33.4 & 33.3 & 32.4 & 33.8 \\
33.9 & 33.8 & 33.8 & 33.2 & 34.1 & 34.2 & 32.8 & 33.2 & 33.4 & 33.4 & 32.4 & 33.8 \\
33.9 & 33.8 & 33.9 & 33.3 & 34.1 & 34.2 & 32.8 & 33.1 & 33.3 & 33.4 & 32.4 & 33.7 \\
33.9 & 33.8 & 33.8 & 33.2 & 34.1 & 34.2 & 32.9 & 33.2 & 33.4 & 33.3 & 32.4 & 33.7 \\
33.9 & 33.8 & 33.9 & 33.3 & 34.1 & 34.2 & 33.0 & 33.2 & 33.4 & 33.4 & 32.3 & 33.8 \\
33.9 & 33.8 & 33.9 & 33.3 & 34.2 & 34.2 & 32.9 & 33.1 & 33.3 & 33.4 & 32.4 & 33.7 \\
34.0 & 33.8 & 33.9 & 33.2 & 34.1 & 34.2 & 32.9 & 33.2 & 33.4 & 33.3 & 32.4 & 33.8 \\
33.9 & 33.8 & 33.8 & 33.3 & 34.1 & 34.3 & 32.9 & 33.1 & 33.4 & 33.4 & 32.4 & 33.7 \\
33.9 & 33.8 & 33.9 & 33.3 & 34.1 & 34.3 & 33.0 & 33.2 & 33.4 & 33.4 & 32.6 & 33.7 \\
33.9 & 33.8 & 33.8 & 33.3 & 34.1 & 34.2 & 32.8 & 33.2 & 33.4 & 33.4 & 32.6 & 33.7 \\
33.9 & 33.8 & 33.8 & 33.3 & 34.1 & 34.2 & 32.8 & 33.2 & 33.4 & 33.3 & 32.6 & 33.7 \\
33.9 & 33.8 & 33.9 & 33.3 & 34.1 & 34.2 & 32.9 & 33.2 & 33.3 & 33.3 & 32.6 & 33.7 \\
33.9 & 33.8 & 33.9 & 33.3 & 34.1 & 34.2 & 33.0 & 33.2 & 33.3 & 33.4 & 32.6 & 33.7 \\
33.9 & 33.8 & 33.8 & 33.2 & 34.1 & 34.2 & 32.9 & 33.2 & 33.3 & 33.4 & 32.6 & 33.7 \\
33.9 & 33.8 & 33.9 & 33.3 & 34.1 & 34.2 & 32.9 & 33.2 & 33.3 & 33.3 & 32.6 & 33.7 \\
33.9 & 33.8 & 33.8 & 33.3 & 34.1 & 34.2 & 32.9 & 33.2 & 33.3 & 33.3 & 32.6 & 33.7 \\
33.9 & 33.8 & 33.8 & 33.3 & 34.1 & 34.2 & 32.8 & 33.1 & 33.3 & 33.3 & 32.6 & 33.8 \\
33.9 & 33.8 & 33.8 & 33.2 & 34.1 & 34.2 & 32.8 & 33.2 & 33.3 & 33.3 & 32.4 & 33.6 \\
33.9 & 33.8 & 33.8 & 33.2 & 34.1 & 34.1 & 32.7 & 33.2 & 33.3 & 33.3 & 32.6 & 33.7 \\
