United Nations ECE/TRANS/WP.11/2016/6

Economic and Social Council

Distr.: General
19 July 2016
English
Original: Russian

Economic Commission for Europe

Inland Transport Committee

Working Party on the Transport of Perishable Foodstuffs

Seventy-second session
Geneva, 4-7 October 2016
Item 6 of the provisional agenda
ATP Handbook

Introduction in the ATP Handbook of definitions to determine the inner and outer heat transfer surface areas of rail wagons other than tank wagons

Transmitted by the Government of the Russian Federation

Summary

Executive summary:	According to ATP, Annex l, Appendix 2, sub-section 2.3.2, the inside and outside surface area of a body of special equipment is determined taking into account its structural peculiarities. However, this ATP provision does not take into account the known peculiarities of the bodies of railway wagons that are not tank wagons and that are used for the transport of perishable foodstuffs. As a result, experts and ATP testing stations may use methods that are unacceptable to other parties to ATP.
Action to be taken:	Using the proposal made by the United Kingdom on methods for measuring the external surface areas of walls of panel van cargo compartments adopted at the seventy-first session of the Working Party (WP.11), introduce provisions in the ATP Handbook for calculating the mean surface area of a wagon body, together with the necessary illustrations.
Related documents:	ECE/TRANS/WP.11/2015/2.

GE.16-12446 (E) 181816 191816

* 1 6 1 2 4 4 6 *

Introduction

1. The bodies, shapes and structural peculiarities of most special equipment can be reduced to a few common types. The United Kingdom has done an analysis of the known body structures of panel van cargo compartments, as a result of which schemes and formulae were proposed for determining the internal and external surface areas of bodies of special equipment on the basis of known mathematical methods. The results of this work are set out in document ECE/TRANS/WP.11/2015/2, adopted by the Working Party at its seventy-first session.

2. According to article 3 of ATP, ATP standards and requirements apply to the carriage of perishable foodstuffs by both road and rail. The test and verification methods and requirements for bodies and thermal appliances of special equipment are the same for road and rail.

3. During the discussion of the proposals by the United Kingdom at the seventieth session of WP.11, the Russian experts expressed the view that the same methods could and should be used to determine the mean surface areas of bodies of railway special equipment. This is especially true of method C, which it would be appropriate to use in particularly difficult situations, when there is not enough information on the design of the body to be able to calculate its mean surface area and a successive iteration method is therefore used.

4. At the seventy-first session of WP.11, the Russian experts drafted proposals for railway wagons, including those with rounded roofs, using the methods proposed by the United Kingdom. However, for technical reasons, an official translation of the proposal by the Russian Federation was not done in English and French within the deadline, which meant that the Working Group could not give the proposal full consideration and put it to the vote at the Working Party's seventy-first session.

5. In view of the above, the Russian Federation is submitting, as an official document, updated proposals for the inclusion in ATP and in the ATP Handbook of the corresponding additions to the proposal made by the United Kingdom, for railway wagons. These proposals are based on the Russian version of the ATP Handbook in use at the time that this official document was drafted, as amended at 30 September 2015, together with the proposal by the United Kingdom adopted at the seventy-first session, contained in document ECE/TRANS/WP.11/2015/2.

Proposals

6. Add the following text at the end of ATP, Annex 1, Appendix 2, sub-section 1.2, as adopted at the seventy-first session of WP.11:

"The above methods shall also apply for calculating the mean surface area of the body of railway wagons other than tank wagons, including those with a rounded roof. In this case the calculations according to the schemes and formulae given below shall be used:

$$S_i = LI \cdot WI + 2 \cdot (LI + WI) \cdot Wi + LI \cdot \frac{PI}{2} + \pi \cdot \frac{WI}{2} \cdot (HI - Wi)$$

$$S_e = LE \cdot WE + 2 \cdot (LE + WE) \cdot We + LE \cdot \frac{PE}{2} + \pi \cdot \frac{WE}{2} \cdot (HE - We)$$

$$PI = 4 \cdot \left(\left(\frac{WI}{2}\right)^x + (HI - Wi)^x\right)^{\frac{1}{x}}$$

$$PE = 4 \cdot \left(\left(\frac{WE}{2}\right)^x + (HE - We)^x\right)^{\frac{1}{x}}$$

$$x = \frac{\ln 2}{\ln \frac{\pi}{2}}$$

Where:

HI is the weighted mean inner height of the body on a central axis X, in m;

$\frac{PI}{2}$ is the length of the inner arc of the rounded roof, in m;

HE is the mean outer height of the body on a central axis, in m;

$\frac{PE}{2}$ is the length of the inner arc of the rounded roof, in m;

$\pi \approx 3.14159$, is the number Pi.