33.9 & 33.8 & 33.8 & 33.2 & 34.1 & 34.1 & 32.8 & 33.2 & 33.4 & 33.2 & 32.6 & 33.7 \\
33.8 & 33.8 & 33.9 & 33.2 & 34.1 & 34.1 & 32.9 & 33.2 & 33.3 & 33.2 & 32.6 & 33.7 \\
33.9 & 33.7 & 33.9 & 33.3 & 34.1 & 34.1 & 32.8 & 33.2 & 33.3 & 33.3 & 32.4 & 33.6 \\
33.9 & 33.7 & 33.6 & 33.2 & 34.1 & 34.2 & 32.8 & 33.1 & 33.3 & 33.3 & 32.4 & 33.7 \\
33.9 & 33.7 & 33.8 & 33.3 & 34.1 & 34.2 & 32.8 & 33.1 & 33.3 & 33.3 & 32.4 & 33.7 \\
33.9 & 33.8 & 33.6 & 33.2 & 34.1 & 34.1 & 32.8 & 33.2 & 33.3 & 33.3 & 32.3 & 33.7 \\
33.9 & 33.8 & 33.8 & 33.2 & 34.1 & 34.1 & 32.9 & 33.2 & 33.3 & 33.2 & 32.4 & 33.6 \\
33.9 & 33.7 & 33.6 & 33.2 & 34.1 & 34.1 & 32.8 & 33.2 & 33.3 & 33.3 & 32.4 & 33.7 \\
33.9 & 33.8 & 33.9 & 33.2 & 34.1 & 34.1 & 32.9 & 33.2 & 33.2 & 33.2 & 32.6 & 33.6 \\
33.9 & 33.7 & 33.6 & 33.2 & 34.1 & 34.1 & 32.8 & 33.1 & 33.2 & 33.2 & 32.4 & 33.6 \\
33.8 & 33.7 & 33.8 & 33.2 & 34.1 & 34.1 & 32.8 & 33.1 & 33.2 & 33.3 & 32.3 & 33.6 \\
33.8 & 33.7 & 33.6 & 33.2 & 34.1 & 34.2 & 32.8 & 33.1 & 33.2 & 33.2 & 32.4 & 33.7 \\
33.9 & 33.7 & 33.6 & 33.2 & 34.1 & 34.1 & 32.8 & 33.1 & 33.3 & 33.2 & 32.3 & 33.7 \\
33.9 & 33.7 & 33.8 & 33.2 & 34.1 & 34.1 & 32.8 & 33.1 & 33.3 & 33.3 & 32.4 & 33.6 \\
33.9 & 33.8 & 33.6 & 33.2 & 34.1 & 34.1 & 32.9 & 33.1 & 33.3 & 33.3 & 32.4 & 33.6 \\
33.9 & 33.7 & 33.8 & 33.3 & 34.1 & 34.1 & 32.8 & 33.2 & 33.3 & 33.2 & 32.4 & 33.7 \\
33.9 & 33.7 & 33.6 & 33.2 & 34.1 & 34.1 & 32.8 & 33.1 & 33.3 & 33.3 & 32.4 & 33.6 \\
33.9 & 33.7 & 33.8 & 33.2 & 34.1 & 34.1 & 32.8 & 33.1 & 33.3 & 33.3 & 32.4 & 33.6 \\
33.9 & 33.7 & 33.6 & 33.2 & 34.1 & 34.1 & 32.8 & 33.1 & 33.2 & 33.3 & 32.4 & 33.6
\end{pmatrix}
$$