The maximum relative error in determining PI and PE in this way shall not exceed 0.3619% (the error is always positive)".

7. Insert the following drawing in the ATP Handbook:

"Figure 6: Estimated values for a body with a rounded roof

Sample calculations

8. Sample Mathcad calculations are given in annexes A and B to this document.

Justification

9. Defining the methods for determining the mean surface area of the body of special equipment, including its inner and outer surface areas, is an important task for ensuring a common understanding by all Contracting Parties, experts and testing stations of the ATP standards and requirements. The use of common, understandable and available methods for calculating the mean surface area of bodies of special equipment will raise the level of mutual trust in ATP certificates and have a positive impact on how the entire system of checking and certifying special equipment functions in general.

10. In practice, the bodies of different types of special equipment may vary considerably in shape and have additional structural elements as well as hidden cavities that must be taken into account during tests and expert checks. If there is an opportunity to take all these structural elements and hidden cavities fully into account, it would be advisable to use method A, proposed by the United Kingdom.

11. In many cases, when special equipment is presented for tests or expert checks, for various reasons the exact design of the body may be difficult to determine or the methods used to calculate mean surface area may be excessively complicated. However, for all known special equipment designs, it is not difficult to measure the inside surfaces of walls, floors and roofs, even for those with complex shapes (examples are given in the proposal by the United Kingdom, document ECE/TRANS/WP.11/2015/2, and can essentially apply to any body design).

If the vehicle manufacturer indicates in the technical documentation the mean thickness of the insulation, it is advisable to use method B, in which the external dimensions of the body of the special equipment are determined in the same way as the internal dimensions, taking into account the mean thickness of the insulation. When this parameter is unknown, the most universal method — method C — should be used.

12. When the experts of the Russian Federation drew up the proposals for the use for railway wagon bodies of the calculation methods proposed by the United Kingdom, the problem arose of how to apply such methods in practice, bearing in mind the design peculiarities of wagon bodies, in particular their roofs.

An analysis of the data contained in a compendium entitled “1520 mm gauge railway cargo wagons”[1] shows that the most common type of design for wagon bodies has vertical walls of equal height, a floor at a right angle and a curved roof. Such wall and floor designs are relatively easy to calculate. When there are more complex shapes, the weighted mean values calculated using the methods proposed by the United Kingdom may be used.

However, such methods cannot be employed when the special equipment has a rounded roof. In such cases, other calculation methods must be used to calculate the roof surface.

13. In cases where the manufacturer provides an exact description that makes it possible to calculate the length of the roof arc, method A should be used to calculate its surface (as all the data are known).

However, such information, even if it is available in the technical documentation, often requires the use of complex mathematics. For the purposes of determining K coefficients with margins of error not exceeding those indicated in ATP, Annex 1, Appendix 2, sub-section 2.3.2, a simpler method may be used, presenting the arc of the rounded roof section as halves of the perimeter of an ellipsis. The perimeter of the ellipsis may be defined by its two axes, corresponding to the width of the body and the height of the rounding of the roof,

[1] Information publication issued by the design and construction bureau of the Russian Railways open (public) corporation.

with a slight margin of error of no more than 0.3619% with an ellipse eccentricity of ~0.979811 (axis ratio ~1/5, corresponding for example with most known wagon designs). It is specifically such a calculation that is being proposed by the experts from the Russian Federation.

The total margin of error for the mean surface area of a wagon roof may be determined using the methods contained in the ATP Handbook (comments to ATP, Annex 1, Appendix 2, sub-section 2.3.2).

14. In the final analysis, it must be verified that calculation method C is applicable for the known body designs of railway wagon other than tank wagons. Taking into consideration the basic objectives of ATP, which are aimed at ensuring the preservation of the quality of perishable foodstuffs during their carriage, it is obvious that the K coefficients calculated according to method C must not be lower than their true values. The calculated mean surface area of a body of special equipment must thus be the minimum which in turn will correspond with the minimum calculated mean thickness of insulation. The insulation thickness in general is dependent on the heat conductivity of the materials used for it (usually, it is of one material, used as the main filler for the insulation space of the special equipment design).