Coverage coefficient for a level of confidence of p = 95% for a measurement of the K coefficient:

$k := 2$

Accuracy class of the electric power consumption meter, % of measured result:

$\delta\_Q := .1$

Instrument margin of error for the wagon body's internal temperature measurement, in K:

$\Delta\_Ti := 0.1$

Instrument margin of error for the wagon body's external temperature measurement, in K:

$\Delta\_Te := 0.1$

External dimensions of the wagon body:

Note: The external dimensions of the wagon body are taken from the technical documentation. The admissible error may be taken as the unit in the highest digit position for this parameter, divided by two.

length, average value of length and the assigned value of error, in m:

$LeD := 15.750$

$mLe := mean(LeD) = 15.750$

$\Delta\_Le := \frac{10^{-3}}{2} = 0.0005$

width, average value of width and the assigned value of error, in m:

$BeD := 2.790$

$mBe := mean(BeD) = 2.790$

$\Delta\_Be := \frac{10^{-3}}{2} = 0.0005$

side wall height, its average value and the assigned value of error, in m:

$HeD := 2.915$

$mHe := mean(HeD) = 2.915$

$\Delta\_He := \frac{10^{-3}}{2} = 0.0005$

central longitudinal axis height, its average value and the assigned value of error, in m:

$HHeD := 3{,}323$

$mHHe := mean(HHeD) = 3.323$

$\Delta\_HHe := \frac{10^{-3}}{2} = 0.0005$

Internal dimensions of the wagon body (cargo compartment):

Note: The internal dimensions of the wagon body are taken from the results of measurements (direct, repeated, uniform measurements) carried out using a 15 m tape measure at various places in the body. The instrument error of the tape measure is 0.005 m (half its graduation). In determining wagon body lengths exceeding the length of the tape

measure, two consecutive measurements were carried out, consequently adding the results obtained; the error was thus doubled.

Instrument error of the measuring tape, in m:

$$\Delta\_tape := \frac{10^{-2}}{2} = 0.005$$

length, average value of the length and margin of error of measurement, in m:

$$LiD := (15.395 \quad 15.405 \quad 15.400 \quad 15.400)$$

$$mLi := mean(LiD) = 15.400$$

$$\Delta\_Li := 2 \cdot \Delta\_tape = 0.010$$

width, average value of the width and margin of error of measurement, in m:

$$BiD := (2.455 \quad 2.450 \quad 2.455 \quad 2.455)$$

$$mBi := mean(BiD) = 2.454$$

$$\Delta\_Bi := \Delta\_tape = 0.005$$

side wall height, its average value and margin of error of measurement, in m:

$$HiD := (2.640 \quad 2.630 \quad 2.640 \quad 2.630)$$

$$mHi := mean(HiD) = 2.635$$

$$\Delta\_Hi := \Delta\_tape = 0.005$$

central longitudinal axis height, its average value and the margin of error of measurement, in m:

$$HHiD := (2.905 \quad 2.900)$$

$$mHHi := mean(HHiD) = 2.902$$

$$\Delta\_HHi := \Delta\_tape = 0.005$$

Calculation of heat output:

power cable length from the measurement instrument to the entry into the vehicle, m:

$$L\_line := 60 - \frac{mLi}{2}$$

specific electrical resistance of the wire in the power cable, in ohm-mm^2/m:

$$\rho := 0.0175$$

rated electrical tension in the grid, in V:

$$U := 220$$

cross-sectional area of the wire in the power cable, in mm^2:

$$s := 2.5$$

Calculated values of heat output, in W:

$$WD := \overrightarrow{\left[QD \cdot \left(1 - \frac{2 \cdot QD \cdot L\_line \cdot \rho}{U^2 \cdot s} \right) \right]} =$$

1800.8
1779.1
1798.8
1784.9
1804.7
1788.8
1802.8
1779.1
1786.9
1804.7
1782.9
1798.8
…

2 Definition of the average area of the heat transfer surface of the wagon body and its combined standard uncertainty

The standard uncertainty of measurement of the internal length of the wagon body, in m, calculated using type A evaluation:

$$uA\_Li := \sqrt{\frac{\sum \left(LiD^T - mLi \right)^2}{cols(LiD) \cdot (cols(LiD) - 1)}} = 0.0020$$

The standard uncertainty of measurement of the internal length of the wagon body, in m, calculated using type B evaluation:

$$uB\_Li := \frac{\Delta\_Li}{\sqrt{3}} = 0.0058$$

The combined standard uncertainty of measurement of the internal length of the wagon body, in W:

$$uC\_Li := \sqrt{uA\_Li^2 + uB\_Li^2} = 0.0061$$

Also, the width and side wall height and central longitudinal axis height of the wagon body, in m:

$$uA\_Bi := \sqrt{\frac{\sum \left(BiD^T - mBi \right)^2}{cols(BiD) \cdot (cols(BiD) - 1)}} \; 0.0012 \qquad uB\_Bi := \frac{\Delta\_Bi}{\sqrt{3}} = 0.0029 \qquad uC\_Bi := \sqrt{uA\_Bi^2 + uB\_Bi^2} = 0.0031$$