At present, bodies of special equipment are often insulated using polyurethane foam, which is characterized by its low heat conductivity coefficient compared with other known heat insulation materials used for such purposes. Industrially produced polyurethane foams can have heat transfer coefficients as low as 0.019 W/(m·K). However, in real conditions, taking into account temperature, humidity and the preparation and application of the polyurethane on the insulated surface, this value is rarely lower than 0.023-0.025 W/(m·K). During operation, as the polyurethane foam ages and is subject to humidity, its heat transfer coefficient only increases. Therefore, the estimated heat transfer coefficient, 0.025 W/(m·K), proposed by the United Kingdom in document ECE/TRANS/WP.11/2015/2 is acceptable and reasonable for special equipment used on the road and in rail wagons.

15. Taking into account the known formula for theoretically determining heat transfer coefficients (without taking into consideration convection and radiation), the estimated mean insulation thickness, d, can be derived using the following equation:

$$\frac{1}{\frac{1}{\alpha_e}+\frac{d}{\lambda}+\frac{1}{\alpha_i}}=\frac{W}{\Delta T\cdot S}$$

where:

αe, αi are the relevant estimated heat capacity coefficients of the outer and inner surfaces of the body of the special equipment (heat capacity has a minor impact on the K coefficient and can be ignored in practical calculations);

λ = 0.025 W/(m·K) is the estimated heat conductivity coefficient;

S, in this case, should be determined as follows:

$$S=\sqrt{S_i\cdot S_e}=\sqrt{S_i\cdot f(S_i,d)}$$

Such an equation may theoretically be solved for the value d using numerical methods (as illustrated in method D, in annex A), or by a method using iterations (method C). Both methods apparently produce identical results, but the iteration method is significantly more accessible (as it does not require the use of special algorithms or software). At the same time, the required number of iterations is not very great and is generally determined with the required accuracy by the sought quantity (in the examples given in annexes A and B, this accuracy is equal to 0.001 m).

Costs

16. No additional costs are required. Defining the methods for calculating the mean surface area of the body of railway wagons other than tank wagons does not entail the use of additional instruments, complicated mathematical methods or other costly procedures.

Feasibility

17. The proposed changes will create better conditions for achieving the fundamental goals and objectives of ATP without any additional costs or the need for a transitional period, and will also increase the level of mutual trust among the Contracting Parties to ATP.

Enforceability

18. No problems with the tests or expert checks are foreseen.

Annex A

Determination of the mean surface area of a railway wagon body (case study of a new thermos wagon manufactured by the Dessau Plant, Germany)

1. Source data

Inner dimensions of the wagon body, in m:

length, in m: $LI := 20.596$

width, in m: $WI := 2.702$

side wall height, in m: $Wi := 2.550$

height on central longitudinal axis, in m: $HI := 3.195$

Declared mean thickness of insulation, in m, in the following elements of the body:

end walls: $d\_LI := 2 \cdot 0.200$

side walls and side doors: $d\_WI := 2 \cdot 0.194$

floor: $d\_Wi\_dn := 0.185$

roof: $d\_Wi\_up := 0.200$

Comment: The mean insulation thickness is indicated in the technical operating documentation for the TH 4-201-90 type thermos wagon.

Estimated heat transfer coefficient of the body's inner walls, in W/(m^2K): $\alpha i := \infty$

Estimated heat transfer coefficient of the body's outer walls, in W/(m^2K): $\alpha e := \infty$

Comment: The above parameters have an insignificant effect on the results of the calculations and for simplicity are ignored as they have been assigned infinite values.

Values of parameters measured in the steady state:

mean heat power, in W: $W := 1080$

mean temperature difference inside and outside the wagon body, in K: $\Delta T := 25$

Estimated heat transfer coefficient of the body's insulation, in W/(mK): $\lambda := 0.025$

2. Calculation according to method A

Mean surface area of the wagon body, in m^2: $S\_A := 262.5$

Comment: The calculation of the mean surface area of the wagon body is too lengthy to be shown.