$$uA\_Hi := \sqrt{\frac{\sum \left(HiD^T - mHi \right)^2}{cols(HiD) \cdot (cols(HiD) - 1)}} = 0.0029 \qquad uB\_Hi := \frac{\Delta\_Hi}{\sqrt{3}} = 0.0029 \qquad uC\_Hi := \sqrt{uA\_Hi^2 + uB\_Hi^2} = 0.0041$$

$$uA\_HHi := \sqrt{\frac{\sum \left(HHiD^T - mHHi \right)^2}{cols(HHiD) \cdot (cols(HHiD) - 1)}} = 0.0025 \qquad uB\_HHi := \frac{\Delta\_HHi}{\sqrt{3}} = 0.0029 \qquad uC\_HHi := \sqrt{uA\_HHi^2 + uB\_HHi^2} = 0.0038$$

The standard uncertainty for the external length and width, the side wall height and the central longitudinal axis height of the wagon body, calculated using type B evaluation:

$$uB\_Le := \frac{\Delta\_Le}{\sqrt{3}} = 0.0003 \qquad uB\_Be := \frac{\Delta\_Be}{\sqrt{3}} = 0.0003 \qquad uB\_He := \frac{\Delta\_He}{\sqrt{3}} = 0.0003 \qquad uB\_HHe := \frac{\Delta\_HHe}{\sqrt{3}} = 0.0003$$

The combined standard uncertainty for the external length, width, the side wall height and the central longitudinal axis height of the wagon body:

$$uÑ\_Le := uB\_Le = 0.0003 \qquad uC\_Be := uB\_Be = 0.0003 \qquad uC\_He := uB\_He = 0.0003 \qquad uC\_HHe := uB\_HHe = 0.0003$$

Calculation of twice the mean length of curvature of the carriage roof and its combined standard uncertainty:

Note: Below is an approximation formula for calculating twice the rounded length of the wagon's roof, based on the assumption that its form is semielliptic. Maximum error of the formula: ~0.3619%, with an ellipse eccentricity of ~0.979811(axis ratio ~1/5). Such a methodic margin of error is always positive.

Empirical parameter:

$$x := \frac{\ln(2)}{\ln\left(\frac{\pi}{2}\right)}$$

Function for calculating twice the rounded length of the wagon's roof:

$$fP(B, H, HH) := 4 \cdot \left[\left(\frac{B}{2}\right)^x + (HH - H)^x\right]^{\frac{1}{x}}$$

Average values for twice the average rounded length of the wagon's roof on the exterior, Pe, and the interior, Pi, in m:

$$mPe := fP(mBe, mHe, mHHe) = 6.117$$

$$mPi := fP(mBi, mHi, mHHi) = 5.211$$

Combined standard uncertainty for twice the rounded length of the wagon's roof on the exterior, uC_Pe, and the interior, uC_Pi, in m:

$$uC\_Pe := \sqrt{\left(uC\_Be \frac{d}{dmBe} fP(mBe, mHe, mHHe)\right)^2 + \left(uC\_He \frac{d}{dmHe} fP(mBe, mHe, mHHe)\right)^2 \ldots + \left(uC\_HHe \frac{d}{dmHHe} fP(mBe, mHe, mHHe)\right)^2 + \frac{\left(\frac{0.3619}{100} \cdot mPe\right)^2}{3}} = 0.0128$$

$$uC\_Pi := \sqrt{\left(uC\_Bi \frac{d}{dmBi} fP(mBi, mHi, mHHi)\right)^2 + \left(uC\_Hi \frac{d}{dmHi} fP(mBi, mHi, mHHi)\right)^2 \ldots + \left(uC\_HHi \frac{d}{dmHHi} fP(mBi, mHi, mHHi)\right)^2 + \frac{\left(\frac{0.3619}{100} \cdot mPi\right)^2}{3}} = 0.0157$$

Definition of the average area of the estimated heat transfer surface of the wagon body:

Function for calculating the wagon body's surface area:

$$fS'(L, B, H, HH, P) := L \cdot B + 2 \cdot (L + B) \cdot H + L \frac{P}{2} + \pi \frac{B}{2} \cdot (HH - H)$$

Function for calculating the average surface area of heat transfer surface of the wagon body:

$$fS(Le, Be, He, HHe, Pe, Li, Bi, Hi, HHi, Pi) := \sqrt{fS'(Le, Be, He, HHe, Pe) \cdot fS'(Li, Bi, Hi, HHi, Pi)}$$

Value of the average wagon body surface, in m^2:

$$mS := fS(mLe, mBe, mHe, mHHe, mPe, mLi, mBi, mHi, mHHi, mPi) = 186{,}953$$

Combined standard uncertainty of the average area of heat transfer surface of the wagon body, in m^2:

$$\begin{aligned} uC\_S := \Bigg[& \left(uC\_Le \frac{d}{dmLe} fS(mLe, mBe, mHe, mHHe, mPe, mLi, mBi, mHi, mHHi, mPi) \right)^2 \dots \\ & + \left(uC\_Be \frac{d}{dmBe} fS(mLe, mBe, mHe, mHHe, mPe, mLi, mBi, mHi, mHHi, mPi) \right)^2 \dots \\ & + \left(uC\_He \frac{d}{dmHe} fS(mLe, mBe, mHe, mHHe, mPe, mLi, mBi, mHi, mHHi, mPi) \right)^2 \dots \\ & + \left(uC\_HHe \frac{d}{dmHHe} fS(mLe, mBe, mHe, mHHe, mPe, mLi, mBi, mHi, mHHi, mPi) \right)^2 \dots \\ & + \left(uC\_Li \frac{d}{dmLi} fS(mLe, mBe, mHe, mHHe, mPe, mLi, mBi, mHi, mHHi, mPi) \right)^2 \dots \\ & + \left(uC\_Bi \frac{d}{dmBi} fS(mLe, mBe, mHe, mHHe, mPe, mLi, mBi, mHi, mHHi, mPi) \right)^2 \dots \\ & + \left(uC\_Hi \frac{d}{dmHi} fS(mLe, mBe, mHe, mHHe, mPe, mLi, mBi, mHi, mHHi, mPi) \right)^2 \dots \\ & + \left(uC\_HHi \frac{d}{dmHHi} fS(mLe, mBe, mHe, mHHe, mPe, mLi, mBi, mHi, mHHi, mPi) \right)^2 \dots \\ & + \left(uC\_Pe \frac{d}{dmPe} fS(mLe, mBe, mHe, mHHe, mPe, mLi, mBi, mHi, mHHi, mPi) \right)^2 \dots \\ & + \left(uC\_Pi \frac{d}{dmPi} fS(mLe, mBe, mHe, mHHe, mPe, mLi, mBi, mHi, mHHi, mPi) \right)^2 \Bigg] \end{aligned} = 0.118$$

3 Calculation of average heat output and its combined standard uncertainty

Average value of heat output, in W:

$$mW := mean(WD) = 1762$$

Standard uncertainty of measurement of heat output, in W, calculated by type A evaluation:

$$uA\_W := \sqrt{\frac{\sum (WD - mW)^2}{rows(WD) \cdot (rows(WD) - 1)}} = 3.5$$

Standard uncertainty of measurement of heat output, in W, calculated by type B evaluation:

$$uB\_W := \frac{\frac{\delta\_Q}{100} \cdot mean(WD)}{\sqrt{3}} = 10.2$$

Combined standard uncertainty of heat output measurement, in W:

$$uC\_W := \sqrt{uA\_W^2 + uB\_W^2} = 10.8$$

Note – The uncertainty of the electrical power losses in the wires is disregarded because it has too little influence on the final result in comparison with the rest of the uncertainties under consideration during the measurement of the K coefficient.