Function for calculating the K coefficient: $fK(W, \Delta T, S) := \frac{W}{S \cdot \Delta T}$

Value of the K coefficient, in W/(m^2K): $coefK\_A := fK(W, \Delta T, S\_A) = 0.165$

3. Calculation according to method B

Nominal external dimensions of the wagon body, in m:

length: $LE := LI + d\_LI = 20.996$

width: $WE := WI + d\_WI = 3.090$

height of the side walls: $We := Wi + d\_Wi\_dn = 2.735$

height along the central longitudinal axis: $HE := HI + d\_Wi\_dn + d\_Wi\_up = 3.580$

Empirical parameter for calculation of the length of the roof arc: $x := \frac{\ln(2)}{\ln\left(\frac{\pi}{2}\right)}$

Function for calculation of the length of the roof arc: $fP(B, H, HH) := 4 \cdot \left[\left(\frac{B}{2}\right)^x + (HH - H)^x\right]^{\frac{1}{x}}$

Function for calculation of the wagon body's outer or inner surface area:

$$fS'(L, B, H, HH) := L \cdot B + 2 \cdot (L + B) \cdot H + L \cdot \frac{fP(B, H, HH)}{2} + \pi \cdot \frac{B}{2} \cdot (HH - H)$$

Function for calculation of the mean surface area of the wagon:

$$fS(LE, WE, We, HE, LI, WI, Wi, HI) := \sqrt{fS'(LE, WE, We, HE) \cdot fS'(LI, WI, Wi, HI)}$$

Mean surface area of the wagon body, in m^2: $S\_B := fS(LE, WE, We, HE, LI, WI, Wi, HI) = 261.982$

Value of the K coefficient, in $W/(m^2K)$: $coefK\_B := fK(W, \Delta T, S\_B) = 0.165$

4. Calculation according to method C

$$
\begin{array}{l}
\mathrm{proc}(LI, WI, Wi, HI, W, \Delta T, \lambda, \alpha e, \alpha i) := \\
\quad \left| \begin{array}{l}
prec \leftarrow 0.001 \\
n \leftarrow 0 \\
d_n \leftarrow 0 \\
LE_n \leftarrow LI + 2 \cdot d_n \\
WE_n \leftarrow WI + 2 \cdot d_n \\
We_n \leftarrow Wi + d_n \\
HE_n \leftarrow HI + 2 \cdot d_n \\
S_n \leftarrow fS\left(LE_n, WE_n, We_n, HE_n, LI, WI, Wi, HI\right) \\
coefK\_C_n \leftarrow \dfrac{W}{\Delta T \cdot S_n} \\
\Delta d \leftarrow \infty \\
\text{while } \Delta d > prec \\
\quad \left| \begin{array}{l}
n \leftarrow n + 1 \\
d_n \leftarrow \left(\dfrac{\Delta T \cdot S_{n-1}}{W} - \dfrac{1}{\alpha e} - \dfrac{1}{\alpha i} \right) \cdot \lambda \\
LE_n \leftarrow LI + 2 \cdot d_n \\
WE_n \leftarrow WI + 2 \cdot d_n \\
We_n \leftarrow Wi + d_n \\
HE_n \leftarrow HI + 2 \cdot d_n \\
S_n \leftarrow fS\left(LE_n, WE_n, We_n, HE_n, LI, WI, Wi, HI\right) \\
coefK\_C_n \leftarrow \dfrac{W}{\Delta T \cdot S_n} \\
\Delta d \leftarrow \left| d_n - d_{n-1} \right|
\end{array} \right. \\
\text{return } (d\ \ LE\ \ WE\ \ We\ \ HE\ \ S\ \ coefK\_C)
\end{array} \right.
\end{array}
$$

Comment: List of additional parameters:

$prec = 10^{-3}$ m is the accuracy of the selection of the mean insulation thickness;

n is the index of the iteration, starting with 0 (specific to the array indexing in Mathcad);

d_n is the mean insulation thickness derived in the nth iteration, in m;

LE_n is the estimated outside length of the wagon body derived in the nth iteration, in m;

WE_n is the same, for width, in m;

We_n is the same, for the height of the side wall, in m;

HE_n is the same, for the height along the central longitudinal axis, in m;

S_n is the estimated mean surface area of the wagon body derived in the nth iteration, in m;

$coefK\_C_n$ is the estimated value of the K coefficient derived in the nth iteration using method C, in $W/(m^2K)$;

Δd is the absolute value for the change in the estimated mean insulation thickness, in m ($\Delta d > prec$).