4 Calculation of the average internal temperature of the wagon body and its combined standard uncertainty

Average values of internal temperatures of the wagon body, in °C:

$$mTiD := \left| \begin{array}{l} \text{for } i \in 0..\text{rows}(TiD) - 1 \\ \quad mTiD_i \leftarrow \text{mean}\left[\left(TiD^T\right)^{\langle i \rangle}\right] \\ \text{return } mTiD \end{array} \right. \qquad mTiD = \begin{array}{|r|} \hline 33.4 \\ 33.4 \\ 33.4 \\ 33.4 \\ 33.4 \\ 33.4 \\ \ldots \\ \hline \end{array}$$

The average value of the internal temperature of the wagon body, in °C within the calculated interval:

$$mTi := \text{mean}\,(mTiD) = 33.5$$

Standard uncertainty of measurement of the internal temperature of the wagon body with one measurement, K, with type A evaluation:

$$uA1\_Ti := \left| \begin{array}{l} \text{for } i \in 0..\text{rows}(TiD) - 1 \\ \quad uA1\_Ti_i \leftarrow \sqrt{\dfrac{\sum\left[\left(TiD^T\right)^{\langle i \rangle} - mTiD_i\right]^2}{\text{cols}(TiD)\cdot(\text{cols}(TiD) - 1)}} \\ \text{return } \max(uA1\_Ti) \end{array} \right. \qquad uA1\_Ti = 0.16$$

Standard uncertainty of measurement of the internal temperature of the wagon body (in a series of measurements), K, with type A evaluation:

$$uA2\_Ti := \sqrt{\frac{\sum (mTiD - mTi)^2}{\text{rows}(TiD)\cdot(\text{rows}(TiD) - 1)}} = 0.01$$

Standard uncertainty of measurement of the internal temperature of the wagon body, K, with type B evaluation:

$$uB\_Ti := \frac{\Delta\_Ti}{\sqrt{3}} = 0.06$$

Combined standard uncertainty of measurement of the internal temperature of the wagon body, in K:

$$uC\_Ti := \sqrt{uA1\_Ti^2 + uA2\_Ti^2 + uB\_Ti} = 0.29$$

5 Calculation of the average external temperature of the wagon body and its combined standard uncertainty

Average value of external temperatures of the wagon body, in °C:

$$mTeD := \left| \begin{array}{l} \text{for } i \in 0..\text{rows}(TeD) - 1 \\ \quad mTeD_i \leftarrow \text{mean}\left[\left(TeD^T\right)^{\langle i \rangle}\right] \\ \text{return } mTeD \end{array} \right. \qquad mTeD = \begin{array}{|r|} \hline 7.1 \\ 7.1 \\ 7.2 \\ 7.1 \\ 7.1 \\ 7.1 \\ \ldots \\ \hline \end{array}$$

The average value of the external temperature of the wagon body, in °C, within the calculated interval:

$$mTe := mean(mTeD) = 6.9$$

Standard uncertainty of measurement of the external temperature of the wagon body with one measurement, K, with type A evaluation:

$$uA1\_Te := \left| \begin{array}{l} \text{for } i \in 0..rows(TeD) - 1 \\ \quad uA1\_Te_i \leftarrow \sqrt{\dfrac{\sum\left[\left(TeD^T\right)^{\langle i \rangle} - mTeD_i\right]^2}{cols(TeD) \cdot (cols(TeD) - 1)}} \\ \text{return } max(uA1\_Te) \end{array} \right. \qquad uA1\_Te = 0.12$$

Standard uncertainty of measurement of the average external temperature of the wagon body (in a series of measurements), K, with type A evaluation:

$$uA2\_Te := \sqrt{\frac{\sum(mTeD - mTe)^2}{rows(TeD) \cdot (rows(TeD) - 1)}} = 0.02$$

Standard uncertainty of measurement of the external temperature of the wagon body, K, with type B evaluation:

$$uB\_Te := \frac{\Delta\_Te}{\sqrt{3}} = 0.06$$

Combined standard uncertainty of measurement of the external temperature of the wagon body, in K:

$$uC\_Te := \sqrt{uA1\_Te^2 + uA2\_Te^2 + uB\_Te} = 0.27$$

6 Evaluation of correlations

Analysis of test schemes for the measurement of the K coefficient makes it possible to conclude that there is a correlation (in time) of the following series of measurements:

(a) Average values of external and internal temperatures of the wagon body;

(b) Values of the heat output and average values of the internal temperature of the wagon body.