Results of the parameter selection (columns: d | LE | Ru En Fr WE | We | HE | S | coefK_C) in iterations (in rows):

$$\mathrm{proc}(LI, WI, Wi, HI, W, \Delta T, \lambda, \alpha e, \alpha i) = \left[\begin{pmatrix}0.000\\0.141\\0.149\\0.149\end{pmatrix} \begin{pmatrix}20.596\\20.878\\20.894\\20.894\end{pmatrix} \begin{pmatrix}2.702\\2.984\\3.000\\3.000\end{pmatrix} \begin{pmatrix}2.550\\2.691\\2.699\\2.699\end{pmatrix} \begin{pmatrix}3.195\\3.477\\3.493\\3.493\end{pmatrix} \begin{pmatrix}243.940\\257.146\\257.854\\257.892\end{pmatrix} \begin{pmatrix}0.177\\0.168\\0.168\\0.168\end{pmatrix}\right]$$

5. Determining the mean insulation thickness according to method D

$$\frac{1}{\frac{1}{\alpha e} + \frac{d}{\lambda} + \frac{1}{\alpha i}} = \frac{W}{\Delta T \cdot fS(LI + 2\cdot d, WI + 2\cdot d, Wi + d, HI + 2\cdot d, LI, WI, Wi, HI)} \text{ solve } \rightarrow 0.1492441624219862096$$

Annex B

Determining the mean surface area of the body of a railway wagon (taking as an example wagon No. 80007990, done in April 2015)

1. Source data

Inner dimensions of the wagon body, in m:

length: $LI := mean((15.340\ \ 15.340\ \ 15.345\ \ 15.340)) = 15.341$

width: $WI := mean((2.470\ \ 2.470\ \ 2.465\ \ 2.465)) = 2.468$

height of the side wall: $Wi := mean((2.635\ \ 2.635\ \ 2.630\ \ 2.620)) = 2.630$

height along the central longitudinal axis: $HI := mean((2.900\ \ 2.900)) = 2.900$

Comment: The inner dimensions of the wagon body are determined by repeated, uniformly precise measurements.

Declared values of the mean insulation thickness, in m, in the following elements of the body:

end walls: $d\_LI := 2 \cdot 0.150 = 0.300$

side walls: $d\_WI\_wl := 2 \cdot 0.150 = 0.300$

side doors: $d\_WI\_dr := 2 \cdot 0.100 = 0.200$

floor: $d\_Wi\_dn := 0.10($

roof: $d\_Wi\_up := 0.15($

Dimensions of the cargo door opening, in m:

width: $b := 2.15($

height: $h := 2.09($

Comment: The mean insulation thickness and the dimensions of the cargo door opening are indicated in technical specification TU 3182601-427-01055865-08

Estimated mean insulation thickness, in m, in the side walls and side doors:

$$d\_WI := \frac{d\_WI\_dr(b \cdot h) + d\_WI\_wl(LI \cdot Wi - b \cdot h)}{LI \cdot Wi} = 0.289$$

Estimated heat transfer coefficient of the wagon’s inner walls, in W/(m²K): $\alpha i := \infty$

Estimated heat transfer coefficient of the wagon’s outer walls, W/(m²K): $\alpha e := \infty$

Comment: The above parameters have an insignificant effect on the results of the calculations and for simplicity are ignored as they have been assigned infinite values.

Values of the measured parameters in the steady state:

mean heat power, in W: $W := 1627$

mean temperature difference inside and outside the wagon body, K: $\Delta T := 25.4$

estimated heat transfer coefficient of the body's insulation, W/(mK): $\lambda := 0.025$

function for calculation of the K coefficient: $fK(W, \Delta T, S) := \frac{W}{S \cdot \Delta T}$

2. Calculation according to method A

Comment: The calculation cannot be carried out owing to insufficient data on the internal structure of the thermal insulation and the design particularities of the body.

3. Calculation according to method B

Nominal external dimensions of the wagon body, in m:

length: $LE := LI + d\_LI = 15.641$

width: $WE := WI + d\_WI = 2.756$

height of side walls: $We := Wi + d\_Wi\_dn = 2.730$

height along the central longitudinal axis: $HE := HI + d\_Wi\_dn + d\_Wi\_up = 3.150$

Empirical parameter for calculation of the length of the roof arc: $x := \frac{\ln(2)}{\ln\left(\frac{\pi}{2}\right)}$

Function for calculation of the length of the roof arc: $fP(B, H, HH) := 4 \cdot \left[\left(\frac{B}{2}\right)^x + (HH - H)^x\right]^{\frac{1}{x}}$

Function for calculation of the wagon body's surface area:

$$fS'(L, B, H, HH) := L \cdot B + 2 \cdot (L + B) \cdot H + L \cdot \frac{fP(B, H, HH)}{2} + \pi \cdot \frac{B}{2} \cdot (HH - H)$$

Function for calculation of the wagon body's mean surface area:

$$fS(LE, WE, We, HE, LI, WI, Wi, HI) := \sqrt{fS'(LE, WE, We, HE) \cdot fS'(LI, WI, Wi, HI)}$$

Area of the wagon body's mean surface, in m^2:
$S\_B := fS(LE, WE, We, HE, LI, WI, Wi, HI) = 182.570$

K coefficient, in $W/(m^2K)$: $coefK\_B := fK(W, \Delta T, S\_B) = 0.351$

4. Calculation according to method C

$$
\begin{array}{l|l}
pro(LI, WI, Wi, HI, W, \Delta T, \lambda, \alpha e, \alpha i) := & prec \leftarrow 0.001 \\
& n \leftarrow 0 \\
& d_n \leftarrow 0 \\
& LE_n \leftarrow LI + 2 \cdot d_n \\
& WE_n \leftarrow WI + 2 \cdot d_n \\
& We_n \leftarrow Wi + d_n \\
& HE_n \leftarrow HI + 2 \cdot d_n \\
& S_n \leftarrow fS(LE_n, WE_n, We_n, HE_n, LI, WI, Wi, HI) \\
& coefK\_C_n \leftarrow \dfrac{W}{\Delta T \cdot S_n} \\
& \Delta d \leftarrow \infty \\
& \text{while } \Delta d > prec \\
& \quad \begin{array}{|l}
n \leftarrow n + 1 \\
d_n \leftarrow \left(\dfrac{\Delta T \cdot S_{n-1}}{W} - \dfrac{1}{\alpha e} - \dfrac{1}{\alpha i} \right) \cdot \lambda \\
LE_n \leftarrow LI + 2 \cdot d_n \\
WE_n \leftarrow WI + 2 \cdot d_n \\
We_n \leftarrow Wi + d_n \\
HE_n \leftarrow HI + 2 \cdot d_n \\
S_n \leftarrow fS(LE_n, WE_n, We_n, HE_n, LI, WI, Wi, HI) \\
coefK\_C_n \leftarrow \dfrac{W}{\Delta T \cdot S_n} \\
\Delta d \leftarrow \left| d_n - d_{n-1} \right|
\end{array} \\
& \text{return } (d \;\; LE \;\; WE \;\; We \;\; HE \;\; S \;\; coefK\_C)
\end{array}
$$

Comment: List of additional parameters:

prec=10^{-3}m is the accuracy of the selection of the mean insulation thickness;

n is the number of the iteration, beginning with 0 (specific to the array indexing in Mathcad);

d_n is the mean insulation thickness derived in the nth iteration, in m;

LE_n is the estimated outside length of the wagon body derived in the nth iteration, in m;

WE_n is the same, for width, in m;

We_n is the same, for the height of the side wall, in m;

HE_n is the same, for the height along the central longitudinal axis, in m;

S_n is the estimated mean surface area of the wagon body derived in the nth iteration, in m;

$coefK\_C_n$ is the estimated value of the K coefficient derived in the nth iteration using method C, in W/(m^2K);

Δd is the absolute value for the change in the estimated mean insulation thickness, in m ($\Delta d > prec$).

Results of the parameter selection (columns: d | LE | WE | We | HE | S | coefK_C) in iterations (in rows):

$$\mathrm{proc}(LI, WI, Wi, HI, W, \Delta T, \lambda, \alpha e, \alpha i) = \left[\begin{pmatrix} 0.000 \\ 0.067 \\ 0.069 \\ 0.069 \end{pmatrix} \begin{pmatrix} 15.341 \\ 15.476 \\ 15.480 \\ 15.480 \end{pmatrix} \begin{pmatrix} 2.468 \\ 2.602 \\ 2.606 \\ 2.606 \end{pmatrix} \begin{pmatrix} 2.630 \\ 2.697 \\ 2.699 \\ 2.699 \end{pmatrix} \begin{pmatrix} 2.900 \\ 3.035 \\ 3.039 \\ 3.039 \end{pmatrix} \begin{pmatrix} 172.785 \\ 177.672 \\ 177.810 \\ 177.814 \end{pmatrix} \begin{pmatrix} 0.371 \\ 0.361 \\ 0.360 \\ 0.360 \end{pmatrix} \right]$$