Estimated coefficient of the correlation of average external and internal temperatures of the wagon body:

$$r\_Te\_Ti := \left| \begin{array}{l} r\_Te\_Ti_0 \leftarrow corr(mTeD, mTiD) \\ \text{for } i \in 1..rows(mTiD) - 1 \\ \quad r\_Te\_Ti_i \leftarrow corr(mTeD, stack(submatrix(mTiD, i, rows(mTiD) - 1, 0, 0), submatrix(mTiD, 0, i - 1, 0, 0))) \\ \text{return } max(r\_Te\_Ti) \end{array} \right.$$

$$r\_Te\_Ti = 0.860$$

Estimated coefficient of the correlation of heat output and average internal temperature of the wagon body:

$$\begin{aligned}&\text{r\_W\_Ti} := \left|\begin{array}{l}\text{r\_W\_Ti}_0 \leftarrow \text{corr}(\text{WD}, \text{mTiD})\\ \text{for } \ i \in 1 .. \text{rows}(\text{mTiD}) - 1\\ \quad \text{r\_W\_Ti}_i \leftarrow \text{corr}(\text{WD}, \text{stack}(\text{submatrix}(\text{mTiD}, i, \text{rows}(\text{mTiD}) - 1, 0, 0), \text{submatrix}(\text{mTiD}, 0, i - 1, 0, 0)))\\ \text{return } \ \max(\text{r\_W\_Ti})\end{array}\right.\\ &\text{r\_W\_Ti} = 0.726\end{aligned}$$

7 Calculation of average K coefficient and its combined standard uncertainty

Function for calculating the K coefficient:

$$\text{fK}(W, Ti, Te, S) := \frac{W}{S \cdot (Ti - Te)}$$

Average value of the K coefficient, in W/(m^{2}K):

$$\text{mK} := \text{fK}(\text{mW}, \text{mTi}, \text{mTe}, \text{mS}) = 0.35$$

Combined standard uncertainty of measurement of the K coefficient, W/(m^{2}K):

$$\begin{aligned}\text{uC\_K} := \ & \left[\left(\text{uC\_W} \cdot \frac{d}{d\text{mW}} \text{fK}(\text{mW}, \text{mTi}, \text{mTe}, \text{mS})\right)^2 + \left(\text{uC\_Ti} \cdot \frac{d}{d\text{mTi}} \text{fK}(\text{mW}, \text{mTi}, \text{mTe}, \text{mS})\right)^2 \ldots \right.\\ & + \left(\text{uC\_Te} \cdot \frac{d}{d\text{mTe}} \text{fK}(\text{mW}, \text{mTi}, \text{mTe}, \text{mS})\right)^2 + \left(\text{uC\_S} \cdot \frac{d}{d\text{mS}} \text{fK}(\text{mW}, \text{mTi}, \text{mTe}, \text{mS})\right)^2 \ldots\\ & \left. + 2 \cdot \left(\left|\frac{d}{d\text{mTe}} \text{fK}(\text{mW}, \text{mTi}, \text{mTe}, \text{mS})\right| \cdot \left|\frac{d}{d\text{mTi}} \text{fK}(\text{mW}, \text{mTi}, \text{mTe}, \text{mS})\right| \cdot \text{uC\_Te} \cdot \text{uC\_Ti} \cdot \text{r\_Te\_Ti} \ldots \right.\right.\\ & \left.\left. + \left|\frac{d}{d\text{mW}} \text{fK}(\text{mW}, \text{mTi}, \text{mTe}, \text{mS})\right| \cdot \left|\frac{d}{d\text{mTi}} \text{fK}(\text{mW}, \text{mTi}, \text{mTe}, \text{mS})\right| \cdot \text{uC\_W} \cdot \text{uC\_Ti} \cdot \text{r\_W\_Ti}\right)\right]\end{aligned} = 0.008$$

8 Calculation of the expanded uncertainty of measurement of the K coefficient

$$\text{U\_K} := \text{uC\_K} \cdot k = 0.017 \qquad \text{or as a percentage:} \qquad \frac{\text{U\_K}}{\text{mK}} \cdot 100 = 4.7$$